Hadron Structure and Weak Interactions in a Gauge Theory*

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We construct an anomaly-free $U(1)\otimes SU(2)$ gauge model of weak and electromagnetic interactions based on the three-triplet model of hadron structure. The model has a large number of interesting experimental consequences and requires a relatively low value for the mass of charged intermediate vector bosons; while the lower bound is 13 GeV, 18 GeV is the preferred value.

Recently, many attempts[1] have been made to synthesize weak and electromagnetic interactions within the framework of a Higgs-Kibble–type theory, along the lines first suggested by Weinberg and Salam. It is evident from an examination of hitherto published models that if one gives oneself the freedom to introduce arbitrary numbers of massive leptons and to postulate a variety of quarks without worrying about their role in hadron dynamics, one can construct an almost infinite number of theories. In the present note we reconsider Weinberg's original scheme,[2] based on the group $U(1)\otimes SU(2)$, with the constraint that incorporation of hadrons into the scheme be carried out within the framework of an acceptible model of hadron structure. (We define a hadron model to be acceptable if it accounts for the low-lying hadron spectrum, predicts the correct amplitude for $\pi^0\to 2\gamma$, and pays whatever respect is due to the semiempirical quark-model relations between masses, cross sections, etc.) The only acceptable models known to us are the three-triplet model[3] with fractional charges [the so-called red, white, and blue (RWB) model] and the Han-Nambu model,[4] both based on the group $SU(3)\otimes SU(3)'$. For the sake of definiteness we will write our formulas in a form appropriate to the RWB model. The reader can verify that most of our conclusions hold for both models.

All currently known hadrons are presumed to be $SU(3)'$ singlets to a high degree of accuracy, say one part in 10^3. (The ground-state wave function of low-lying baryons can therefore be endowed with the correct symmetry without running afoul of the spin-statistics theorem.)

Our model has the following features:

(i) The lower limit on m_W, the mass of charged intermediate bosons, is $m_0\cong 13$ GeV. On the other hand, there is *no* lower limit for m_Z, the mass of the neutral intermediate boson.

(ii) The processes $\nu_\mu+p\to\nu_\mu+p$ and $\nu_\mu+e^-\to\nu_\mu+e^-$ are scaled by a *common* factor and vanish exactly for $m_W=m_0\sqrt{2}\cong 18$ GeV. Furthermore, a knowledge of experimental upper bounds on the cross sections for these processes suffices to bound m_W *both above and below*:

$$\frac{2m_0^2}{1+|R|}\lesssim m_W^2\lesssim\frac{2m_0^2}{1-|R|}\quad(|R|<1),\tag{1}$$

where

$$R^2=\frac{\sigma(\nu_\mu e^-\to\nu_\mu e^-)}{\sigma_{FG}(\bar\nu_e e^-\to\bar\nu_e e^-)}\quad(E_\nu\ll 13\text{ GeV})$$

$$=2\frac{d\sigma(\nu_\mu p\to\nu_\mu p)/dt}{d\sigma_{FG}(\bar\nu_e p\to e^+n)/dt}\quad(E_\nu\gg 1\text{ GeV}).\tag{2}$$

Here the subscript FG implies cross sections in the Feynman–Gell-Mann theory and $R\equiv(1-2\times\sin^2\xi)(m_W^2/m_Z^2\cos^2\xi)$, ξ being the Weinberg angle defined below. Also E_ν is the neutrino energy and t is the momentum transfer squared.

On the basis of the Gargamelle data[5] on $\nu_\mu+e^-\to\nu_\mu+e^-$, our theory predicts that

$$14.3\text{ GeV}<m_W<31.7\text{ GeV},$$

and hence that W bosons may be observable at Cern's Intersecting Storage Rings, if not at the National Accelerator Laboratory.

(iii) For the process $\bar\nu_e+e^-\to\bar\nu_e+e^-$ our theory predicts that

$$\sigma(\bar\nu_e e^-\to\bar\nu_e e^-)/\sigma_{FG}=1-|R|\epsilon(m_W^2-2m_0^2)+R^2,\tag{3}$$

ϵ being the sign function $[\epsilon(x)\equiv x/|x|]$.

(iv) Adler's neutrino sum rule[6] may be modified by a factor of 18 at large energies. (On the other hand, the Adler-Weisberger sum rule remains valid.)

(v) The right-hand side of the Gross–Llewellyn Smith[6] sum rule vanishes, just as if one had used field algebra instead of the quark model. Other neutrino sum rules are also affected.

(vi) The lepton spectrum includes two neutral heavy leptons.

(vii) The model is free from anomalies.

(viii) The amplitudes for neutral $\Delta S=1$ decay such as $K_L \to \bar{\mu}\mu$ and nonleptonic $\Delta S=2$ transitions are of order $G_F\alpha\delta$, where δ is a measure of SU(3)′ breaking.

