# Study of Scaling in the Inclusive Electroproduction Reactions $e^- + p \to e^- + \pi^{\pm} + X$†

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(Received 15 January 1973)

We report measurements of the inclusive electroproduction reactions $e^- + p \to e^- + \pi^{\pm} + X$. The measurements were made at two data points with the same value of the Bjorken scaling parameter, $\omega = 4.1$, and with $Q^2$ values of 1.20 and 2.02 $\text{GeV}^2$. The virtual-photoproduction cross sections are studied as a function of missing-mass variables and Feynman scaling variables. The results suggest a simple scaling behavior.

We report here measurements of the inclusive electroproduction reactions

$$e^- + p \to e^- + \pi^+ + X^0, \tag{1}$$

$$e^- + p \to e^- + \pi^- + X^{++}, \tag{2}$$

carried out at the Wilson Synchrotron Laboratory at Cornell University. Measurements were made at two points with the same value for the Bjorken scaling parameter $\omega$ in order to study scaling in these inclusive reactions.

Electroproduction can be treated as photoproduction by a virtual photon whose mass $Q^2$, energy $\nu$, direction, and polarization parameter $\epsilon$ are tagged by the detected electron.[1] The cross section for Reactions (1) and (2) is written

$$\frac{d\sigma}{d\Omega_e dE_e d\Omega_\pi^* d(\text{MM})^2} = \Gamma \frac{d\sigma}{d\Omega_\pi^* d(\text{MM})^2},$$

with $\Gamma$ the flux of virtual photons. $d\sigma/d\Omega_\pi^* d(\text{MM})^2$ is the cross section for the production of a pion by a virtual photon; it is a function of $Q^2$, $\nu$, and $\epsilon$, the virtual-photon variables, and the variables describing the produced pion in the virtual-photon–target-proton center-of-mass system, $\theta^*$, $\varphi$, and $(\text{MM})^2$.

Two spectrometers were used to detect the scattered electron and the electroproduced pion. A Cherenkov counter and a lead-acetate shower counter identified electrons. Pions were identified by a threshold gas Cherenkov counter and by time of flight. The data have been corrected for random coincidences, for pion decay, for absorption in the counters, and for target-wall background. No radiative corrections have been applied to the data. The errors quoted here are statistical; there is an additional overall normalization uncertainty estimated to be ±7%. The values of the virtual-photon parameters ($W$, $Q^2$, $\epsilon$, $\omega$) were centered at (2.14 GeV, 1.20 $\text{GeV}^2$, 0.94, 4.10) for datum point $A$ and at (2.66 GeV, 2.02 $\text{GeV}^2$, 0.82, 4.07) for datum point $B$, respectively.

Figure 1 shows plots of the cross section $d\sigma/d\Omega^* d(\text{MM})^2$ for Reaction (1) at data points $A$ and $B$. The cross section has been averaged over a forward-angle bin and all $\varphi$. The similarity in shape

