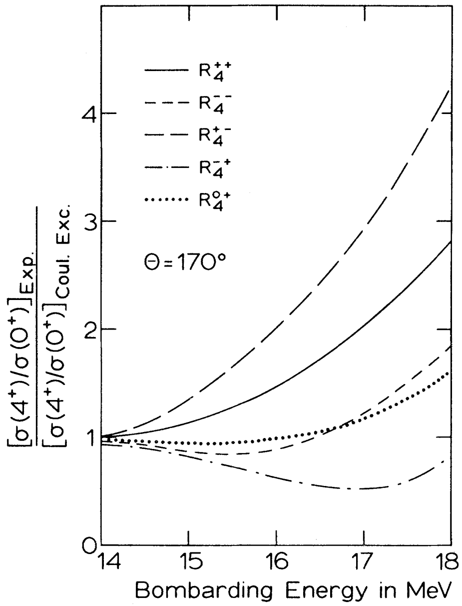

FIG. 2. Calculated excitation probabilities of the 4^+ level for various charge and optical-model deformations (see text). The results are normalized to the expectations of the pure Coulomb-excitation theory.

charge and optical-model deformation parameters. The pure Coulomb-excitation measurements are almost insensitive to the sign of β_4^C as indicated by the convergence of the various R_4 functions at 14 MeV.

The β_2^N and β_4^N obtained in our measurement together with the results of Ref. 1 are given in Table II. These values are different from β_2^C and β_4^C since the radius parameters are unequal. However, the products $\beta_\lambda R$ should be equal[1] if the two sets of data are consistent. Using this criterion, the nuclear deformation parameters were calculated to be $\beta_2^N = 0.279 \pm 0.009$ and $\beta_4^N = 0.050 \pm 0.009$ for a radius parameter $R = 6.093$ fm. These values are within the uncertainties in agreement with β_2^C and β_4^C.

*On leave from the Centre de Recherches Nucléaires, Strasbourg, France.

†On leave from the Weizman Institute for Science, Rehovot, Israel.

[1]D. L. Hendrie *et al.*, Phys. Lett. 26B, 127 (1968).

[2]A. A. Aponick *et al.*, Nucl. Phys. 159A, 267 (1970).

[3]F. S. Stephens *et al.*, Phys. Rev. Lett. 27, 1151 (1971).

[4]T. K. Saylor *et al.*, to be published.

[5]W. Bertozzi *et al.*, Phys. Rev. Lett. 28, 1711 (1972).

[6]W. Brückner *et al.*, in Proceedings of the Osaka Conference on Nuclear Moments, 1972 (to be published).

[7]G. Jansen, Ph. D. thesis, Heidelberg University, 1969 (unpublished).

[8]P. Winkler, Nucl. Phys. 168A, 139 (1971).

[9]K. Alder *et al.*, Phys. Lett. 32B, 645 (1970).

[10]A. Winther and J. de Boer, in *Coulomb Excitation*, edited by K. Alder and A. Winther (Academic, New York, 1966).

[11]S. Devons and I. Duerdoth, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 2.

[12]J. Raynal, Ph. D. thesis, Centre d'Etudes Nucléaires de Saclay Report No. DOC/68-3/DD (unpublished).

[13]M. Samuel *et al.*, Comput. Phys. Commun. 2, 455 (1971).

[14]E. M. Bernstein *et al.*, Phys. Rev. 121, 841 (1961).

Higher-Order Correlation Properties of a Laser Beam*

S. Chopra and L. Mandel
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 9 November 1972)

The intrinsic third-order intensity correlation function of a laser beam has been measured by a digital correlation technique, for a He:Ne laser operating near threshold. The results are compared with some recent calculations of Cantrell, Lax, and Smith and are found to be in good agreement.

In recent years there has been considerable interest in the correlation properties of laser light, particularly when the laser is operating near its threshold of oscillation. However, with one exception,[1] all correlation measurements have been confined to the second-order intensity correlation function, and the higher-order correlation properties have received little or no attention.[2] The reason is not hard to find. Correlation experiments of the third or higher order involve measurements of the arrival times of three or more photons, and require an order of magnitude more electronic data storage and analysis than the corresponding second-order problem. At the same time great measurement accuracy is needed if the intrinsic higher-order correlations are not to be masked by lower-order ones. The theoretical problem of calculating the higher-order correlations is correspondingly more formidable.

Recently Cantrell, Lax, and Smith[3] succeeded in computing the intrinsic higher-order correlations of the laser field near threshold, on the basis of the nonlinear oscillator model of the laser,[4,5] in which the optical field is taken to obey a Langevin equation of motion driven by Markoffian random forces. Experimental confirmation of the results of these calculations provides a test of the laser model that is significantly more searching than previous tests.[6]

We have recently developed a new technique[7] for making higher-order correlation measurements that is considerably more accurate than previous methods, such as those of Ref. 1. With the help of this technique we have succeeded in determining the third-order intrinsic intensity correlation function of a laser beam, at different working points of the laser, corresponding to values of the pump parameter a of 0, -1, and -2. The results constitute the first direct verification of the higher-order predictions of the nonlinear oscillator theory of the laser.