We label the triplets as $\mathcal{P}_i$, $\mathcal{N}_i$, λ_i $(i=1,2,3)$. For any fixed i we have a three-spinor of SU(3), for any fixed isospin and hypercharge we have a three-spinor of SU(3)′. The charge-raising isospin current, the hypercharge current, and the electromagnetic current in this model are

$$J_\mu^{(+)}=\sum_{i=1}^{3}\bar{\mathcal{P}}_i\gamma_\mu\mathcal{N}_i, \tag{4}$$

$$J_\mu^{(Y)}=\sum_{i=1}^{3}\tfrac{1}{3}(\bar{\mathcal{P}}_i\gamma_\mu\mathcal{P}_i+\bar{\mathcal{N}}_i\gamma_\mu\mathcal{N}_i-2\bar{\lambda}_i\gamma_\mu\lambda_i), \tag{5}$$

$$J_\mu^{\mathrm{EM}}=\sum_{i=1}^{3}\tfrac{1}{3}(2\bar{\mathcal{P}}_i\gamma_\mu\mathcal{P}_i-\bar{\mathcal{N}}_i\gamma_\mu\mathcal{N}_i-\bar{\lambda}_i\gamma_\mu\lambda_i). \tag{6}$$

It is clearly necessary to define these currents as SU(3)′ singlets; otherwise the charges corresponding to these currents would not represent the total isospin, hypercharge, and electric charge, respectively. Generalizing to the level of chiral SU(3) ⊗ SU(3), we shall require that the currents and charges which occur in the Gell-Mann algebra be defined as SU(3)′ singlets.

The covariant derivative appropriate to the group U(1) ⊗ SU(2) is $\partial_\mu-ig\vec{W}_\mu\cdot\vec{T}-\frac{1}{2}ig'B_\mu Y$, where $(\vec{T},\frac{1}{2}Y)$ are the four matrix generators of the group. $T_3+\frac{1}{2}Y=Q$ is the electric charge. It is convenient to introduce Weinberg's angle[2] by the expression $\tan\xi=g'/g$; the neutral massive field will then be $Z_\mu=\cos\xi W_{3\mu}-\sin\xi B_\mu$.

We assign the quarks to the representations of U(1) ⊗ SU(2) as follows: four doublets,

$$(\mathcal{P}_1,\mathcal{N}_1(\theta))_L,\quad(\mathcal{P}_2,\lambda_1(\theta))_L,$$

$$(\mathcal{P}_3,\mathcal{N}_2)_R,\quad(\mathcal{P}_2,\lambda_3)_R;$$

and ten singlets,

$$\mathcal{P}_{1R},\mathcal{N}_{1R},\lambda_{1R},\mathcal{N}_{2L},\lambda_{2L},\lambda_{2R},\mathcal{P}_{3L},\mathcal{N}_{3L},\mathcal{N}_{3R},\lambda_{3L}.$$

Here we have used the conventional definition $\mathcal{N}_1(\theta)=\mathcal{N}_1\cos\theta+\lambda_1\sin\theta$ and $\lambda_1(\theta)=-\mathcal{N}_1\sin\theta+\lambda_1\times\cos\theta$, with θ the Cabibbo angle.

The leptons ν_e, e^-, ν_μ, and μ^-, together with two heavy neutral leptons E^0 and M^0, are assigned as follows: doublets, $(\frac{1}{3}\nu e\pm\frac{1}{3}\sqrt{8}E^0,e^-)_L$, $(E^0,e^-)_R$; singlet, $(\mp\frac{1}{3}\sqrt{8}\nu e+\frac{1}{3}E^0)_L$; and similarly for the muon system with $\nu e\to\nu_\mu$, $e^-\to\mu^-$, $E^0\to M^0$. The mixing introduced in the left-handed doublet is necessary for universality between β decay and μ decay since the matrix element of the weak current between currently known hadrons H and H' is

$$\langle H'|\mathcal{P}_{1L}\gamma_\mu\mathcal{N}_{1L}(\theta)|H\rangle=\tfrac{1}{3}\sum_{i=1}^{3}\langle H'|\bar{\mathcal{P}}_{iL}\gamma_\mu\mathcal{N}_{iL}(\theta)|H\rangle. \tag{7}$$