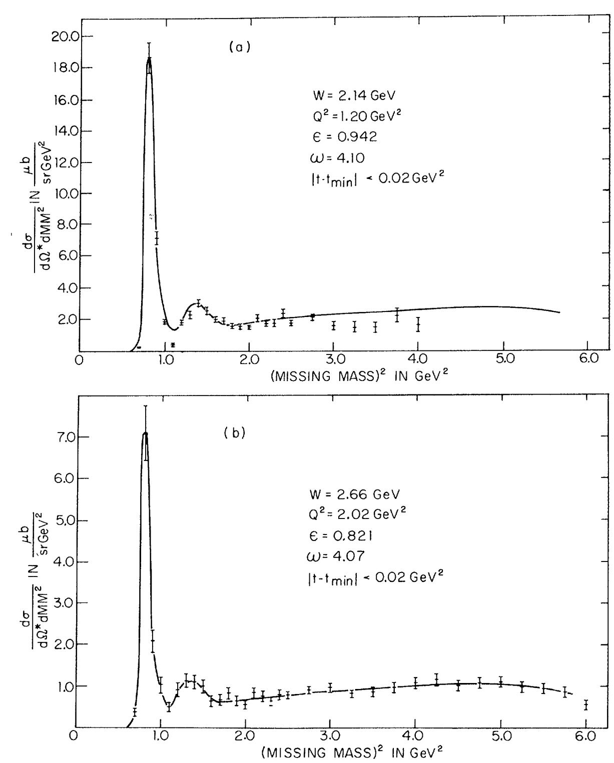


FIG. 1. Virtual-photoproduction cross section for the reaction $\gamma_v + p \to \pi^+ + X^0$ at (a) datum point $A$ and (b) datum point $B$. $t$ is the four-momentum transfer squared from the virtual photon to the produced pion. The curve is described in the text.

of the two spectra is striking. The curve shown in Fig. 1(a) was obtained by multiplying the ordinate of the hand-drawn curve in Fig. 1(b) by the measured ratio of the cross sections at the $e^- + p \to e^- + \pi^+ + n$ peak. A dispersion-theory model for the $\pi^+ n$ reaction shows that the ratio of the cross sections for the $\pi^+ n$ reaction is dominated by the scalar component and is approximately proportional to the square of the pion form factor.[2,3] The data suggest that this may be true for all missing-mass squared less than roughly 2.5 $\mathrm{GeV}^2$.

Another way to present the electroproduction data is in terms of the Feynman scaling variables $x$ and $p_\perp^2$ through the equation[4,5]

$$\frac{d\sigma}{d\Omega_e\, dE_e\, dP_\perp^2\, dx} = \Gamma \frac{d\sigma}{dx\, dP_\perp^2}. \tag{3}$$

Here $P_\perp$ is the transverse momentum of the pion, and $x$ is defined in terms of the c.m. momenta by

$$x = P_\parallel^* / P_{\max}^*, \tag{4}$$

where $P_\parallel^*$ is the component of the momentum along the direction of the virtual photon, and $P_{\max}^*$ is the maximum momentum for the pion. The variable $x$ has the disadvantage that the $x$ corresponding to the large peak for the reaction $e^- + p \to e^- + \pi^+ + n$ changes as $P_\perp^2$ increases and that for $x$ near 1 the range in $P_\perp^2$ is a function of $x$. To avoid this dependence we have chosen instead to use $x'$ where

$$x' = P_\parallel^* / [(P_{\max}^*)^2 - P_\perp^2]^{1/2}. \tag{5}$$

We have analyzed the data in terms of the invariant structure function

$$F(x', P_\perp^2) = \frac{1}{\sigma_{\text{tot}}} \frac{1}{\pi} \frac{E^*}{[(P_{\max}^*)^2 - P_\perp^2]^{1/2}} \frac{d\sigma}{dP_\perp^2\, dx'} = \frac{E}{\sigma_{\text{tot}}} \frac{d^3\sigma}{dP^3}, \tag{6}$$

where $\sigma_{\text{tot}}$ is the total virtual-electroproduction cross section for the $W$ and $Q^2$ of the reaction. $\sigma_{\text{tot}}$ was taken from a fit to measurements by groups at Stanford Linear Accelerator Center and Massachusetts Institute of Technology of $\nu W_2$ with the assumption that $\sigma_S / \sigma_T = Q^2/\nu^2$.[6]

Figure 2 shows the $P_\perp^2$ distribution for two regions of $x'$ at both data points $A$ and $B$. Also given in Fig. 2 are the coefficients $B$ for a fit to the data of the form $\exp(-BP_\perp^2)$. A similar $P_\perp^2$ dependence is observed for the reaction $\gamma_v + p \to \pi^+ + n$ ($x' = 1$); $B = 8.09 \pm 0.65\ \mathrm{GeV}^{-2}$ at datum point $A$ and $B = 6.59 \pm 1.10\ \mathrm{GeV}^{-2}$ at datum point $B$. These data suggest that the $P_\perp^2$ distribution is not a strong function of $x'$ or $Q^2$ for fixed $\omega$.[7]

Figure 3(a) shows the invariant structure function for the $\pi^+$ inclusive reaction with $P_\perp^2 < 0.02$

