scattered into the crescents. Presumably this tends to straighten out the layers. McMillan[9] observed an increase in layer spacing in a smectic A with decrease in temperature. For fixed slide spacing, a decrease in temperature would thus produce a compressive stress, and we do observe a decrease of scattered intensity on cooling our samples. On the other hand, stresses of opposite sign, such as allowing the slides to move apart, are always accompanied by a dramatic increase in the scattered intensity and, therefore, distortion of the layers. Both reversible and irreversible distortions upon expansion have been observed. For sufficiently weak and slow expansion, the excess scattered intensity disappears when the expansion is stopped. However, continued alternate squeezing and expansion of the sample always results in a gradually increasing level of static scattering, which, microscopic study shows, is associated with enhanced static surface defects. These observations suggest the possibility of acoustic detection by expansion-induced deformations and surface defects in the smectic and cholesteric plane texture.

We would like to acknowledge the continued interest and assistance of R. Meyer and of Z. Muljiani and D. Dolphin who prepared the compounds studied here. We have profited from discussions with G. Durand concerning this effect.[10]

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Magnetic-Field–Induced One-Dimensional Behavior in the Specific-Heat Transition in Dirty Bulk Superconductors

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The specific heat is calculated in the transition region for an extreme type-II bulk superconductor [zero-temperature Ginzburg-Landau (GL) coherence length $\xi(0)\approx 48$ Å, dirtiness parameter $\xi_0/l \approx 120$, GL parameter $\kappa \approx 67$] for various values of applied magnetic field H, and compared with previously published data. Nonlocality is neglected and the quartic term in the GL free-energy functional is treated in a Hartree approximation. The calculated and measured transitions broaden with increasing H, the latter in a manner suggestive of an approach to the theoretically predicted one-dimensional form.

There has recently been both theoretical[1] and experimental[2] interest in the question of a possible magnetic-field–induced reduction of effective dimensionality in superconductors. As noted by Lee and Shenoy,[3] application of a magnetic field to a bulk superconductor can lead to effective one-dimensional behavior in the fluctuation specific heat near the critical temperature $T_{c2}(H)$. In the presence of the field the order parameter is expanded in Landau orbitals (instead of plane waves) indexed by "quantum" numbers n and Q_z. Near the transition the $n=0$ term dominates the fluctuation contribution to the thermodynamic potential, leaving only Q_z to be summed, as in a one-dimensional (1D) system. The specific-heat calculation of Ref. 3, however, neglected the quartic term, describing fluctuation interactions, in the Ginzburg-Landau free-energy functional $F[\psi]$. The resulting expression therefore diverged near the critical temperature, preventing comparison of theory and experiment in the vicinity of the critical region. The simple theory presented here extends the previous calculation[3] into the critical region by treating the quartic term in $F[\psi]$ in a self-consistent Hartree approximation investigated by Marčelja[4] and others.[5-7] (The alternative "Hartree-Fock" approximation[8] leads to difficulties[6] in the specific heat and was therefore rejected.) The calculated curves are then compared with previously published data of Barnes and Hake.[9,10] (We note that in zero magnetic field, the quartic term can be treated exactly for a one-dimensional system.[11] Our concern here, however, is with the influence of the field in broadening the transition in a bulk superconductor; that is, with three-dimensional systems which "look one-dimensional" in a neighborhood of the upper critical field curve.)

Two recent papers[12,13] have emphasized that in the presence of strong magnetic fields, higher orders in $\vec{p}-2e\vec{A}/c$ must be included in $F[\psi]$, in contrast to the usual expansion to second order. However, it was also found[12,13] that such nonlocality effects are unimportant in very dirty superconductors, it being suggested in Ref. 13 that nonlocality effects are measured by $h/(1+\xi_0/l)^2$, where[3]

$$h=\xi(0)^2(2eH/\hbar c)=H[-T_{c0}(\partial H_{c2}/\partial T)_{T=T_{c0}}]^{-1}, \tag{1}$$

in which ξ_0 is the BCS coherence length, l is the mean free path, $\xi(0)$ is the GL zero-temperature coherence length, T_{c0} is the zero-H transition temperature, and H_{c2} is the upper critical field. For the system considered here,[9,10] $h(1+\xi_0/l)^{-2}\approx 10^{-5}$ [$\xi(0)\approx 48$ Å, $\xi_0\approx 600$ Å, $l\approx 5$ Å, $(\partial H_{c2}/\partial T)_{T_{c0}}=-34.1$ kG/K, $H_{c2}(0)\approx 63$ kG, $T_{c0}=4.246$ K, $h\approx 6.9\times 10^{-6}\,H$]. Hence we have used the usual local theory.[3,14,15]