In order for μ decay to have the observed magnitude, g must be related to the Fermi constant by $(8/\sqrt{2})G_F=g^2/9m_W^2$. This differs from the usual expression by a factor of $\frac{1}{9}$. On the other hand, the photon coupling is $e=g\sin\xi$. Hence $m_W>13$ GeV. With our assignment it is necessary to have a complex doublet and a real spin-zero triplet in order to give mass to the quarks and the leptons via the Higgs mechanism. The presence of the scalar triplet removes the lower bound to m_Z which now satisfies an upper bound

$$m_Z^2\cos^2\xi<m_W^2. \tag{8}$$

The current which couples to Z_ρ, when evaluated between the familiar hadrons and leptons, takes the form

$$J_Z^{\ \rho}=(g/2\cos\xi)[\tfrac{1}{9}(\bar{\nu}_e\gamma^\rho\nu_e+\bar{\nu}_\mu\gamma^\rho\nu_\mu)+J_{\mathrm{EM}}^{\ \rho}(1-2\sin^2\xi)], \tag{9}$$

which leads to features (ii) and (iii). Note that the special value of $\xi=45°$ or $R=0$ is consistent with present experimental indications. Contrast this with the situation with Weinberg's model.[7] We remark that all this is made possible by the presence of right-handed doublets as well as left-handed doublets, which also ensures the absence of anomalies.

Next we consider current-algebra sum rules in our theory. Since the photon is an SU(3)′ singlet (this is one feature of the RWB model that does not[8] carry over to the Han-Nambu model), it cannot excite a nucleon into a state which lies outside the manifold of low-lying states; all Compton and electroproduction sum rules[9] are therefore unaffected. Similarly, since the pion is an SU(3)′ singlet, the Adler-Weisberger sum rule is unaffected. [The quantity g_A in this sum rule can be identified with the ratio of coupling constants in β decay in the SU(3)′ limit.]

The situation changes dramatically, however, when we come to sum rules which are intrinsically weak-interaction sum rules in the sense that their derivation is based upon some model for the commutator $[J_\mu^{(+)\mathrm{wk}}(x),J_\nu^{(-)\mathrm{wk}}(0)]\,\delta(x_0)$. Most interesting of these is the Adler sum rule for neutrino-induced processes:

$$\lim_{E_\nu \to \infty} \sum_{H_1, H_2} \left(\frac{m_W^2 - q^2}{m_W^2}\right)^2 \left(\frac{d\sigma(\bar{\nu}_\mu t \to \mu^+ H_1)}{d|q^2|} - \frac{d\sigma(\nu_\mu t \to \mu^- H_2)}{d|q^2|}\right) = C_t, \tag{10}$$

when t is the target hadron, H_1 and H_2 any hadron states with the appropriate charge, q the momentum transfer from ν to μ, and C_t a constant determined by the expectation value of the time-time part of the above commutator between t states. In the usual theory,

$$C_p = (G_F^2/\pi)(1 + \sin^2\theta),$$
$$C_n = (G_F^2/\pi)(-1 + 2\sin^2\theta). \tag{11}$$

These values survive in our theory if the summation over H_1 and H_2 is restricted to SU(3)′ singlet states; for sufficiently large E_ν, however, it is difficult to implement this restriction experimentally without invoking a demon who will be willing to sit at the end of the neutrino beam and check whether an emerging hadron is an SU(3)′ singlet! For H unrestricted,

$$C_p = 18G_F^2/\pi,$$
$$C_n = 0. \tag{12}$$

The Gross–Llewellyn Smith sum rule is based on the space-space part of the aforementioned commutator. The contribution of left-handed and right-handed quarks now cancel against each other in the SU(3)′ limit, hence the result (v) stated above. The neutrino sum rule in Ref. 10 would also be changed. The consistency of that sum rule, as stated in Ref. 10, with the current experimental status[5] would suggest that SU(3)′ nonsinglet states have not yet been produced. We may emphasize that features (i), (ii), (vi), (vii), and (viii) do not depend on the production of such states. Finally, feature (viii) is based on standard arguments.[11]

We now conclude with the following remarks:

(a) The process $\nu + p \to \nu + H$ is related directly to electroproduction in our model. Also, the mass of M^0 can be constrained within fairly narrow limits by considering its contribution to the anomalous moment of the muon. These points will be discussed elsewhere.

(b) There are, of course, many possible variations on our theme if one is willing to sacrifice some of our listed features.

(c) Our model reproduces all the known features of weak interactions, except for the $\Delta I = \frac{1}{2}$ rule and CP nonconservation. In the present state of the art, we can appeal to ideas of octet enhancement and soft-pion theorems for the $\Delta I = \frac{1}{2}$ rule and put in CP nonconservation by hand.[12]

(d) Perhaps the most interesting aspect of our model is the testability of many of its predictions with existing machines and within the framework of experimental programs currently under way. Our model may have the dubious distinction of being an early casuality in a confrontation of gauge theories with experiment.

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[12]A kinematical $\Delta I = \frac{1}{2}$ rule and CP nonconservation may be incorporated in this model by enlarging the gauge group. M. A. B. Bég, to be published.