From the Fokker-Planck equation for the laser field in the steady state it follows that the joint probability density $p_3(V_1,t;\,V_2,t+\tau;\,V_3,t+\tau')$ for the field to have complex amplitudes V_1 at time t, V_2 at time $t+\tau$, and V_3 at time $t+\tau'$ is given by

$$p_3(V_1,t;V_2,t+\tau,V_3,t+\tau')=p_1(V_1)G(V_2,V_1,\tau)G(V_3,V_2,\tau'-\tau), \tag{1}$$

where $p_1(V)$ is the probability density for the field at any time, and $G(V_1,V_2,\tau)$ is Green's function for the process. The latter function can be expanded in terms of the eigenvalues and eigenfunctions of a certain one-dimensional Schrödinger equation,[4,5] and then evaluated. Once $p_3(V_1,t;V_2,t+\tau;V_3,t+\tau')$ is known, the third-order intensity correlation function follows from the integral

$$\langle I(t)I(t+\tau)I(T+\tau')\rangle=\int|V_1|^2|V_2|^2|V_3|^2p_3(V_1,t;V_2,t+\tau;V_3,t+\tau')\,d^2V_1\,d^2V_2\,d^2V_3. \tag{2}$$

In order to measure $\langle I(t)I(t+\tau)I(t+\tau')\rangle$ for the laser beam, we make use of the fact that when the beam falls on a photodetector, the joint threefold probability $P_3(t,t+\tau,t+\tau')\,dt\,d\tau\,d\tau'$ of photoelectric emissions at times t, $t+\tau$, and $t+\tau'$, within dt, $d\tau$, and $d\tau'$, respectively, is given by[8-11]

$$P_3(t,t+\tau,t+\tau')\,dt\,d\tau\,d\tau'=(\alpha cS)^3\langle I(t)I(t+\tau)I(t+\tau')\rangle\,dt\,d\tau\,d\tau', \tag{3}$$

where α is the quantum efficiency of the detector and S its illuminated surface area. It is clear, therefore, that by repeatedly recording the pair of time intervals τ,τ', and making a two-dimensional histogram of the observed values, we can obtain the third-order intensity correlation function. In practice, not all the observed correlations of arrival times are due to intrinsic third-order processes. If we introduce the normalized correlation functions defined by

$$\lambda^{(2)}(\tau)\equiv\langle\Delta I(t)\Delta I(t+\tau)\rangle/\langle I\rangle^2,\quad \lambda^{(3)}(\tau,\tau')\equiv\langle\Delta I(t)\Delta I(t+\tau)\Delta I(t+\tau')\rangle/\langle I\rangle^3, \tag{4}$$

we can express Eq. (3) in the form

$$P_3(t,t+\tau,t+\tau')=(\alpha cS\langle I\rangle)^3[1+\lambda^{(2)}(\tau)+\lambda^{(2)}(\tau')+\lambda^{(2)}(\tau'-\tau)+\lambda^{(3)}(\tau,\tau')], \tag{5}$$

which shows that $\lambda^{(3)}(\tau,\tau')$ can be derived from measurements of $P_3(t,t+\tau,t+\tau')$ only after the effects of second-order correlations have been subtracted out.

The apparatus used for the experiments consists of a He:Ne laser operated at 6328 Å controlled by a photoelectric feedback arrangement that allows it to be operated at any working point below, at, or above threshold.[12] The laser beam is filtered and passed to a beam splitter, after which the two beams are detected by two photomultiplier tubes. After amplification and pulse shaping the photoelectric pulses are passed to the inputs 1 and 2 of the correlator.

The appearance of a pulse at input 1 at time t, the start pulse, initiates a counting sequence, such that the arrival times $t+\tau$, $t+\tau'$, $t+\tau''$, etc. of subsequent pulses at input 2 are recorded. In effect the pulse at input 1 starts a clock, whose output pulses are counted by a set of binary-coded decimal counters, each of which is stopped in turn when pulses arrive at input 2. The counters therefore provide a measure of the time intervals $\tau,\tau',\tau'',\ldots$ in digital form. The two-dimensional array of pairs of numbers τ,τ'; τ,τ''; τ',τ''; etc., is then mapped onto a one-dimensional array, which is stored in the memory of a multichannel analyzer. Thus, the combination 3,7, for example, is stored as the single

