Observation of Intermediate Quantum-Resistance States of Type-I Superconducting Films in Parallel Magnetic Fields*

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We have found several new conducting states of constant and quantized resistances in a superconducting Pb film in parallel magnetic fields between H_{c2} and H_{c3}. These states can be explained qualitatively as due to the conduction of coherent quasiparticles in the normal lamina penetrated by the magnetic field.

It has been reported previously that the quasiparticles in the surface bound states of a bulk type-I superconductor can be excited into the BCS continuum by means of microwave absorption.[1-3] In this Letter, we wish to report some experimental evidence which shows that dc current may conduct directly through the bound states in a pure type-I superconducting film. The experimental evidence which manifests this effect is in the form of several linear current branches "hidden" between the two well-known (zero-resistance and Ohmic) sections of the I-V characteristic of the film.

The samples investigated are high-purity (99.9999%) Pb films with dimensions 0.3 mm in width, 5 mm in length, and 4000 Å in thickness. The films are evaporated onto room-temperature glass substrates in a vacuum of 10^{-6} Torr. The I-V characteristics are traced out by using a low-noise constant-current source which can be manipulated in either the increasing- or the decreasing-current direction near any point on the I-V curve. The conventional four-terminal arrangement is used to measure the current and voltage along the length of the film.

When the I-V curve is traced out in a conventional way (by simply increasing the bias current monotonically from the zero voltage to the Ohmic section and then decreasing continuously back to the zero voltage), the I-V characteristic of the superconducting film in a parallel magnetic field is generally shaped like the insert in Fig. 1. However, if the bias current is allowed to decrease very slowly, the film does not make a smooth transition from the Ohmic state directly back to the superconducting state. Rather, it skips discontinuously along the returning section of the I-V curve and momentarily stops at several intermediate points. Starting from one of these points, a stable current branch of constant slope can be traced out. Repeating the same procedure many times but each time starting from a different intermediate point, an I-V characteristic showing these new intermediate states can be obtained. An example of such an I-V characteristic at low voltage is shown in Fig. 1.

The most striking feature of these intermediate quantum-resistance (IQR) states is that they have well-defined quantum resistances which are less

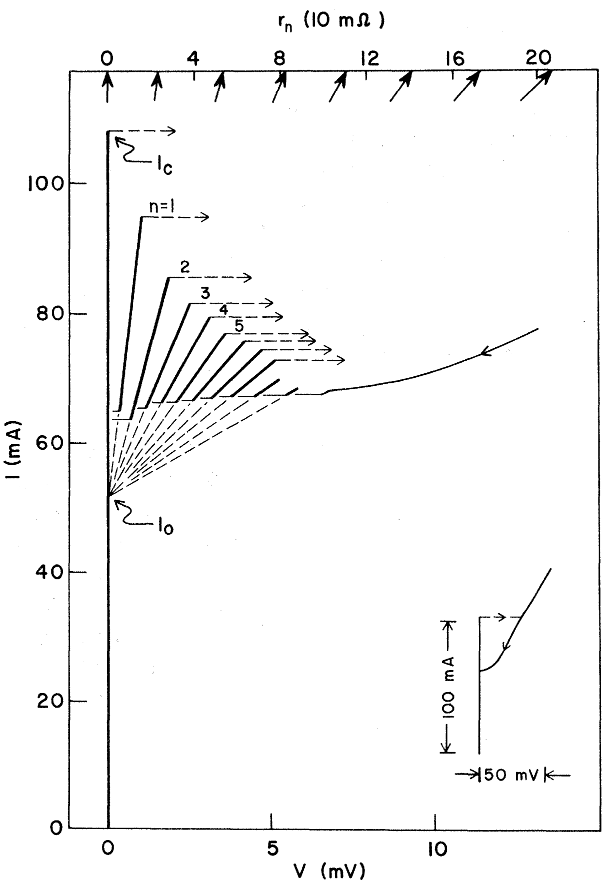

FIG. 1. The I-V characteristic of a Pb film in a parallel magnetic field $H_{\parallel}=1.4$ kG and at the temperature $T=4.2$ K. The normal-state resistance of the film is $R_N=0.36$ Ω. The upper scale is for reading the quantum resistances which are indicated by the arrows extrapolated from the linear current branches.

