states are determined by the thickness of the normal lamina. If the magnetic field has a component perpendicular to the film surface, some complications arise due to the additional flux-flow voltage.[13] When the magnetic field is oriented at about 10° to 20° relative to the film surface (depending on the magnetic field), the current branches become so small that the flux flow determines the main features of the I-V characteristics.

Several films of different thicknesses have been studied to look for thickness dependence. For a thin film ($t \sim \xi$), the quantum resistances do not have as strong a field dependence as shown in Fig. 3. For a thicker film ($t > 3\xi$), the current branches representing the IQR states are not straight lines. In addition, they are not stable. This shows that when $d \ll t$, the normal lamina is not well defined nor stable. The best experimental results are obtained when the thickness is in the range of $\xi < t < 3\xi$. A more detailed account of this work will be reported elsewhere.

Sugahara[14] has recently investigated the step structures in the I-V characteristics of a type-II superconducting wire of diameter much greater than the coherence length. He attributed his constant-voltage steps also to the coherent bound states (but of different nature). Since our current branches have constant slopes instead of constant voltages, a direct comparison of the two results cannot be made.

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Fluctuation-Induced Diamagnetism in Dirty Superconductors

Kazumi Maki*
Department of Physics, University of California, Los Angeles, California 90024
(Received 13 November 1972)

An explicit evaluation is made of the fluctuation-induced diamagnetism from zero-point fluctuations, which have been neglected in a previous calculation by Maki and Takayama. It is shown that in fact this contribution has another scaling field $\sim \varphi_0/l^2$, as calculated previously by Lee and Payne and by Kurkijärvi, Ambegaokar, and Eilenberger. However, this term gives rise to an almost constant susceptibility (of the same order as Landau diamagnetism) in the field region of experimental interest [i.e., $H \sim H_{c2}(0)$, the upper critical field at $T = 0$ K].

Fluctuation-induced diamagnetism in superconductors above the transition temperature has been studied recently both theoretically and experimentally. In the pure limit, where the effect of nonlocality is extremely important, the deviation from the simple theory[1-5] (which is worked out in the framework of the Ginzburg-Landau theory), found experimentally by Gollub, Beasley, and Tinkham,[5] is well accounted for in terms of microscopic theories proposed by Lee and Payne (LP)[6] and by Kurkijärvi, Ambegaokar, and Eilenberger (KAE).[7] In the dirty limit the theoretical situation appears a matter of controversy for the moment. It has been recognized that, in the dirty limit, the nonlocal effect is of no importance, but time-dependent fluctuations play a primary role in the induced diamagnetism.[8] Although a number of recent experiments[5,9,10]

seem to support the prediction of the theory proposed by Maki and Takayama (MT),[8] some doubts have been cast over the theoretical foundation.[6] In particular, LP and KAE proposed an alternative calculation for the dirty limit, which not only disagrees entirely with experiments,[5,9,10] but also predicts a large scaling field H_s' which is completely unphysical.

The purpose of this note is twofold. Firstly, we would like to clarify the difference between our approach and those of LP and KAE; secondly, we would like to point out that the zero-point fluctuation term, which has been deliberately neglected in MT, does not contribute to the observed fluctuation-induced diamagnetism, but to a small diamagnetism almost proportional to H. Strictly speaking, this term has a scaling field of the order of $H_s' \sim \varphi_0/l^2$ $[\sim (\xi_0/l)H_{c2}(0)]$, where l is the electron mean free path and $\varphi_0 = hc/2e$ as calculated by LP and KAE.[6,7] However, in the field region of experimental interest (i.e., $H \sim H_{c2}(0) \sim \varphi_0/l\xi_0$, where ξ_0 is the BCS coherence length), this term gives rise to a constant diamagnetic susceptibility, which is subtracted in all experiments mentioned above. In the following we confine ourselves to the dirty limit ($l \ll \xi_0$) for simplicity, although the present analysis can be easily extended to a more general situation.

