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Nonrenormalization Theorem in the Chiral Symmetry Limit*

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We show that the form factors of the strangeness-changing current are renormalized in first order, rather than second order, in the symmetry-breaking parameters if the chiral symmetry is realized by Nambu-Goldstone bosons. The first-order renormalization is explicitly calculated for the meson and baryon form factors.

In this note we re-examine the celebrated nonrenormalization theorem of Ademollo and Gatto[1] and Behrends and Sirlin[2] in the light of recent observations on perturbation theory about a Nambu-Goldstone symmetry.[3] The nonrenormalization theorem asserts that the vector form factors of currents at zero momentum transfer, whose associated charges are the generators of a symmetry group, remain unrenormalized up to second order in the symmetry breaking. The implication of this theorem—that the renormalization effects are therefore small—is crucial to establishing contact between the Cabibbo theory of weak interactions and experiment.

This theorem is certainly true in the case that the symmetry limit is realized in the usual way in which the states are irreducible representations of the symmetry group. If, however, the symmetry limit is realized by ground-state Nambu-Goldstone bosons, we find that the form factors are renormalized to *first* order in the symmetry-breaking parameters.

We now illustrate this remark using the language of the $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$ model, although the result is actually model independent. Consider the Hamiltonian density

$$H(x) = H_0(x) + \epsilon_0 u_0(x) + \epsilon_8 u_8(x),$$

where $H_0(x)$ is chiral $SU(3) \otimes SU(3)$ invariant and $u_{0,8}(x)$ transform like members of the $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$ representation. We assume that as the symmetry-breaking parameters ϵ_0 and ϵ_8 vanish, the symmetry of H_0 is realized by an octet of massless ground-state pseudoscalar bosons. If $f_+(0)$ is the K_{l3} decay form factor of the strangeness-changing current between a π and K, then with $f_+(0) = 1 + \epsilon_8^2 F(\epsilon_0, \epsilon_8)$ we find $F(\epsilon_0, 0) \to r/\epsilon_0 + s/\epsilon_0^{1/2} + \cdots$ as $\epsilon_0 \to 0$, with r a known constant. Consequently $f_+(0) = 1 + O((\epsilon_8/\epsilon_0)\epsilon_8)$ and the renormalization occurs in first order.

It is apparent from these remarks that if we consider perturbation theory in ϵ_8 about SU(3) then the correction is of $O(\epsilon_8^2)$ in accord with the usual theorem. However, if we consider perturbation theory about $SU(3) \otimes SU(3)$, which may be as good a symmetry as SU(3) $(\epsilon_8/\epsilon_0 \simeq -\sqrt{2})$, then the correction is properly of order $(\epsilon_8/\epsilon_0)\epsilon_8 \sim O(\epsilon_8)$—first order.

We indicate below how this result is obtained. We also calculate the renormalization, to first order, in a model-independent way. For the $f_+(0)$ form factor one obtains, with $f_\pi \simeq 0.96\mu_\pi/\sqrt{2}$,

$$f_+(0) = 1 - \frac{5(\mu_K^2 - \mu_\pi^2)^2}{384\pi^2 f_\pi^2 (2\mu_K^2 + \mu_\pi^2)} = 0.98 \qquad (1)$$

which is numerically only a 2% renormalization. We have also examined the renormalization for the baryon form factors, where this first-order renormalization is at most 7%. Our results do not alter the beautiful agreement of the Cabibbo theory with experiment and support the usual assumption of ignoring renormalization effects.