than the normal-state resistance of the film. In addition, the extrapolations (shown as dashed lines) of these current branches lead to the same intercept at zero voltage. This current intercept I_0 is less than the total critical current I_c of the film, and it has a magnetic field dependence similar to that of I_c. The I-V characteristics of these IQR states can be simply expressed by

$$I = I_0 + V/r_n, \quad n = 1, 2, \ldots, \tag{1}$$

where r_n is the quantum resistance of the nth state. As can be seen from the arrows pointing to the upper scale of Fig. 1, the values of r_n satisfy

$$r_n = r_0(n - \alpha), \tag{2}$$

where $r_0 = 0.03\ \Omega$ and $\alpha \sim 0.2$ (the magnetic field dependences of r_0 and α to be described later).

Equation (1) does not indicate the allowable range of current (hereafter called current amplitude) for each IQR state. However, the pattern of the current amplitudes shown in Fig. 1 is typical of the experimental results. Namely, the current amplitudes decrease successively with increasing n. Because of this reverse order in magnitude, these current branches could thus be "hidden" if the conventional way were used to trace out the I-V curve. Only near the parallel upper critical field, H_{c3}, or when the magnetic field is not parallel to the film, do the current amplitudes become irregular in order and steplike structures appear at the top.

The magnetic field dependences of the IQR states are as follows. These IQR states are not observable for $H_{\parallel} < 0.8H_{c2}$, where H_{c2} is the perpendicular critical field of the Pb film. The first ($n = 1$) IQR state appears near $0.8H_{c2}$ ($H_{c2} \sim 1.0$ kG). However, the current amplitude of this branch is rather small at this magnetic field. Its magnitude grows with increasing magnetic field. Meanwhile, more current branches become observable in the increasing order of n. These magnetic field dependences are illustrated in Fig. 2 in which the current amplitudes of the first two IQR states are shown as functions of magnetic field. It is also seen in Fig. 2 that the current amplitudes decrease rapidly after $1.5H_{c2}$, indicating the diminishing of superconductivity in the film.

As shown in Fig. 3, when the first two IQR states appear initially near $0.8H_{c2}$, their resistances are quite large. With increasing magnetic field, they quickly drop to their respective limiting values. Even beyond $1.5H_{c2}$, the quantum resistances do not change their values signifi-

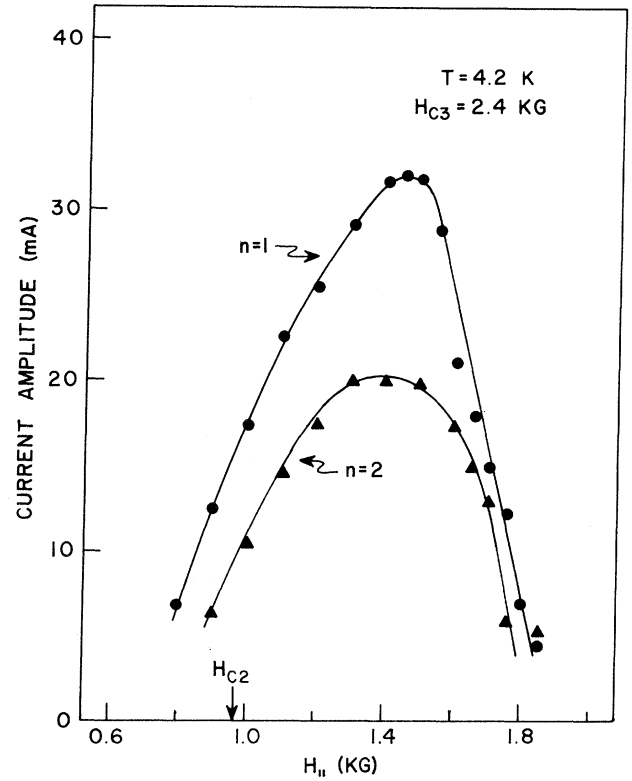

FIG. 2. The magnetic field dependences of the current amplitudes for the first two quantum-resistance states.

cantly. The magnetic field dependences of α and r_0 can also be obtained from Fig. 3 and Eq. (2). It turns out that α is not strongly field dependent and is within the range of $\frac{1}{5} < \alpha < \frac{1}{2}$. Therefore, the magnetic dependence of r_0 is very similar to that of r_1 since $r_0 = r_1/(1 - \alpha)$.