The fluctuation-induced diamagnetism is obtained from the thermodynamical potential due to fluctuations, which is given in the presence of magnetic field H by[8]

$$\frac{F}{V} = -T\frac{eH}{\pi}\sum_{\nu=-\infty}^{\infty}\sum_{n=0}^{\infty}\int\frac{dk}{2\pi}\ln[\mathfrak{D}(n, k, \omega_\nu)], \tag{1}$$

where $\mathfrak{D}(n, k, \omega_\nu)$ is the fluctuation propagator in the presence of a magnetic field $\boldsymbol{H}$ (in the dirty limit)[11]:

$$\mathfrak{D}(n, k, \omega_\nu) = N(0)^{-1}\left[\psi\left(\frac{1}{2}+\frac{|\omega_\nu|+D[4eH(n+\frac{1}{2})+k^2]}{4\pi T}\right)-\psi\left(\frac{1}{2}-\frac{\epsilon_0}{4\pi T}\right)\right]^{-1}, \tag{2}$$

where $N(0)$ is the electron state density (for one spin direction)[2] at the Fermi level, D is the diffusion constant, $\psi(z)$ is the digamma function, and ϵ_0 is determined from

$$-\ln(T/T_{c0}) = \psi(\tfrac{1}{2}-\epsilon_0/4\pi T)-\psi(\tfrac{1}{2}), \tag{3}$$

with T_{c0} the transition temperature for $H=0$. In the expression (1) the summation over the Matsubara frequency ω_ν can be transformed into an integral over ω by analytical continuation, so that

$$\frac{F}{V} = \frac{eH}{\pi^3}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}dk\int_0^{\infty}d\omega\,[\tfrac{1}{2}+(e^{\beta\omega}-1)^{-1}]\,\mathrm{Im}\{\ln[\mathfrak{D}^{-1}(n, k, i\omega)]\}, \tag{4}$$

where $\beta = 1/T$ is the inverse of the temperature. In the previous treatment (MT), the term with the $\frac{1}{2}$ in square brackets was neglected. On the other hand, in LP-KAE, both terms in the first bracket were retained, since they performed a brute-force summation over ω_ν starting from Eq. (1), which resulted in a larger scaling field H_s' in the dirty limit. We will now study the term with the $\frac{1}{2}$ (i.e., the zero-point fluctuation term), which constitutes the basic difference between MT and LP-KAE[6,7]:

$$\frac{F_0}{V} = \frac{eH}{2\pi^3}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}dk\int_0^{\infty}d\omega\,\mathrm{Im}\{\ln[\mathfrak{D}^{-1}(n, k, i\omega)]\}. \tag{5}$$

Making use of the Euler-MacLaurin expansion, we can expand Eq. (5) in powers of H:

$$\frac{F_0}{V} = \frac{1}{8\pi^3}\int_{-\infty}^{\infty}dk\int_0^{\infty}d\omega\,\mathrm{Im}\Big\{\int_0^{\infty}dx\,f[-i\omega+D(x+k^2)]+\tfrac{1}{6}D(2eH)^2f^{(1)}(-i\omega+Dk^2+2DeH)$$
$$-\tfrac{1}{6}D^2(2eH)^3f^{(2)}(-i\omega+Dk^2+2DeH)+\tfrac{23}{360}D^3(2eH)^4f^{(3)}(-i\omega+Dk^2+2DeH)\Big\}+O(H^5), \tag{6}$$

where

$$f(x) = \ln\{N(0)[\psi(\tfrac{1}{2}+x/4\pi T)-\psi(\tfrac{1}{2}-\epsilon_0/4\pi T)]\}, \tag{7}$$

and $f^{(n)}(x)$ is the nth derivative of $f(x)$. Since the first term in Eq. (6) is independent of field, the zero-point fluctuation term gives rise to an almost constant susceptibility as long as we neglect the term of

order H^3. The integral over ω is then easily carried out, and we have

$$\frac{F_0}{V}=\text{const}+\frac{D(2eH)^2}{48\pi^3}\int_{-\infty}^{\infty}dk\,\mathrm{Re}[f(-i\tau^{-1}+Dk^2+2DeH)-f(Dk^2+2DeH)]$$

$$+\frac{D^2(2eH)^3}{48\pi^3}\int_{-\infty}^{\infty}dk\,f^{(1)}(Dk^2+2DeH)-\frac{23}{2880\pi^3}(2eH)^4\int_{-\infty}^{\infty}dk\,f^{(2)}(Dk^2+2DeH)+\cdots. \tag{8}$$