We follow Fubini and Furlan.[4] If we consider the matrix element of $[F_+, F_-] = I_3 + \frac{3}{2}Y$, $F_\pm = F_4 \pm iF_5$ with F_a a generator of SU(3), between ground-state mesons states $|M\rangle$, then we obtain a sum rule of the form

$$(I_3+\tfrac{3}{2}Y)_{MM}=\sum_{M'}[|F_-^{M\to M'}(0)|^2-|F_+^{M\to M'}(0)|^2]+\int_{s_0}^{\infty}ds\,(s-\mu_M^2)^{-2}\sum_n(2\pi)^3\,\delta^4(P-P_n)$$
$$\times[|\langle n|\partial_\mu V_\mu^-(0)|M\rangle|^2-|\langle n|\partial_\mu V_\mu^+(0)|M\rangle|^2], \tag{2}$$

where $P^2=s$. Here $F_\pm^{M\to M'}(0)$ are the form factors we wish to consider; they arise from saturating the commutator $[F_+,F_-]$ with the pole terms. The continuum contribution is a weighted integral over the cross-section difference of the scattering of current divergences from the meson M and represents the renormalization effect. That the continuum term contains the square of matrix elements of $\partial_\mu V_\mu^\pm$, which are first order in symmetry breaking, is the origin of the usual nonrenormalization theorems.

We have found that in the chiral SU(3) ⊗ SU(3) limit, in which the ground-state mesons become massless, there is a class of states contributing to the continuum which is uniquely singular. These states are represented by a Cutkosky diagram and are illustrated in Fig. 1.

The singularity is due to the combination of two effects: (i) The positions of the two meson poles approach $k^2=0$ in the chiral limit, and (ii) the boundary of the physical region of k^2 approaches zero in the chiral limit because $M_a^2\to q^2=0$. If the ground-state meson M_a were replaced by an object of fixed mass $\sqrt{s_1}\neq 0$, then the meson poles would not approach the physical region and there would be no singularity. Hence, we would not expect a singularity if the divergences of vector currents were replaced by the divergences of axial-vector currents, nor would we expect to increase the singularity by considering several meson poles, as in a multiperipheral model.

From the diagram in Fig. 1 we need consider only $|n\rangle=|n';M_a\rangle$, so that

$$\langle n';M_a|\partial_\mu V_\mu^\pm(0)|M\rangle=\sum_{M_b}C_\pm^{M_bM_a}\langle n'|J_{M_b}(0)|M\rangle/(k^2-M_b^2),$$

where $C_\pm^{M_bM_a}=\langle M_a|\partial_\mu V_\mu^\pm(0)|M_b\rangle$ can be parametrized in terms of the known ground-state meson mass splittings. Here $J_{M_b}(0)$ is the meson source. After explicitly extracting factors proportional to SU(3) breaking (ϵ_8^2), we will evaluate the continuum integral in the SU(3) symmetry limit $(\epsilon_8\to 0)$ and then take the SU(3) ⊗ SU(3) limit $(\epsilon_0\to 0)$. As we will see, the continuum integral diverges like $1/\epsilon_0\sim 1/\mu^2$.

After some manipulation one finds for the contribution of the states shown in Fig. 1 to the integral in the sum rule (2)

$$\int_{(3\mu)^2}^{\infty}\frac{ds}{(s-\mu^2)}\sum_{M_b,M_c}B_{M_b,M_c}\int_{(2\mu)^2}^{\infty}\frac{ds_2\,\pi\lambda^{1/2}(s,s_2,\mu^2)}{4s}\int_{-1}^{+1}\frac{dz}{(k^2-\mu^2)^2}$$
$$\times\sum_{n'}\delta^4(k+p-p_{n'})\langle n'|J_{M_c}(0)|M\rangle^*\langle n'|J_{M_b}(0)|M\rangle. \tag{3}$$

