Shulhof and co-workers[7] have analyzed their results successively with

$$\Omega_- = A + B(q/K_{\parallel})^2$$

and

$$\Omega_- = B'(q/K_{\parallel})^2.$$

In fact the values of the q vector spanned by neutron diffraction experiments is always such that $B(q/K_{\parallel})^2 \gg A$ and it is thus difficult to obtain a good numerical value for A. On the contrary, in acoustic experiments, the wave vector q is so small that $B(q/K_{\parallel})^2 \ll A$. Heller[12] has already proposed an explanation for the two contributions to Ω_-. The first part (A) must be related to the adiabatic longitudinal relaxation. The second part $[B(q/K_{\parallel})^2]$ must be related to the isothermal longitudinal relaxation; its characteristic time $1/\Gamma_{\parallel}$ behaves as $1/q^2$ because in the low-temperature phase the magnetization is coupled with the density of energy which itself easily relaxes over small distances (large q) and follows a diffusion law. Following that interpretation, the relaxation time measured in our experiments corresponds to adiabatic fluctuations of the order parameter. It is a common feature in ultrasonic experiments.

In conclusion, the results presented here appear to be complementary to those obtained from neutron diffraction data; they show that the Landau-Khalatnikov effect may also operate in solids.

*Associated with the Centre National de la Recherche Scientifique.

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$E2$ Transition Systematics in the $1f$-$2p$ Shell: A Constraint on the Effective Interaction

Satish Sharma and Kumar Bhatt
Physical Research Laboratory, Ahmedabad-9, India

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We discuss a constraint put by the $E2$ transition systematics in the $1f$-$2p$ shell on the effective two-body interaction in the shell. Some simple modifications in the Kuo-Brown effective interaction are made which tend to make it satisfy this constraint.

For a nucleus in the first half of a shell, not suffering from excessive neutron excess, the low-energy states would to a large extent be projectable from a deformed intrinsic state. In this case the magnitudes of the $E2$ transitions between such states tend to be proportional to the magnitude of the electric quadrupole moment of the intrinsic state; hence the magnitudes of the experimental values of the quadrupole transition rates can provide some information on the quadrupole moment of the intrinsic state of the nucleus.

The deformed Hartree-Fock (HF) or Hartree-Fock-Bogoliubov (HFB) state of the nucleus generated by the effective interaction provides a reasonable intrinsic state of the nucleus. The effective interaction should therefore be expected to give rise to intrinsic states for different nuclei such that the quadrupole moments of these have, as a function of the neutron and proton number, a trend similar to that implied by the experimen-

TABLE I. Experimental $B(E2)$ $(0^+ \rightarrow 2^+)$ values and the calculated mass quadrupole moments using different effective interactions for Ti, Cr, and Fe isotopes.

		$B(E2)$ $(0 \rightarrow 2)$ [a]	Mass quadrupole moments [b]				
Nucleus	A	Expt	KB	MWH_1	MWH_2	KB_1	H
Ti	48	7.0 ± 1.4	23.9	19.9	19.1	17.9	20.51
	50	3.2 ± 0.8	28.5	−10.7	−9.8	−9.6	10.93
	52	···	20.8	21.1	18.7	18.9	···
Cr	50	12 ± 0.8	26.4	24.3	23.6	21.9	24.30
	52	6.7 ± 0.7	36.2	35.3	13.0	11.8	13.67
	54	10 ± 0.7	29.1	27.1	25.5	25.0	25.98
Fe	52	···	28.6	21.2	20.6	19.0	20.84
	54	5.1 ± 0.5	39.9	42.4	10.6	9.6	10.92
	56	9.0 ± 1	28.4	25.5	23.5	23.3	24.46

[a] The $B(E2)$ values are in units of $10^{-50} e^2$ cm^4.
[b] The quadrupole moments are in units of the harmonic-oscillator parameter $b^2 = \hbar/M\omega$.

tal $E2$ transition rates.