Here we cut off the upper limit of the integral at $\omega=\tau^{-1}$, the inverse of the electron lifetime, since Eq. (2) is valid only for $\omega\tau\ll1$. A more precise treatment, using the fluctuation propagator valid for all ω, shows that the integral is cut off around $\omega\tau\cong1$ because of $\omega\tau\ll1$; D also depends on ω so as to guarantee the convergence of the integral. The susceptibility arising from the zero-point fluctuation is given from Eq. (8) as

$$\chi_0=-\frac{\partial^2}{\partial H^2}\left(\frac{F_0}{V}\right)=-\frac{D}{3\pi^3}e^2\langle k\rangle, \tag{9}$$

where

$$\langle k\rangle=\int_0^{\infty}dk\,\mathrm{Re}[f(-i\tau^{-1}+Dk^2+2DeH)-f(Dk^2+2DeH)]. \tag{10}$$

Here we neglected the contribution from the last two terms in Eq. (8), which are smaller by a factor of $l(H/\varphi_0)^{1/2}$, and thus are completely negligible for $H\lesssim H_{c2}(0)$.

In the dirty limit ($l\ll\xi_0$), Eq. (10) can be transformed by a partial integration to yield

$$\langle k\rangle=-\frac{D}{2\pi T}\int_0^{\infty}k^2\,dk\,\mathrm{Re}\left\{\frac{\psi^{(1)}[\frac{1}{2}-i(4\pi\tau T)^{-1}+D(k^2+2eH)/4\pi T]}{\psi[\frac{1}{2}-i(4\pi\tau T)^{-1}+D(k^2+2eH)/4\pi T]-\psi(\frac{1}{2}-\epsilon_0/4\pi T)}\right.$$

$$\left.-\frac{\psi^{(1)}[\frac{1}{2}+D(k^2+2eH)/4\pi T]}{\psi[\frac{1}{2}+D(k^2+2eH)/4\pi T]-\psi(\frac{1}{2}-\epsilon_0/4\pi T)}\right\}$$

$$=\langle k\rangle_1+\langle k\rangle_2, \tag{11}$$

where

$$\langle k\rangle_1\cong\frac{2}{(\tau D)^{1/2}}\mathrm{Re}\int_0^{\infty}dx\left[\frac{1}{\ln(Ax^2)}-\frac{1}{\ln[A(x^2-i)]}\right],$$

$$\langle k\rangle_2\cong\frac{2}{(\tau D)^{1/2}}\mathrm{Re}\left[-i\int_0^{\infty}dx\frac{1}{x^2-i}\frac{1}{\ln[A(x^2-i)]}\right],$$

$$\ln A=\ln\{(4\pi\tau T)^{-1}[1+\tfrac{4}{9}(l^2eH)^2]^{1/2}\}-\psi(\tfrac{1}{2}-\epsilon_0/4\pi T). \tag{12}$$

Here we approximated the polygamma functions containing Dk^2 as a variable in Eq. (11) by their asymptotic expressions for large variables, since the contribution to the integral arises mainly from large k values (i.e., $k\cong l^{-1}$). Furthermore, it is easy to see that

$$\langle k\rangle_1=[2/(\tau D)^{1/2}][O((\ln A)^{-2})],$$

while

$$\langle k\rangle_2=[\pi/\sqrt{2}\,(\tau D)^{1/2}][(\ln A)^{-1}+O((\ln A)^{-2})]; \tag{13}$$

we have therefore

$$\langle k\rangle=\frac{\sqrt{3}\,\pi}{\sqrt{2}\,l}\left\{\left[\ln\left(\frac{1}{4\pi\tau T}[1+\tfrac{4}{9}(l^2eH)^2]^{1/2}\right)-\psi\left(\frac{1}{2}-\frac{\epsilon_0}{4\pi T}\right)\right]^{-1}+O((\ln A)^{-2})\right\}. \tag{14}$$