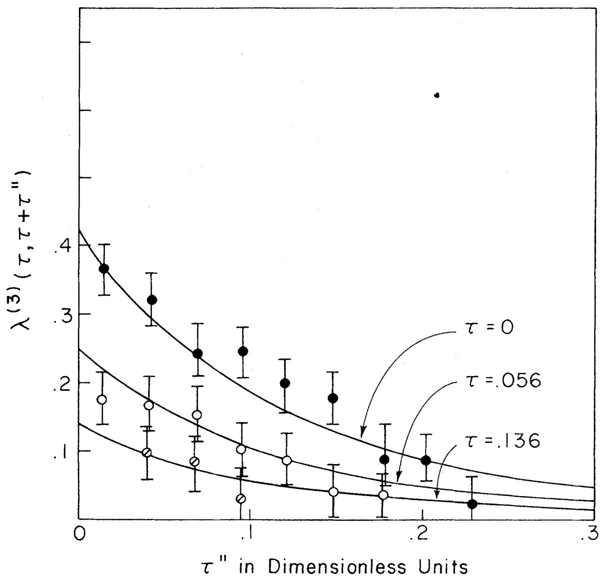

FIG. 1. Results of measurements of $\lambda^{(3)}(\tau, \tau+\tau'')$ together with their standard deviations, for the laser operating at threshold ($a=0$). The solid curves were calculated by Cantrell, Lax, and Smith (Ref. 3).

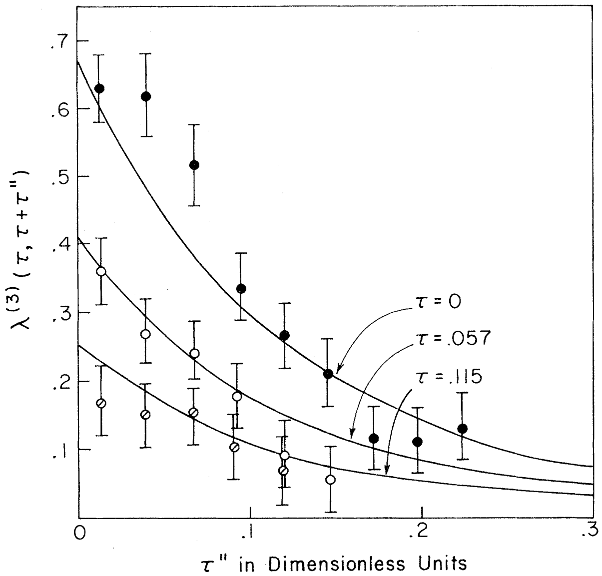

FIG. 2. Same as Fig. 1, but with $a=-1$.

number 73. The counters are then cleared and the sequence is ready to be initiated by another start pulse. The apparatus is described more fully elsewhere.[7] In practice the time intervals were registered in multiples of 5 μsec, which is just about adequate for the correlation times of the laser involved.

When the dark counting rates r_1 and r_2 of the photodetectors are taken into account, it may be shown[7] that the expected number of events $n(\tau,\tau')$ registered in the channel corresponding to a pair of time intervals τ,τ' is given by

$$\langle n(\tau,\tau')\rangle = n_s(R_2\Delta\tau)^2[1+\theta_1\theta_2\lambda^{(2)}(\tau)+\theta_1\theta_2\lambda^{(2)}(\tau')+\theta_2^2\lambda^{(2)}(\tau'-\tau)+\theta_1\theta_2^2\lambda^{(3)}(\tau,\tau')]. \tag{6}$$

Here n_s is the number of times the counting sequence is initiated, $\Delta\tau$ is the digitizing time interval, R_1 and R_2 are the mean counting rates in the two channels, and

$$\theta_1 = (R_1 - r_1)/R_1, \quad \theta_2 = (R_2 - r_2)/R_2. \tag{7}$$

Some limitations on the counting rates need to be observed with this equipment,[7] but they are far less severe than the limitations imposed by the use of time-to-amplitude converters.[13]

In order to make contact with the computed results of Cantrell, Lax, and Smith,[3] who expressed all times in dimensionless form, as is customary, it was necessary first to measure the intensity correlation time T_c of the laser at threshold ($a=0$). As usual, the laser threshold is identified by the fact that the value of the normalized second-order correlation function $\lambda^{(2)}(0)$ at zero time delay is $\pi/2-1=0.571$ at threshold.[4,5] With the laser set to work at threshold, the correlation time $T_c\,(a=0)$ was then determined from the measured values of $\lambda^{(2)}(\tau)$. This provided the required time scaling parameter, according to the relation[4,5]

$$\text{normalized time} = (\text{measured time})\times 0.171/T_c(a=0). \tag{8}$$

The value of the pump parameter a for a different setting of the laser was identified from the mean counting rates of the detectors via the relation[4,5]

$$\frac{\langle I(a)\rangle}{\langle I(0)\rangle} = \frac{\sqrt{\pi}\,a}{2} + \frac{\exp(-a^2/4)}{1+\Phi(\frac{1}{2}a)}, \tag{9}$$

where $\Phi(x)$ is the Gaussian error integral.