The most dramatic feature of the experimental $E2$ transition systematics in the $1f$-$2p$ shell[1] is the drop in the known $B(E2)$ $(0 \rightarrow 2)$ values when N or $Z = 28$. As shown in Table I, the $B(E2)$ $(0 \rightarrow 2)$ values for the nuclei with $N = 26$ and 30 are significantly larger than those for the $N = 28$ nuclei. This implies that the intrinsic quadrupole moment for the $N = 28$ isotope must be *smaller* than that for the $N = 26$ or 30 isotope.

In contrast, the most consistent feature of the deformed HF and HFB calculations[2] in the $1f$-$2p$ shell carried out with different interactions—except the one used by Chandra[3]—is that the lowest-energy HF or HFB intrinsic states obtained from various interactions have large deformation. Hence, the intrinsic quadrupole moment for the $N = 28$ isotope is significantly *larger* than those for the $N = 26$ and 30 isotopes. In all the calculations we discuss, the one-body part of the Hamiltonian is taken from the Ca^{41} spectrum.

In view of the reliability of the experimental $B(E2)$ data, this clear contradiction suggests that we should modify the effective interactions most commonly used.

The reason for the anticorrelation between the "effective-interaction-generated" and the "experimentally implied" intrinsic quadrupole moment trends can be understood by looking at the sketch [Fig. 1(a)] of the sequence of Nilsson-like levels

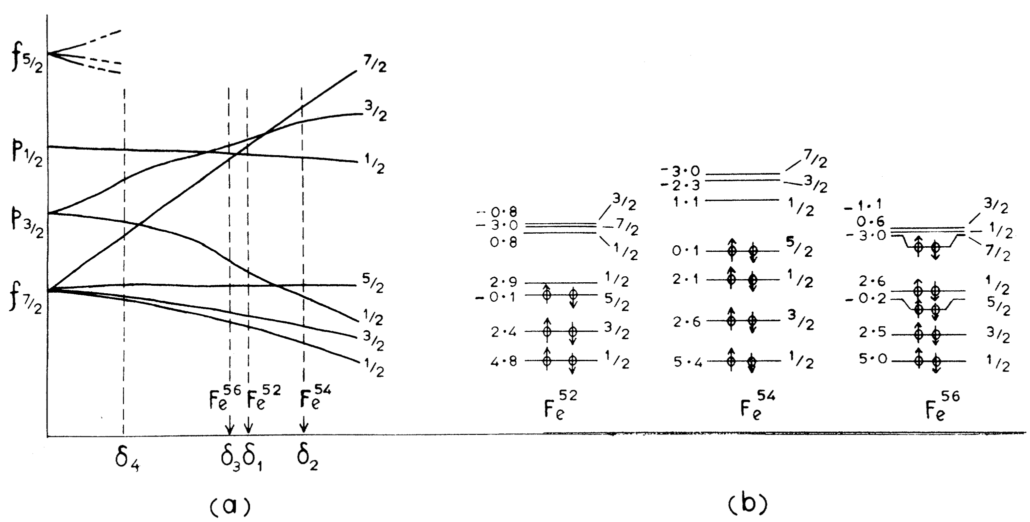

FIG. 1. Comparison of the deformations of the HF states of the $N = 26$, 28, and 30 isotopes of Fe. In (b), for each of the HF states, the quadrupole moments $\langle q_0^2 \rangle$ of the HF orbits are given on the left in units of $b^2 = \hbar/M\omega$. The values of $k = \langle j_z \rangle$ are given on the right.

as a function of the deformation δ. Figure 1(b) shows the sequence of the occupied and some of the unoccupied neutron orbits in the HF states of the nuclei Fe^{52}, Fe^{54}, and Fe^{56} as obtained by using the Kuo-Brown (KB) effective interaction.[4] A comparison of Figs. 1(a) and 1(b) shows that for $N=26$ the deformation is like δ_1, for $N=28$ it is larger and is like δ_2, while for $N=30$ it is like δ_3. Note that in Fe^{52} the first unoccupied orbit is the second $k=\frac{1}{2}$ orbit with a rather large prolate quadrupole moment. It is occupied by the two additional neutrons in Fe^{54}. Hence, the quadrupole moment of the Fe^{54} HF state is larger than that for Fe^{52}. Similarly, the orbits occupied by the two additional neutrons in Fe^{56} have negative quadrupole moments and, hence, the quadrupole moment of the HF state of Fe^{56} is smaller than that of Fe^{54}.