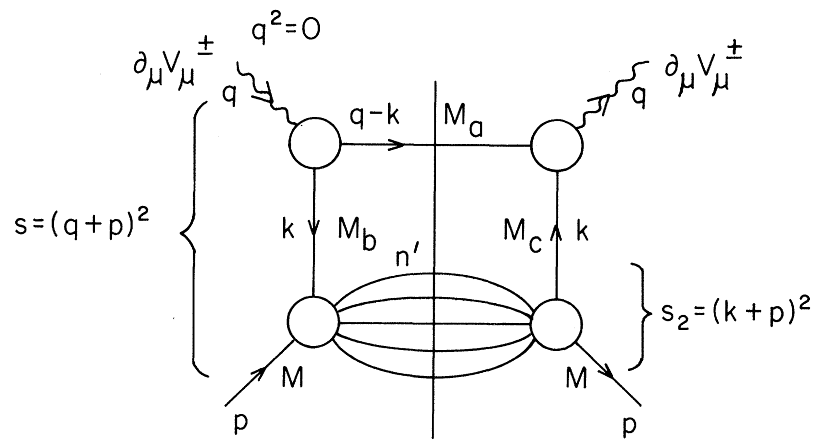

FIG. 1. Cutkosky graph contributing the most singular part of the renormalization in the chiral limit. The lines labeled M represent ground-state mesons which all become massless in the symmetry limit.

The kinematics are illustrated in Fig. 1 and

$$\lambda(s_1, s_2, s_3) = s_1^2 + s_2^2 + s_3^2 - 2s_1s_2 - 2s_1s_3 - 2s_2s_3; \quad s_2 = (k+p)^2, \quad s = (q+p)^2, \quad q^2 = 0, \quad p^2 = \mu^2;$$

$$k^2 = \mu^2 + s_2 - (s+\mu^2)(s+s_2-\mu^2)/2s + (s-\mu^2)\lambda^{1/2}(s, s_2, \mu^2)z/2s.$$

Here $\mu^2 = \frac{1}{3}(2\mu_K^2 + \mu_\pi^2)$ is the average meson mass (having eliminated μ_η^2 by the Gell-Mann–Okubo formula) and

$$B_{M_b M_c} = \sum_{M_a} (C_-^{M_b M_a} C_-^{M_c M_a *} - C_+^{M_b M_a} C_+^{M_c M_a *}).$$

In the above expression we recognize

$$\operatorname{Im} A^{M_b M_c}(s_2, k^2) = \sum_{n'} \delta^4(k+p-p_{n'})\langle n' | J_{M_c}(0) | M\rangle^* \langle n' | J_{M_b}(0) | M\rangle$$

as the absorptive part of the forward, virtual meson–meson scattering amplitude. If we expand this amplitude, $\operatorname{Im} A^{M_b M_c}(s_2, k^2) = \operatorname{Im} A^{M_b M_c}(s_2, 0) + O(k^2)$, we find that the terms of $O(k^2)$ do not contribute to the most singular part of the renormalization (for the same reason we have ignored any k^2 dependence of $C_\pm^{M_b M_a}$). With $\operatorname{Im} A^{M_b M_c}(s_2, k^2) \simeq \operatorname{Im} A^{M_b M_c}(s_2, 0)$ we can explicitly do the z integration in (3) with the result

$$\frac{\pi}{2\mu^2} \int_{(2\mu)^2}^\infty ds_2 \sum_{M_b, M_c} B_{M_b M_c} \operatorname{Im} A^{M_b M_c}(s_2, 0) \int_{(\sqrt{s_2}+\mu)^2}^\infty \frac{ds\, \lambda^{1/2}(s, s_2, \mu^2)}{(s-\mu^2)^4}.$$

The important factor of $1/\mu^2$ has already appeared as a result of the z integration, which is done exactly, over the singular meson propagators $(k^2 - \mu^2)^{-2}$. For the integral over s in the above expression one obtains $\frac{1}{6}(s_2 - \mu^2)^{-2}[1 + O(\mu)]$, so that the final result for the singular piece as $\mu^2 \to 0$ is

$$\frac{\pi}{12\mu^2} \sum_{M_b, M_c} B_{M_b M_c} \int_{(2\mu)^2}^\infty \frac{ds_2 \operatorname{Im} A^{M_b M_c}(s_2, 0)}{(s_2 - \mu^2)^2}.$$

The remaining integral in this expression is precisely the integral in the Adler sum rule for meson-meson scattering,[5] and hence we can evaluate it exactly. It is this feature that permits a direct evaluation of the renormalization effects to first order.