The zero-point fluctuations give rise to a diamagnetic susceptibility which depends weakly on the temperature and the field. We can compare this term with the Landau (or the orbital) diamagnetism in a

normal metal, χ_L. In fact we have

$$\chi_0/\chi_L \cong (1/\sqrt{6})[\ln(4\pi\tau T)^{-1} - \psi(\tfrac{1}{2} - \epsilon_0/4\pi T)]^{-1}$$
$$\cong (1/\sqrt{6})[\ln(T_{c0}/\tau\Delta_{00}T)]^{-1} \text{ for } H \ll H_{c2}(0)\xi_0/l\,, \tag{15}$$

where Δ_{00} is the BCS energy gap at $T=0$ and $H=0$. Therefore, this new susceptibility is weakly temperature and field dependent for $T \cong T_{c0}$ and $H \cong H_{c2}(0)$. However, in much higher fields [i.e., $H_s' \sim \varphi_0/l^2 \sim (\xi_0/l)H_{c2}(0)$] or at higher temperatures [$T \sim (\xi_0/l)T_{c0}$] this term will be suppressed in accordance with LP-KAE.[6,7]

For the magnetization, this term gives rise to a contribution (almost) linear in a magnetic field. On the other hand, the term with the Bose distribution function, which is considered in (MT), yields the magnetization[8] for large fields [$H \gtrsim H_{c2}(0)$],

$$-M/\sqrt{H} \cong (4e)^{3/2}(T^2/48)(4DeH)^{-1}b_1(\gamma), \tag{16}$$

$$b_1(\gamma) = -(\sqrt{2}-1)\zeta(\tfrac{1}{2}) + \sum_{n=0}^{\infty}\left[\frac{1}{(n+\frac{1}{2})^{1/2}} - \frac{1}{(n+\frac{1}{2}+\gamma)^{1/2}} + \frac{\gamma}{(n+\frac{1}{2}+\gamma)^{3/2}}\right], \quad \gamma = \epsilon_0/4eDH, \tag{17}$$

where $\zeta(z)$ is Riemann's ζ function. The above magnetization $(-M)$ vanishes like $H^{-1/2}$ in the high-field region [$H \gtrsim H_{c2}(0)$].

We may conclude that, in the dirty limit, the fluctuations give rise to a magnetization which consists of two distinct terms: One has a scaling field $H_s' \sim \varphi_0/l^2$, while the other has a scaling field $H_s \sim H_{c2}(0)$. In the field region of experimental interest [i.e., $H \sim H_{c2}(0)$], the former contribution to the magnetization rises linearly in H, while the latter decreases like $H^{-1/2}$ as the magnetic field increases. Therefore, the separation of these two terms, which has been done in MT, but not in LP and KAE, is essential to obtain the correct scaling field. It is no wonder that more recent experiments[5,9,10] support our treatment (MT), since in those experiments the magnetization linear in H is carefully subtracted.

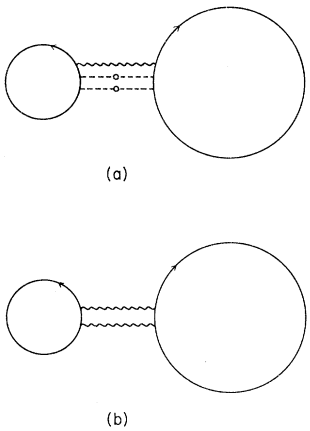

FIG. 1. Lowest-order correction to the Landau diamagnetism of a normal metal due to the electron-phonon interaction shown diagramatically for (a) dirty metals, and (b) pure metals. Solid lines, electron propagator in the presence of a magnetic field; wavy lines, phonon propagator; broken lines with open circles indicate the impurity scattering.