It is clear that if the interaction is such that it gives rise to the HF states with deformations like δ_4 for $N=28$, the second $k=\frac{1}{2}$ orbit with a positive quadrupole moment will be the fifth orbit to be occupied. In this case the HF state with $N=30$ will have a larger quadrupole moment than that for the $N=28$ state. Also, since in the HF state corresponding to $N=26$, the $k=\frac{7}{2}$ orbit with a large negative quadrupole moment ($\epsilon=-3b^2$) is the first unoccupied orbit, its quadrupole moment will be larger than that for the $N=28$ HF state.

In fact, in a previous analysis[5] of the $B(E2)$ systematics, in terms of the underlying HF single-particle structure, it was suggested that, to be consistent with the experimental $B(E2)$ $(0\rightarrow 2)$ systematics, the *proper* intrinsic states for the nuclei should be such that the second $k=\frac{1}{2}$ orbit with a larger prolate quadrupole moment is occupied only after the orbits with $k=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ are filled.

We now turn this suggestion into a criterion for modifying the Kuo-Brown effective interaction. The effective interaction should be modified in such a way that the second $k=\frac{1}{2}$ orbit is the fifth orbit to be occupied; consequently, the deformation of the HF state for $N=28$ which the modified interaction gives rise to is smaller than that given by the original interaction. The modification should not give rise to harmful side effects.

The decrease in the deformation required to bring the $k=\frac{7}{2}$ orbit below the second $k=\frac{1}{2}$ orbit can be brought about if either of the following obtains:

(i) The center of gravity of the interaction between the two $1f_{7/2}$ nucleons is made more attractive. This attraction would tend to increase the $(f_{7/2})^n$ component in the intrinsic state and thus reduce its deformation.

(ii) The center of gravity of the interaction between an $f_{7/2}$ nucleon and a nucleon in a shell $j\neq\frac{7}{2}$ is made more repulsive.[6] Since the $f_{7/2}$ single-particle state is lowest in energy, the majority of the nucleons are in the $f_{7/2}$ state. Hence an increase in the repulsion mentioned above will tend to increase the $(f_{7/2})^n$ component and again reduce the deformation.

It is interesting that in their shell-model study of calcium isotopes, McGrory, Wildenthal, and Halbert (MWH)[7] also found it necessary to modify in a similar way the $T=1$ Kuo-Brown interaction *to obtain the best fit to the experimentally observed* energy spectra. To be specific, they considered the following two different modifications of the $T=1$ Kuo-Brown matrix elements.

Modification 1.—The $\langle f_{7/2}^2|V|f_{7/2}^2\rangle$, $J=0$ and 2 matrix elements were made more attractive by about 300 keV, and the $\langle f_{7/2}p_{3/2}|V|f_{7/2}p_{3/2}\rangle$ center of gravity was raised by about 300 keV.

Modification 2.—In addition to the above changes, every $\langle f_{7/2}j|V|f_{7/2}j\rangle$ matrix element, where $j=1f_{5/2}$ and $2p_{1/2}$, was also raised by 250 keV.

We shall refer to these two modified Kuo-Brown interactions of McGrory, Wildenthal, and Halbert as MWH_1 and MWH_2 interactions. Note that in both the MWH_1 and MWH_2 interactions only the *$T=1$ component* of the Kuo-Brown interaction was modified.

