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Anomalous Pinch Effect in a Tokomak

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We demonstrate the existence of an anomalous pinch effect if turbulent fields are generated by current instabilities. This effect is similar to the Ware effect, but the stationary electric force is replaced by the drag force resulting from asymmetry of the turbulent fields.

It is now well known[1] that a toroidal electric field can give rise to a radial convection velocity in a geometry of the tokomak type (Fig. 1). Then it is expected that the same effect will occur if the stationary electric force is replaced by the drag force resulting from turbulent fields. However, two conditions must be fulfilled. First, the effective mean free path must be larger than the length of the torus. Second, the turbulence must be generated by the toroidal current density in order to provide an asymmetry of the turbulent fields. Then it is the purpose of this Letter to specify the conditions of instability for a toroidal current carrying plasma, and to demonstrate the existence of an anomalous pinch effect, when these current instabilities are excited. In contrast to recently published works,[2,3] we limit our analysis to the case where

$$\left|\frac{1}{u_0}\frac{\partial u_0}{\partial r}\right| < 4\frac{V_e^2}{c^2}\frac{1}{\lambda_{D_e}},\ \frac{V_i}{\Omega_{c_i}}.$$

Here u_0 is the current drift velocity, $V_{e,i}$ are respectively the electron and ion thermal velocies, c the speed of light, λ_D the Debye length, and Ω_{c_i} the ion cyclotron frequency. For simplicity we shall restrict our computations to the banana regime where trapped particles exist, though the results are easily extended to the plateau regime.

It is necessary to inspect again the conditions for instabilities because of the existence of trapped particles which do not carry the current.[4] All of the modes of interest have large parallel wave numbers so that $k_\parallel Rq > 1$, where R is the major radius of the torus and q is the usual safety fac-

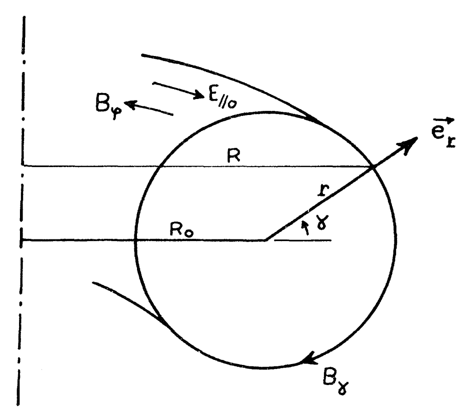

FIG. 1. Coordinate system.

tor,

$$q = rB_\varphi/RB_\gamma.$$

The parallel phase velocities, V_φ, are always assumed to be larger than V_i and smaller than V_e. Moreover, V_φ and the current drift velocity u_0 have to be of the same sign for instability, and they will be chosen positive in the following. So we shall consider the high-frequency ion-acoustic (h.f.a.) mode with

$$\omega^2 = \frac{k^2 C_s^2}{1 + k^2\lambda_D^2} > \omega_{bT_e}^2,$$

the low-frequency ion-acoustic (l.f.a.) mode with $\omega = k_\parallel C_s < \omega_{bT_e}$, and the current drift mode with $\omega = \omega_e^* < \omega_{bT_e}$, where $C_s^2 = T_e/M_i$ $(T_e > T_i)$, ω_e^* is the electron-density-gradient drift frequency, and and $\omega_{bT_e} = (r/R)^{1/2} V_e/Rq$ is the bounce frequency of trapped electrons.

At first we look for the growth rates of these modes when collisions can be neglected. We notice that if the parallel phase velocity V_φ is larger than $(r/R)^{1/2}V_e$, all the resonant particles will be passing particles and the toroidal effects will be negligible. Then if $u_0 > (r/R)^{1/2}V_e$, the instability conditions will not be changed for the h.f.a. mode and the drift mode. In the following we shall only consider cases where $u_0 < (r/R)^{1/2}V_e$, which is the realistic experimental condition in most of the plasma. In such conditions, the growth rates γ are given by

$$\frac{\gamma}{|\omega|_{\text{h.f.a.}}} = \sqrt{\pi}\left[\frac{u_0}{V_e}\left(\frac{V_\varphi}{(r/R)^{1/2}V_e}\right)^5 - \frac{V_\varphi}{V_e} - \left(\frac{T_e}{T_i}\right)^{1/2} \exp\left(-\frac{T_e}{T_i}\right)\right],$$

$$\frac{\gamma}{|\omega|_{\text{D}}} = \sqrt{\pi}\,\frac{u_0}{V_e}\left(\frac{V_\varphi}{(r/R)^{1/2}V_e}\right)^5 - \frac{V_\varphi}{V_e}\frac{3}{2}\frac{\partial \ln T_e}{\partial \ln n}\left(\frac{\omega}{\omega_{bT_e}}\right)^2,$$