Putting all this together we find for the sum rule (2)

$$(I_3 + \tfrac{3}{2}Y)_{MM} = \sum_{M'} [|F_-^{M\to M'}(0)|^2 - |F_+^{M\to M'}(0)|^2] + \frac{5}{192\pi^2 f_\pi^2} \frac{(\mu_K^2 - \mu_\pi^2)^2}{(2\mu_K^2 + \mu_\pi^2)} (I_3 + \tfrac{3}{2}Y)_{MM} + O\left(\frac{\epsilon_8^2}{\epsilon_0^{1/2}}\right).$$

This result implies that all the meson decay form factors are renormalized from their symmetric values $f_+^0(0)$ according to

$$\frac{f_+(0)}{f_+^0(0)} = 1 - \frac{5(\mu_K^2 - \mu_\pi^2)^2}{384\pi^2 f_\pi^2 (2\mu_K^2 + \mu_\pi^2)} + O\left(\frac{\epsilon_8^2}{\epsilon_0^{1/2}}\right). \tag{4}$$

This is a 2% effect; about the higher-order corrections $O(\epsilon_8^2/\epsilon_0^{1/2})$ we have nothing to say.

A similar procedure yields the leading-order renormalization of the baryon form factors. The major difference between the baryon case and meson case is the presence of baryon-pole terms in the sum on the states n' in the Cutkosky diagram. This pole piece, however, can be explicitly evaluated in terms of the f and d coupling of the pseudoscalar mesons to the baryons. The sum on all other states n' can be again reduced to the Adler-Weisberger integral[5] and explicitly evaluated. Our results are

$$-(\tfrac{2}{3})^{1/2} F_1(\Lambda \to p) = 1 + C[1 + \tfrac{1}{15}g_A^2(45 - 60\alpha + 56\alpha^2)], \quad -F_1(\Sigma^- \to n) = 1 + C[1 + \tfrac{1}{15}g_A^2(45 - 180\alpha + 144\alpha^2)],$$

$$F_1(\Xi^0 \to \Sigma^+) = 1 + C[1 + \tfrac{1}{15}g_A^2(45 - 36\alpha^2)], \quad (\tfrac{2}{3})^{1/2} F_1(\Xi^- \to \Lambda) = 1 + C[1 + \tfrac{1}{15}g_A^2(45 - 120\alpha + 116\alpha^2)]; \tag{5}$$

$$C = -5(\mu_K^2 - \mu_\pi^2)^2 / 384\pi^2 f_\pi^2 (2\mu_K^2 + \mu_\pi^2),$$

and α is related to the f/d ratio of the meson-baryon coupling by $f/d = (1-\alpha)/\alpha$, $\alpha \simeq 0.633 \pm 0.012$.[6]

The corrections to (5) are of $O(\epsilon_8^2/\epsilon_0^{1/2})$. To this order Eqs. (5) imply a sum rule

$$\sqrt{6}\,[F_1(\Lambda \to p) + F_1(\Xi^- \to \Lambda)] + F_1(\Sigma^- \to n) + F_1(\Xi^0 \to \Sigma^+) = 0. \tag{6}$$

Numerically these renormalizations are at most 7%. We find from (5) that $(\frac{2}{3})^{1/2}F_1(\Lambda\to p) = -0.934$, $F_1(\Sigma^-\to n) = -1.000$, $F_1(\Xi^0\to\Sigma^+) = 0.930$, and $(\frac{2}{3})^{1/2}F_1(\Xi^-\to\Lambda) = 0.957$.

The major conclusion of this study is that there is a class of dangerous box Cutkosky diagrams (shown in Fig. 1) which introduce several singular ground-state meson propagators in the chiral-symmetry limit. These diagrams are the only ones that in this application can produce the $1/\mu^2$, or equivalently $1/\epsilon_0$, singularity to destory the naive nonrenormalization theorem. Such effects are, however, not large numerically.

Other applications of these ideas will be presented in a more detailed report.

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