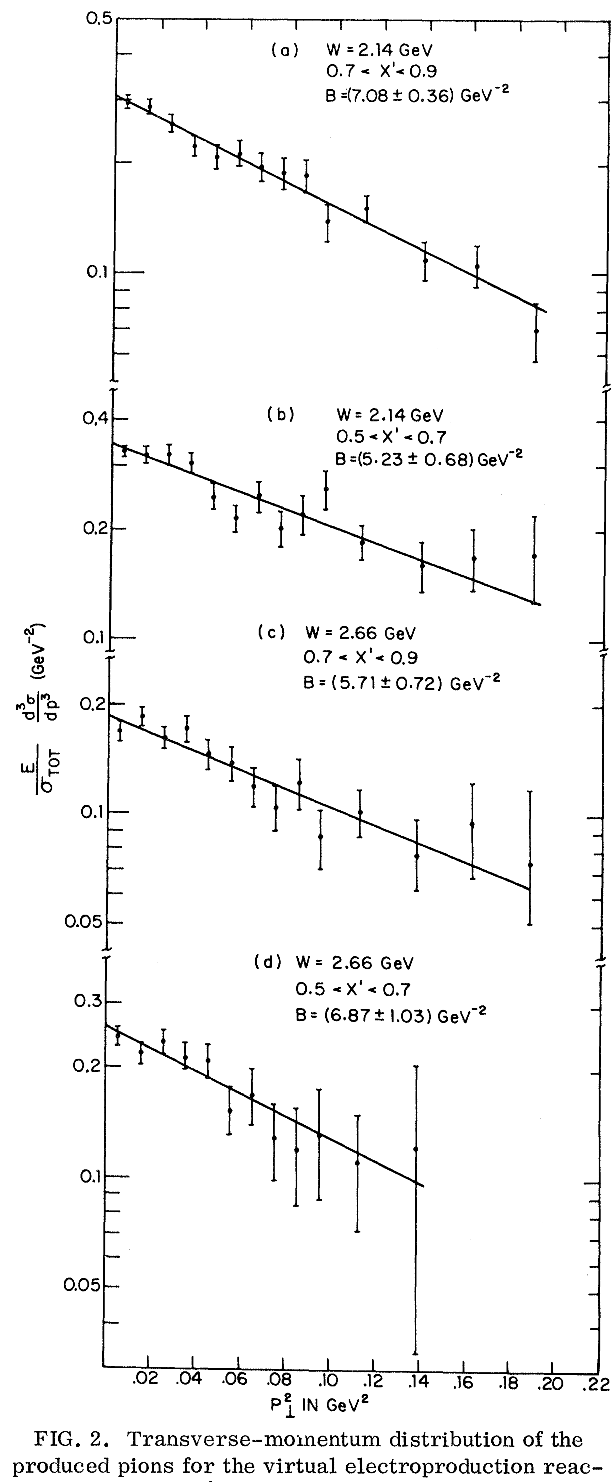


FIG. 2. Transverse-momentum distribution of the produced pions for the virtual electroproduction reaction $\gamma_v + p \to \pi^+ + X^0$ at data points $A$ and $B$ for two regions of longitudinal pion momentum. The lines are fits of the form $A \exp(-BP_\perp^2)$.