$$\frac{\gamma}{|\omega|_{\text{l.f.a.}}} = \sqrt{\pi}\left[\frac{u_0}{V_e}\left(\frac{V_\varphi}{(r/R)^{1/2}V_e}\right)^5 - \frac{V_\varphi}{V_e}\left(\frac{\omega}{\omega_{bT_e}}\right)^2 - \frac{T_e}{T_i}\exp\left(-\frac{T_e}{T_i}\right)\right],$$

where $u_0 = |eE_\parallel/M_e\nu_{ei}|$ is the average electron-ion collision frequency. Here we have taken the energy dependence of the collision frequency into account. The parallel velocity of resonant particles is smaller than $(r/R)^{1/2}V_e$, and they must be circulating particles in order to carry the current. Then their energy, ϵ, is smaller than $T_e[V_\varphi/(r/R)^{1/2}V_e]^2$. As the collision frequency is proportional to $\epsilon^{-3/2}$, such low-energy particles will contribute very weakly to the current so that there is a further reduction in the growth rate. Moreover, the resonant trapped particles damp the wave heavily. Then it is impossible to find any ion-acoustic current instability with $u_0 < (r/R)^{1/2}V_e$ if collisions are not taken into account. As regards the drift mode, we have a possible instability either if $\partial \ln T_e/\partial \ln n < 0$, or if $\partial \ln T_e/\partial \ln n$ is small. The first case is not of interest here since then the instability is not generated by the current. On the other hand, the second case may happen if there is a skin effect which gives rise to peaks of temperature where the density gradient exists.

The collisions will have two effects. At first they will untrap the low-energy electrons which will then carry the current, and so will contribute to the growth rate. Secondly, they widen the resonance which now concerns the particles with parallel velocity larger than the phase velocity. As collisions are increased, the instability condition varies continuously from $u_0 > (r/R)^{1/2}V_e$ to $u_0 > V_\varphi > C_s$. This last limit is reached when $\nu_{ei}R/r \sim \omega_{bT_e}$, i.e., at the end of the banana regime where the growth rates are given by

$$\frac{\gamma}{|\omega|_{\text{h.f.a.}}} = \sqrt{\pi}\left[\frac{u_0}{V_e}\left(\frac{\nu_{ei}}{\omega_{bT_e}}\frac{R}{r}\right)^{5/4} - \frac{V_\varphi}{V_e} - \left(\frac{T_e}{T_i}\right)^{1/2}\exp\left(-\frac{T_e}{T_i}\right)\right], \quad \text{if } \frac{R}{r}\frac{\nu_{ei}}{\omega_{bT_e}} > \left(\frac{V_\varphi}{(r/R)^{1/2}V_e}\right)^4,$$

$$\frac{\gamma}{|\omega|_{\text{D}}} = \sqrt{\pi}\left[\frac{u_0}{V_e}\left(\frac{\nu_{ei}}{\omega_{bT_e}}\frac{R}{r}\right)^{5/4} - \frac{V_\varphi}{V_e}\frac{3}{2}\frac{\partial \ln T_e}{\partial \ln n}\frac{\omega_{bT_e}R}{\nu_{ei}r}\right], \quad \omega < \frac{R}{r}\nu_{ei} < \omega_{bT_e},$$

$$\frac{\gamma}{|\omega|_{\text{l.f.a.}}} = \sqrt{\pi}\left[\frac{u_0}{V_e}\left(\frac{\nu_{ei}}{\omega_{bT_e}}\frac{R}{r}\right)^{5/4} - \frac{V_\varphi}{V_e}\frac{\omega_{bT_e}R}{\nu_{ei}r} - \left(\frac{T_e}{T_i}\right)^{1/2}\exp\left(-\frac{T_e}{T_i}\right)\right], \quad \omega < \frac{R}{r}\nu_{ei} < \omega_{bT_e}.$$

It is important to notice that in all cases the trapped particles absorb the energy from the wave, though the circulating particles do not contribute to damping.