We attempt to explain these IQR states qualitatively in terms of coherent quasiparticle bound states.[4,5] The origins of such bound states in a superconducting-normal (SN) binary film, a SNS sandwich, or in the normal lamina of an intermediate-state superconductor have been discussed in many theoretical articles.[4-12] In our case, the bound states are due to the coherent reflections of quasiparticles (and holes) into quasiholes (and particles) by the pair-potential boundaries which separate the magnetic-field-penetrated normal lamina from the two superconducting surface sheaths. While each quasiparticle is characterized by a set of quantum numbers (k_x, k_y, $k_{\perp n}$), where k_x and k_y are the two components parallel to the film and $k_{\perp n}$ is the component perpendicular to the film, a bound state is characterized by $k_{\perp n}$ only. For a rec-

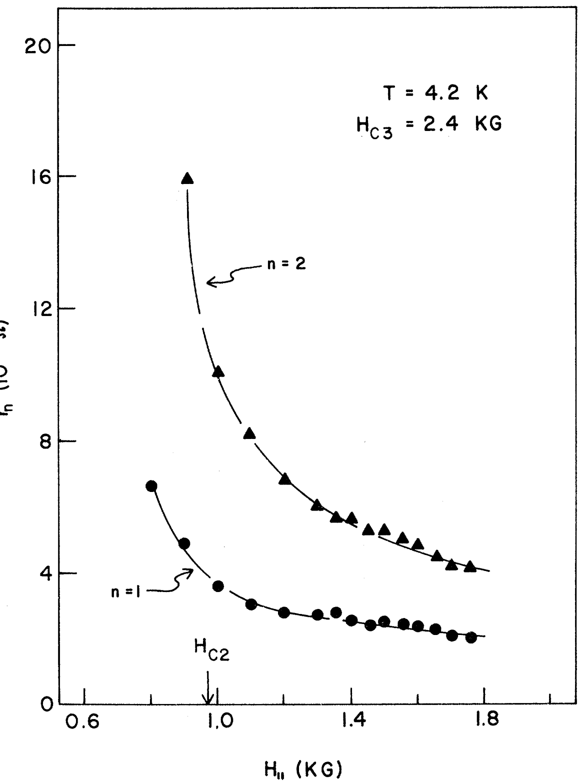

FIG. 3. The magnetic field dependences of the first two quantum resistances.

tangular pair potential well, it is known that $k_{\perp n} = (\pi/d)(n-\alpha)$, where d is the width of the well, n an integer, and α a constant. If we consider those quasiparticles whose Fermi velocities are nearly parallel to the plane of the film, the nth bound state consists of all the quasiparticles with different k_{Fx} and k_{Fy} but the same $k_{\perp n}$.

Such a bound state, having an energy within the energy gap, is a macroscopic phase coherent state like the ground state. As these particles move nearly parallel to the film they are subjected to transverse reflections with the same frequency $\nu_n = \hbar k_{\perp n}/md$. Our explanation of the IQR states is as follows. When the biased current is smaller than the critical current of the two superconducting sheaths, it is conducted through the two superconducting sheaths alone. If the current is greater than the critical value, it may split into two components: One, equal to I_0, remains in the two superconducting sheaths, and the other, equal to V/r_n, is diverted into the normal lamina and conducted via the nth bound state. We assume that the resistance of the normal lamina (in this metastable state) is proportional to the frequency of the transverse reflection. For a rectangular potential well, this assumption leads to $r_n \propto v_n \propto k_{\perp n} \propto n-\alpha$. The common intercept I_0 is also the consequence of this particular shape of potential well since the effective thickness of the two superconducting sheaths is the same for all the bound states.