One may wonder why the magnetization arising from zero-point fluctuations depends only weakly on temperature and field since it is, after all, due to superconducting fluctuations. It is because we can think of this term as the lowest-order correction to the Landau diamagnetism for a normal metal due to the electron-phonon interaction (see Fig. 1). In fact a simple transformation yields

$$\chi_0/\chi_L \cong (\lambda/\sqrt{6})[1 - \lambda\ln(\tfrac{1}{4}\tau\omega_D)]^{-1},$$

where $\lambda = |g|N(0)$ is the BCS coupling constant and ω_D is the Debye frequency. Furthermore, the scaling field H_s' associated with the zero-point term (which is essentially calculated by LP and KAE[6,7]) does not depend on ξ_0, but is expressed entirely in terms of l; H_s' reflects only the normal properties of the metal. It is important to stress that this correction appears only in the dirty limit. In pure metals, the lowest-order correction of this type appears only in second-order terms in λ.

It will be of some interest to make absolute measurements of the susceptibility at high fields, which will certainly reveal the presence of the zero-point fluctuation contribution. In particular, the weak field and temperature dependences of this term will provide a means of separating it from Pauli paramagnetism and Landau diamagnetism in the normal state.

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*Permanent address: Department of Physics, Tohoku

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Absorption and Transport Study of the Pseudogap in Amorphous Ge

M. L. Knotek* and T. M. Donovan
Physics Division, Michelson Laboratory, China Lake, California 93555
(Received 7 December 1972)

Absorptance and spectral response of the photoconductivity data indicate the absorption edge in a-Ge is near $h\nu = 0.6$ eV and not at 1 eV as suggested by other studies. The position of the optical threshold is obtained from *in situ* measurements on ultrahigh-vacuum–deposited films and is within experimental error of the thermal activation energy determined as the sum of the separation of the Fermi energy from the valence band and the high-temperature activation energy of the dc conductivity.

There have been a number of studies of the optical properties of amorphous Ge (a-Ge), which give either broad[1,2] or sharp[3-5] optical thresholds, depending on deposition conditions. Broad thresholds are sharpened by annealing,[2] allowing the interpretation that broad edges are associated with defect states and annealing reduces the number of these states. Annealing has also been reported to shift the threshold to energies higher than the direct edge in crystal Ge (c-Ge).[2] Large shifts (on the order of 0.5 eV) in the optical threshold are difficult to understand if one considers the effect of annealing on film structure, i.e., there is no apparent change in the nearest- and next–nearest-neighbor atomic distance which could account for such a shift.[6] Also, no large-scale annealing effects are observed in *in situ* photoemission results[7] which might be expected to reflect large changes in the density of states in the optical gap.

We have grown films so as to have either broad or sharp optical thresholds as grown. We find on annealing in $<10^{-6}$-Torr vacuum that the edge after anneal for either case, broad or sharp, is near 0.6 eV. Most important, we find that by depositing and measuring the photoresponse *in situ* at 10^{-10} Torr a sharp edge near $h\nu = 0.56$ eV is obtained. The energy of this optical threshold is equal numerically to the sum of the separation of the Fermi energy from the valence band, $E_F = 0.31$ eV, as determined by photoemission studies[8] and the high-temperature activation energy of the dc conductivity, $E_A = 0.25$ eV, reported here, both results being *in situ* results obtained in ultrahigh vacuum (UHV). This places the "band" gap in a-Ge near the indirect edge in c-Ge. Thus, the thermal and optical band gaps in uncontaminated films are in agreement.

The optical absorptance A was measured both by the reflectance and transmittance methods[3] and by measuring the photoresponse of the samples in the photoconductive mode.[9] The two methods agree very closely where direct comparison has been possible, as reported by Fischer and Donovan,[9] the photoresponse tracking A from threshold to ~3 eV. For example, in Fig. 1 we plot absorptance $A = 1 - R - T$ (where R is the corrected reflectance[3] and T the corrected transmittance[3]) and the optical absorption coefficient α derived from the absorptance data for a 2200-Å film deposited at 10^{-6} Torr, in an oil-pumped vacuum system, at a rate of ~1 Å/sec on a substrate held at 150°C. Also plotted in Fig. 1 is the photoresponse $\Delta\sigma$, normalized to incident photon flux. The structure in both $\Delta\sigma$ and A is due to interference effects and is removed in the analysis of the R and T data yielding α. The film shown in Fig. 1 exhibits a sharp threshold near 0.55 eV,