The calculation proceeds from the functional integral expression for the partition function, $Z=Z_0\int D\psi \times\exp(-\beta F[\psi])$, where in the absence of a field[6]

$$\beta F[\psi]=N_0Vk_BT\sum_Q|\psi_Q|^2[\epsilon+\xi(0)^2Q^2]+(bk_BT/2)\sum_{Q_i}\psi_{Q_1}{}^*\psi_{Q_2}{}^*\psi_{Q_3}\psi_{Q_1+Q_2-Q_3}\quad (i=1,2,3). \tag{2}$$

The ψ_Q are Fourier components of the order parameter, N_0 is the single-spin-state density at the Fermi surface, V is the volume, $\epsilon=\ln(T/T_{c0})$, and $b=N_0V[7\zeta(3)/8\pi^2]=0.106N_0V$. The Hartree approximation reduces Eq. (2) to

$$\beta F[\psi]=N_0Vk_BT\sum_Q|\psi_Q|^2[\epsilon+\xi(0)^2Q^2+0.106\sum_{Q'}\langle|\psi_{Q'}|^2\rangle]-(bk_BT/2)(\sum_Q\langle|\psi_Q|^2\rangle)^2. \tag{3}$$

We define the renormalized temperature shifts

$$\eta=\epsilon+0.106\sum_Q\langle|\psi_Q|^2\rangle. \tag{4}$$

The self-consistency condition accompanying Eq. (3) then becomes

$$\eta-\epsilon=(0.106/N_0Vk_BT)\sum_Q[\eta+\xi(0)^2Q^2]^{-1}. \tag{5}$$

If one uses Eq. (3) in the calculation of Z, then the thermodynamic potential, $-k_BT\ln Z$, and specific-heat difference $\Delta C=C_s-C_n$ may be obtained. The latter is given by

$$\Delta C(T)=(k_B/V)[\bar{s}(T)^{-1}+0.106/N_0Vk_BT]^{-1}, \tag{6}$$

where

$$\bar{s}(T)=\sum_Q[\eta+\xi(0)^2Q^2]^{-2}. \tag{7}$$

In the presence of a magnetic field the Q summation in $\bar{s}(T)$ is expressed as $\sum_Q\to(2AeH/2\pi\hbar c)\sum_{n,Q_z}$ (A is the area of the sample transverse to the field H) and $\xi(0)^2Q^2$ is replaced by $\xi(0)^2Q_z{}^2+2h(n+\frac{1}{2})$. Evaluating the Q_z sum as an integral yields

$$\bar{s}(T,h)=\frac{Vh}{8\pi\xi(0)^3}\sum_{n=0}^{\infty}(\eta_h+2hn)^{-3/2}\equiv[Vh/8\pi\xi(0)^3]s(T,h) \tag{8}$$

in which $\eta_h = \eta + h$. The specific heat, Eq. (6), and the renormalized temperature shift, Eq. (5), become

$$\Delta C(T,h) = \frac{hk_B}{8\pi\xi(0)^3}\left[s(T,h)^{-1} + \frac{0.106h}{8\pi N_0\xi(0)^3k_BT}\right]^{-1}, \tag{9}$$

$$\eta_h = \epsilon + h + \frac{0.106h}{4\pi N_0\xi(0)^3k_BT}\sum_{n=0}^{\infty}(\eta_h + 2hn)^{-1/2}. \tag{10}$$

Equations (8), (9), and (10) are the basic results of the present theory. Neglecting the fluctuation interaction by setting $b=0$ in Eq. (2) reduces Eq. (9) to Eq. (8) of Ref. 3. The field-induced reduction of effective dimensionality is manifest in the h dependence of $s(T,h)$: (1) For $h \ll \eta$, $s(T,h)$ converts to an integral, $s(T,h) \to h^{-1}\eta^{-1/2}$, and ΔC reduces to the zero-field 3D result; (2) for $h \gg \eta$, the $n=0$ term dominates the sum, $s(T,h) \to \eta_h^{-3/2}$, and ΔC acquires the temperature dependence of a 1D superconductor.

At this point we note an important limitation of the present theory: The critical-field curve $H_{c2}(T)$ is reproduced only for weak fields, $h \ll 1$. This results from our use of $\epsilon + \xi(0)^2Q^2$, a small-Q approximation to the zero-frequency pair propagator,[16] in the quadratic term of the free-energy functional, Eq. (2). In particular, the critical temperature in this and previous[3] approximations is given by $\epsilon(T_{c2}) + h = 0$ or $T_{c2} = T_{c0} \times \exp(-h)$, which is clearly incorrect for large h. Furthermore, the present theory neglects the experimentally observed paramagnetic limitation,[9,10] which tends to reduce[10] $T_{c2}(H)$. Thus in order to compare the shapes of the predicted transitions with those measured, we have solved Eq. (10) using $\epsilon + h = \ln(T/T_{c2})$, with T_{c2} the experimentally measured value, obtained by an entropy fitting procedure,[9] as opposed to the incorrect theoretical value. We do not believe that the widths of the predicted transitions are significantly changed thereby. The discrepancy between the theoretical and experimental $T_{c2}(h)$ values reached a maximum of 8% at 29 kG, of which about 4% is apparently due to paramagnetic limitation.