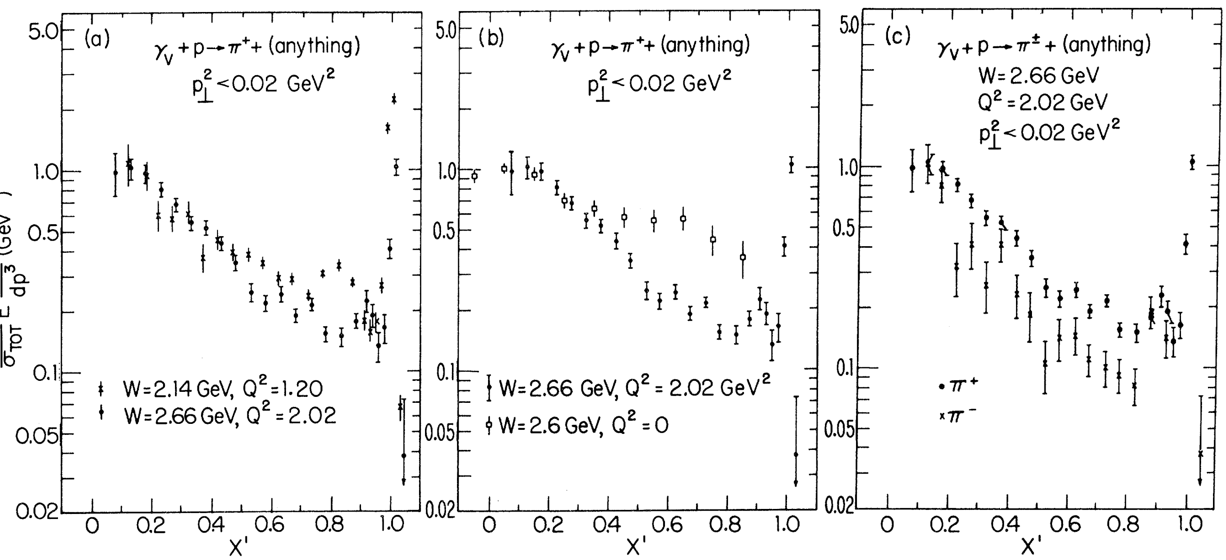


FIG. 3. Longitudinal distribution of the invariant structure function for pions produced by virtual photons. (a) $\pi^+$ at data points *A* and *B*. (b) $\pi^+$ at datum point *B* compared with photoproduction measurements of Struczinski *et al.* (Ref. 8). (c) $\pi^+$ and $\pi^-$ at datum point *B*.

GeV$^2$ at data points *A* and *B*. The invariant structure function is the same at the two data points below the region of prominent two-body channels. This suggests that in this region the data show the same type of scaling as is found in hadron reactions.

Figure 3(b) shows a comparison between the $\pi^+$ invariant structure function for datum point *B* and that for photoproduction with the same c.m. energy *W*.[8] The $\pi^+n$ channel has been excluded from the photoproduction data. The spectra are similar at small $x'$ and depart at large $x'$. A more detailed analysis shows that the removal of $\pi^+$ due to $\rho$ decay in the photoproduction data gives a very similar spectrum to that observed in electroproduction.[9]

Figure 3(c) shows a comparison of the invariant structure functions for the $\pi^+$ and $\pi^-$ reactions at datum point *B*. The $\pi^-$ data appear to be lower than the $\pi^+$ data by roughly a factor of 2 over most of the $x'$ region. This agrees qualitatively with the results of Dakin *et al.*,[10] who measure $h^+/h^-$, where *h* is any detected hadron.

We wish to acknowledge the support of Professor Boyce McDaniel, and the staff of the Wilson Synchrotron Laboratory, and also the staff of the Harvard Cyclotron Laboratory. Professor Nancy Hicks made a significant contribution to the early part of the data analysis.

†Research supported in part by the U. S. Atomic Energy Commission, and in part by the National Science Foundation.

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[4]R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).

[5]It can be shown that in the Bjorken limit scaling in the Feynman variables implies scaling in the variables introduced by S. D. Drell and T.-M. Yan [Phys. Rev. Lett. 24, 855 (1970)]. For a discussion of this point see F. W. Colglazier and F. Ravndal, Phys. Rev. D 7, 1537 (1973). We have chosen to use the Feynman variables because of their wide use in hadron physics.

[6]F. W. Brasse *et al.*, Nucl. Phys. B39, 421 (1972).

[7]E. Lazarus *et al.* [Phys. Rev. Lett. 29, 743 (1972)] find a somewhat larger value for *B* in the region of $x'$ around 0.

[8]W. Struczinski *et al.*, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, 1972 (to be published).

[9]G. Wolf, DESY Report No. 72/61, 1972 (unpublished).

[10]J. T. Dakin *et al.*, Phys. Rev. Lett. 29, 746 (1972).