The preceding discussion was necessary in order to show that current instabilities can exist in typical tokomak regimes. Now we discuss the effect of such instabilities on the radial transport. We assume that the fluctuation level remains so small that trapped particles still exist. We use the same methods and approximations as in a neoclassical transport computation[5,6] including both collisions and fluctuations. The flux driven by fluctuations reads

$$\Gamma=\oint \frac{d\gamma}{2\pi}\frac{R}{R_0}\int d_3V\, V_r\delta F_0=\oint \frac{d\gamma}{2\pi}\frac{R}{R_0}\int d_3V\,\frac{MV_\parallel}{eB_\gamma}\left(-C(\delta F_0)+\frac{e}{M}\sum_k{}' \frac{\tilde{E}_\parallel{}^k\partial\tilde{f}_{-k}}{\partial\vec{V}_\parallel}\right)+\sum_k\frac{\tilde{E}^k\times\vec{B}}{B^2}\cdot\vec{e}_r\,\tilde{n}_{-k}, \tag{1}$$

where δF_0 is the modification of the quasistationary distribution function due to the fluctuations; $\tilde{f}_k$, $\tilde{n}_k$, and $\tilde{E}^k$ are the fluctuating distribution functions, densities, and fields; and $C_{e,i}(\delta F_0)$ is the collision operator for electrons or ions. The last term is the usual radial flux which is obtained also in straight magnetic fields and which results from the electric drift in the fluctuating fields. The integral does not exist without toroidal effects since the stationary condition implies that the integrand vanishes. On the other hand, the parity of the trapped-particle distribution function entails that $C(\delta F_0)$ does not contribute to the integral for trapped particles. Then the toroidal contribution to the flux reduces to

$$\Gamma=\oint \frac{d\gamma}{B_\gamma}\frac{R}{R_0}\int\frac{d\epsilon\, d\mu B}{|V_\parallel|}\left(1-\frac{V_\parallel}{\langle V_\parallel\rangle}\right)\int_0^\mu d\mu'\sum_k \tilde{E}_\parallel{}^{-k}\frac{\partial \tilde{f}_k{}^{e^-}}{\partial\epsilon}, \tag{2}$$

with $\mu=V_\perp{}^2/2B$, $\langle V_\parallel\rangle=\oint(d\gamma/2\pi)(R/R_0)V_\parallel$, and where the term in $V_\parallel/\langle V_\parallel\rangle$ is included only over the velocity space corresponding to the untrapped particles. Equation (2) shows clearly that we could have computed the radial flux of trapped particles by assuming that they experience a radial drift $\tilde{E}_\parallel{}^k/B_\gamma$. Here we may notice that this radial flux does not exist if the instability is not driven by the current since in such a case $\tilde{f}_k{}^{e^-}$ would be an even function of $k_\parallel$. We must also emphasize that, contrary to what happens in straight magnetic fields, the flux still remains when quasilinear effects on the circulating electrons have suppressed the growth rate. If we assume that the linear approximation remains valid for the electrons, we obtain

$$\Gamma=\sum_k -\frac{n|e|}{T_e}\frac{\tilde{E}_k}{k}\frac{\tilde{E}_\parallel{}^{-k}}{B_\gamma}\frac{V_\varphi}{V_e}$$

for h.f.a.,

$$\Gamma=\sum_k -\frac{n|e|}{T_e}\frac{\tilde{E}_k}{k}\frac{\tilde{E}_\parallel{}^{-k}}{B_\gamma}\frac{V_\varphi}{V_e}\frac{3}{2}\frac{\partial\ln T_e}{\partial\ln n}\frac{\omega_{bT_e}}{\nu_{ei}}\frac{R}{r}$$

for drift waves,

$$\Gamma=\sum_k -\frac{n|e|}{T_e}\frac{\tilde{E}_k}{k}\frac{\tilde{E}_\parallel{}^{-k}}{B_\gamma}\frac{C_s}{V_e}\frac{\omega_{bT_e}R}{\nu_{ei}r}$$

for l.f.a. waves, where the sums are over those k for which $k_\parallel>0$. It is seen that the flux is always inward for ion-acoustic modes. This is easily understood since we know that trapped particles absorb energy from the waves and then their momentum increases when they travel in the direction of the current drift velocity, as it would be under the action of the equilibrium electric field $E_\parallel{}^0$. We can estimate the maximum value of the inward flux by noticing that particles remain trapped if $(e/M)\sum_k\tilde{E}_\parallel{}^k\partial\tilde{f}_{-k}{}^{e^-}/\partial\vec{V}_\parallel$ is smaller than ω_{bT_e}. Then we find

$$|\Gamma|\leq\left|n\left(\frac{r}{R}\right)^{1/2}\frac{V_e{}^2}{\Omega_{c_e}}\frac{1}{r}\right|,$$

where Ω_{c_e} is the electron cyclotron frequency.

In conclusion, we have shown that current-driven instabilities could give rise to large radial inward velocities. Then the resulting effect will not be a diffusion, but rather an inward convection for the regions bearing large currents.

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