The calculated $\Delta C(T,h)$ curves are shown in Fig. 1 along with the experimental $C_H(T)$ curves of Barnes and Hake[9] for Ti-16-at.% Mo. The first 100 terms of the n sum were included in $s(T,h)$. The η_h of Eq. (10) was calculated using only the $n=0$ term of the sum, an approximation which was checked at 3 kG and was found to introduce only a slight discrepancy in comparison with the η_h values resulting from keeping 100 terms of the sum. Aside from the discrepancy in the magnitudes of ΔC_{max}, the experimentally observed field-induced increase in the transition width and the shapes of the curves as the transition is approached from above T_{c2} are in fairly good agreement with the present and previous[3] theories. The width of the critical region at H

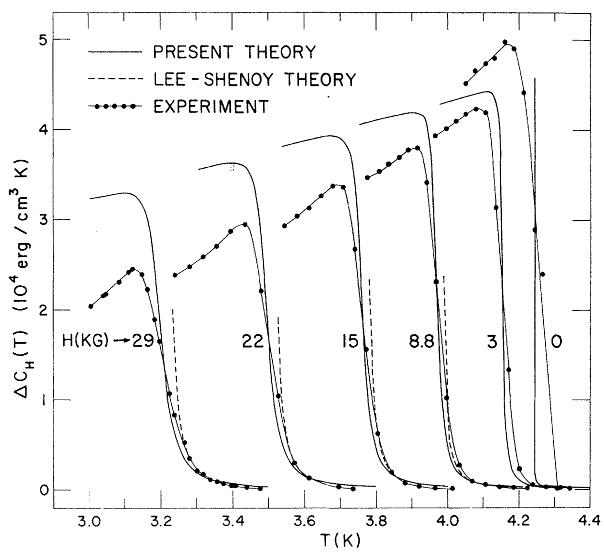

FIG. 1. Specific heat difference, $\Delta C = C_s - C_n$, versus temperature T for various values of applied magnetic field H, as given by the present theory, the theory of Ref. 3, and the data of Ref. 9. The dashed line (Lee-Shenoy theory) merges with the heavy solid line (present theory) for T well above $T_{c2}(H)$. The zero-H curve given in the present theory stops abruptly at $\Delta C = 1.43C_n$ at a slightly shifted zero-H transition temperature $T_c = 4.2456$ K (critical exponent $\alpha = -1$; see Ref. 6). The data points are the observed superconducting-state specific heat C_s values minus the normal-state specific heat $C_n = \gamma T + \beta T^3$, where γ and β were determined by a least-squares fit for data taken in applied fields sufficient to quench superconductivity. All theoretical curves have been scaled so as to center the transitions at the experimental $T_{c2}(H)$ as discussed in the text. The experimental values of $T_{c2}(H)$ for $H = 29$, 22, 15, 8.8, 3, and 0 kG are, respectively, 3.216, 3.509, 3.772, 3.980, 4.159, and 4.246 K, obtained by an entropy-fitting procedure (see Ref. 9, Table I, footnote c). The $H = 0$ data were taken without compensation of Earth's magnetic field.

$=29$ kG may be approximated by[3] $\epsilon_c \approx h^{2/3}(k_F\xi_0)^{-1/3} \times (k_F l)^{-1} \approx 0.006$, using[10] $k_F \approx \hbar^{-1} m^* v_F \approx 1.2\times 10^8$ cm^{-1}, where m^* is the thermal effective mass. Thus the Lee-Shenoy theory[3] should be applicable over a considerable range above $T_{c2}(H)$.

The measured width of the zero-field transition, ≈ 0.1 K, is unaccounted for in the present theory. Although this width is about a factor of 2 less than that observed by others[17] in similar-concentration Ti-Mo specimens, it may still be due to sample inhomogeneity. The measured[17] T_{c0} versus c (c is the at.% concentration) suggest that the observed width of about 0.1 K might indicate an alloy concentration spread of at most 3 at.%. The measured[18] H_{c2} versus c and[9] $H_{c2}(T)$ would then imply a weakly field-dependent inhomogeneity broadening, increasing to ≈ 0.13 K at 29 kG. Thus an approximate 0.1-K inhomogeneity broadening might be subtracted from the widths of the experimental finite-field curves, leading to a somewhat better fit with the present theory. It should be remarked that the magnetic field inhomogeneity[9] of at most $\pm 0.1\%$ would account for a transition broadening of only ≈ 0.002 K at 29 kG. The lack of agreement of $\Delta C_{\max}$ (from 5% at 3 kG to 35% at 29 kG) may derive from the previously mentioned small-Q approximation, neglect of vortex lattice structure below H_{c2}, and/or our cavalier use of the Hartree approximation in strong magnetic fields.

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