The results of the measurements are presented in Figs. 1, 2, and 3, together with the theoretically predicted values calculated by Cantrell, Lax, and Smith.[3] The curves are plots of the

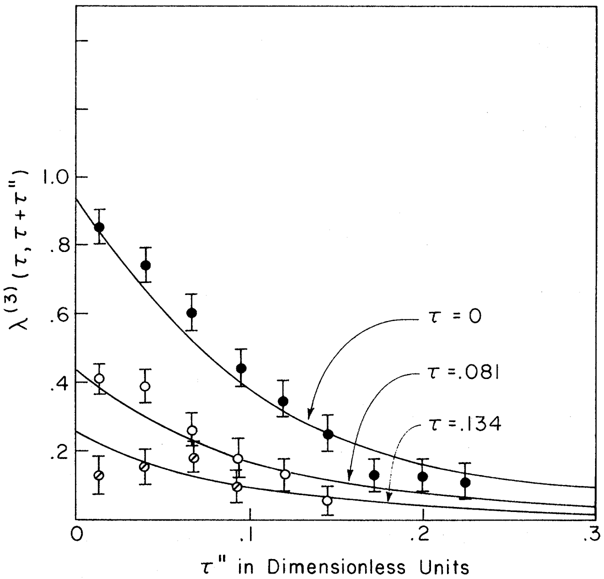

FIG. 3. Same as Figs. 1 and 2, but with $a = -2$.

function $\lambda^{(3)}(\tau, \tau+\tau'')$ against τ'', for several different values of τ. In practice $\langle n(\tau,\tau')\rangle$ was measured to an accuracy of about 1%, and the much larger statistical uncertainties of $\lambda^{(3)}(\tau,\tau')$ are the result of the multiple subtractions that are required in order to obtain $\lambda^{(3)}(\tau,\tau')$ from Eq. (6). Theoretical values of $\lambda^{(2)}(\tau)$ were used in Eq. (6), but the validity of the predicted form[4,5] of $\lambda^{(2)}(\tau)$ has been confirmed in separate experiments.[14]

Within the statistical uncertainties the experimental and theoretical values are in good agreement, and the higher-order p"edictions of the nonlinear oscillator theory of the laser are well confirmed.

*Work supported by the Air Force Office of Scientific Research and the National Science Foundation.

[1]F. Davidson and L. Mandel, Phys. Lett. 27A, 579 (1968).

[2]In principle, measurements of the photoelectric counting probabilities or moments, of the kind reported by R. F. Chang, V. Korenman, C. O. Alley, Jr., and R. W. Detenbeck [Phys. Rev. 178, 612 (1969)], and D. Meltzer, W. Davis, and L. Mandel [Appl. Phys. Lett. 17, 242 (1970)], also give some information on the higher-order correlation properties, but not the form of the correlation function.

[3]C. D. Cantrell, M. Lax, and W. A. Smith, in Proceedings of the Conference on Coherence and Quantum Optics, Rochester, New York, June 1972 (Pergamon, New York, to be published), and Phys. Rev. A (to be published); see also C. D. Cantrell and W. A. Smith, Phys. Rev. 37A, 167 (1971).

[4]H. Risken and H. D. Vollmer, Z. Phys. 204, 240 (1967).

[5]R. D. Hempstead and M. Lax, Phys. Rev. 161, 350 (1967).

[6]The experimental results obtained in Ref. 1 were slightly at variance with those predicted by Cantrell and Smith (Ref. 3).

[7]S. Chopra and L. Mandel, to be published.

[8]L. Mandel, Proc. Phys. Soc., London 72, 1037 (1958), and in *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1963), Vol. II, 181.

[9]R. J. Glauber, Phys. Rev. 130, 2529 (1963), and 131 2766 (1963).

[10]L. Mandel, E. C. G. Sudarshan, and E. Wolf, Proc. Phys. Soc., London 84, 435 (1964).

[11]P. L. Kelley and W. H. Kleiner, Phys. Rev. 136, A316 (1964).

[12]The arrangement has been described more fully by D. Meltzer and L. Mandel, IEEE J. Quantum Electron. 6, 661 (1970).

[13]F. Davidson and L. Mandel, J. Appl. Phys. 39, 62 (1968); F. Davidson, Phys. Rev. 185, 446 (1969).

[14]S. Chopra and L. Mandel, in Proceedings of the Conference on Coherence and Quantum Optics, Rochester, New York, June 1972 (Pergamon, New York, to be published), and IEEE J. Quantum Electron. 8, 324 (1972).