In Table I, column 3, we have listed the *mass quadrupole moments* of the lowest-energy HF states of the $N=26$, 28, and 30 isotopes, generated by the Kuo-Brown interaction.[8] Note that in this case the quadrupole moment of the $N=28$ isotope is *larger* than those for the $N=26$ and 30 isotopes.

The results for the interactions MWH_1 and MWH_2 are given in columns 4 and 5 of Table I. The interaction MWH_1 which gives quite a reasonable fit to the spectra of the Ca isotopes also succeeds in giving the right trend for the magnitudes of the quadrupole moments of the Ti isotopes. However, for the Cr and Fe nuclei, the quadrupole moments for the $N=28$ isotopes are still the largest. The interaction MWH_2, with increased repulsion between the $f_{7/2}$ and the other shells, does give HF states which show quadrupole-moment trends qualitatively consistent with the experimental $B(E2)$ systematics. Note, however, that the HF state of Ti^{50} as given by these interactions is oblate. The energy of the prolate state is about 400 keV higher than the oblate state, and the quadrupole moment is between $8.0b^2$ and $9.5b^2$.

In addition to the MWH interactions, we have also considered a number of other choices for the modifications (i) and (ii) of the Kuo-Brown interaction. In column 6 of Table I we give the result for an interaction labeled KB_1. In this interaction the center of gravity of the entire set ($T=1$ as well as $T=0$) of the $\langle f_{7/2}{}^2 | V | f_{7/2}{}^2 \rangle$ Kuo-Brown matrix elements was lowered by 200 keV by making all the matrix elements more attractive by 200 keV; that of the $\langle f_{7/2} p_{3/2} | V | f_{7/2} p_{3/2} \rangle$ matrix elements was raised by 200 keV in a similar way. All the other matrix elements were left unchanged. This interaction also gives the proper trend for the intrinsic quadrupole moments.

In column 7, labeled H, we have quoted the quadrupole moments of the HF states of these nuclei as given by Chandra. His interaction also gives the proper trend for the quadrupole moments. Note also that for Ti^{50} the HF state is prolate, but the oblate state is only 130 keV higher than the prolate state.

Since enhanced quadrupole collectivity is the dominant feature of low-lying states of nuclei, it appears that among the different interactions which give the dip in the intrinsic quadrupole moment at $N=28$, the best one is likely to be that which gives rise to the maximum quadrupole moment for all the nuclei. According to such a criterion, it appears that the interaction MWH_2 is the best of the various modified Kuo-Brown interactions we have examined. The Chandra interaction seems from this point of view to be somewhat better than the MWH_2 interaction. It would be interesting to use this interaction in a shell-model calculation to examine its spectral properties in some detail.

It seems difficult to find a good interaction which gives larger intrinsic quadrupole moments than the MWH_2 interaction. The difficulty is that for an interaction giving large quadrupole moments, the deformation for the $N=28$ isotope becomes large enough for the second $k=\frac{1}{2}$ orbit to be occupied before the $k=\frac{7}{2}$ orbit, thereby making the intrinsic quadrupole moment for the $N=28$ nuclei larger than those for the $N=26$ or 30 nuclei. Thus, the MWH_2 interaction is optimum in the sense of giving the largest possible quadrupole moment, while keeping the $k=\frac{7}{2}$ orbit below the $k=\frac{1}{2}$ orbit.

We would like to mention that we have also considered HFB rather than the HF intrinsic states for these nuclei. It has not been possible, by making small changes of the type mentioned above in the Kuo-Brown interaction, to obtain an interaction which gives HFB intrinsic states having "proper" quadrupole-moment trends for all the nuclei considered. The details of our HFB calculations shall be reported elsewhere.

We have tried here to indicate that the tendency for the $1f_{7/2}$ subshell closure at $N=28$, which is best reflected in the experimental $B(E2)$ systematics, could serve as a useful and sensitive constraint for obtaining an improved effective interaction in the $1f$-$2p$ shell.

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