The magnetic field dependences can be understood as mainly due to the change of the normal-layer thickness.[4] At $H_\parallel \lesssim H_{c2}$, the magnetic field starts to penetrate into a normal layer of thickness roughly equal to the coherent length ξ, which for the Pb film in question is about 1500 Å. As the magnetic field is further increased, the normal layer expands to the whole film which has a thickness of $t \sim 4000$ Å. The increase of the effective thickness of the normal layer leads to a larger current amplitude. In addition, a larger d means smaller $k_{\perp n}$'s, consequently smaller r_n's. Other experimental evidence also supports the preceding argument: First, Fig. 3 shows that both r_1 and r_2 are reduced from the maxima to the minima by a factor of about 3, which is roughly the ratio of t/ξ. Second, when the first IQR state becomes observable at $H_\parallel \sim 0.8$ kG, the current branch is not a perfect straight line but is rounded (not shown here). This roundness of the current branch indicates that the boundaries of the normal layer are not well defined when the magnetic field first penetrates into a thin layer. The current branch becomes almost a perfect straight line in a large field as shown in Fig. 1. Finally, it is noted that for $H_\parallel > 1.4$ kG, while the current amplitudes (in Fig. 2) start to drop because of the destruction of superconductivity, the quantum resistances (in Fig. 3) still remain at the limiting values, meaning that the final boundaries are close to the film surfaces.

It is worth noting that when the film is in an IQR state, not all the current is conducted through the coherent quasiparticle state. Equation (1) and Fig. 1 indicate that a component equal to I_0 is still in the two superconducting sheaths. Therefore, I_0 signifies the critical current of the superconducting sheaths. Since the voltage is developed along the whole film, the superconducting sheaths should also be biased by the same voltage. The electrodynamic properties of these voltage-biased superconducting sheaths remain to be investigated.

The experimental results for the different orientations of the magnetic fields are essentially the same so long as the field is parallel to the film surface. This further proves that the bound states are determined by the thickness of the normal lamina. If the magnetic field has a component perpendicular to the film surface, some complications arise due to the additional flux-flow voltage.[13] When the magnetic field is oriented at about 10° to 20° relative to the film surface (depending on the magnetic field), the current branches become so small that the flux flow determines the main features of the I-V characteristics.

Several films of different thicknesses have been studied to look for thickness dependence. For a thin film ($t\sim\xi$), the quantum resistances do not have as strong a field dependence as shown in Fig. 3. For a thicker film ($t>3\xi$), the current branches representing the IQR states are not straight lines. In addition, they are not stable. This shows that when $d\ll t$, the normal lamina is not well defined nor stable. The best experimental results are obtained when the thickness is in the range of $\xi<t<3\xi$. A more detailed account of this work will be reported elsewhere.

Sugahara[14] has recently investigated the step structures in the I-V characteristics of a type-II superconducting wire of diameter much greater than the coherence length. He attributed his constant-voltage steps also to the coherent bound states (but of different nature). Since our current branches have constant slopes instead of constant voltages, a direct comparison of the two results cannot be made.

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Fluctuation-Induced Diamagnetism in Dirty Superconductors

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An explicit evaluation is made of the fluctuation-induced diamagnetism from zero-point fluctuations, which have been neglected in a previous calculation by Maki and Takayama. It is shown that in fact this contribution has another scaling field $\sim\varphi_0/l^2$, as calculated previously by Lee and Payne and by Kurkijärvi, Ambegaokar, and Eilenberger. However, this term gives rise to an almost constant susceptibility (of the same order as Landau diamagnetism) in the field region of experimental interest [i.e., $H\sim H_{c2}(0)$, the upper critical field at $T=0$ K].

Fluctuation-induced diamagnetism in superconductors above the transition temperature has been studied recently both theoretically and experimentally. In the pure limit, where the effect of nonlocality is extremely important, the deviation from the simple theory[1-5] (which is worked out in the framework of the Ginzburg-Landau theory), found experimentally by Gollub, Beasley, and Tinkham,[5] is well accounted for in terms of microscopic theories proposed by Lee and Payne (LP)[6] and by Kurkijärvi, Ambegaokar, and Eilenberger (KAE).[7] In the dirty limit the theoretical situation appears a matter of controversy for the moment. It has been recognized that, in the dirty limit, the nonlocal effect is of no importance, but time-dependent fluctuations play a primary role in the induced diamagnetism.[8] Although a number of recent experiments[5,9,10]