Stabilization of Explosive Instabilities by Nonlinear Frequency Shifts

V. N. Oraevskii and V. P. Pavlenko
Institute of Nuclear Research, Academy of Sciences of the Ukrainian SSR, Kiev, U. S. S. R.

and

H. Wilhelmsson
Institute for Electromagnetic Field Theory, Chalmers University of Technology, Göteborg, Sweden

and

E. Ya. Kogan
Kiev University, Kiev, U. S. S. R.
(Received 24 October 1972)

It is found that nonlinear frequency shifts proportional to the square of the amplitudes of the waves may lead to a soliton-type solution in time for stabilized explosive instabilities.

In the field of nonlinear wave problems the phenomenon of explosive instability is of great interest.[1-20] Instabilities of this type are due to nonlinear interaction between waves having negative and positive wave energies.[1-10] They may lead to unlimited field amplitudes in a finite time. Beam-plasma systems,[1,2] some magnetized plasmas,[1,9] as well as plasmas where the neutrals or ions have levels of inverse population,[7] may exhibit such phenomena. Several interesting efforts have been made[16-20] to study the problem of stabilization of explosive instabilities. In particular Fukai, Krishan, and Harris[16] considered the influence of nonlinear frequency shifts in a coherent phase description and gave an upper limit for the amplitude reached during "explosion." However, we think that this problem needs further investigation and, in particular, an analytic solution which also covers the time interval beyond the "explosion."

The equations of motion for the field amplitudes, including terms to third order in amplitudes of the expansion, and assuming coherent wave interaction, are[5]

$$i(\partial A_k/\partial t)\exp(-i\omega_k t)=\sum V_{k,k',k''}A_{k'}A_{k''}\exp[-i(\omega_{k'}+\omega_{k''})t]+\sum W_{k,k_1,-k_2,k}|A_{k_1}|^2A_k\exp(-i\omega_k t)$$
$$+\sum W_{k,k_1,k_2,k}A_kA_{k_1}A_{k_2}\exp[-(\omega_{k_1}+\omega_{k_2}+\omega_k)t]. \qquad (1)$$

As a consequence of the oscillatory nature of the last sum in Eq. (1), it may be neglected as compared to the other terms of third order. The coefficients $V_{k,k',k''}$ and $W_{k,k_1,-k_2,k}$ are real quantities in the absence of dissipation.

For resonant three-wave interactions, satisfying decay conditions for the frequencies and wave numbers ($\omega_0=\omega_1+\omega_2$, $k_0=k_1+k_2$), and assuming the form $A_i=u_i\exp(i\Phi_i)$ for the complex amplitudes, we have three equations for the amplitudes,

$$\begin{aligned}\partial u_0/\partial t&=Vu_1u_2\cos\Phi,\\ \partial u_1/\partial t&=Vu_0u_2\cos\Phi,\\ \partial u_2/\partial t&=Vu_0u_1\cos\Phi;\end{aligned} \qquad (2)$$

and for the combination $\Phi=\Phi_0-\Phi_1-\Phi_2+\frac{1}{2}\pi$ of the phases, we have

$$\frac{\partial\Phi}{\partial t}+\delta\omega=-V\left[\frac{u_1u_2}{u_0}+\frac{u_0u_2}{u_1}+\frac{u_0u_2}{u_2}\right]\sin\Phi, \qquad (3)$$

where

$$\delta\omega=\sum\beta_iu_i^2 \quad (i=0,1,2) \qquad (3a)$$

is the frequency shift[21] and V is the coupling coefficient. In the expression (3a) for the frequency shift, the quantities $\beta_i=\alpha_i^0-\alpha_i^1-\alpha_i^2$ $(i=0,1,2)$ and $\alpha_i^k=W_{k,i,-i,k}$ $(i,k=0,1,2)$. Equations (2) have the following invariants:

$$n_0-n_1=M_1, \quad n_0-n_2=M_2, \qquad (4)$$

where we have introduced the notation $n_i=u_i^2$. For simplicity we choose $M_1=M_2=0$ here. We then obtain from Eqs. (2)–(4) $(n_i=n)$

$$\partial n/\partial t=2Vn^{3/2}\cos\Phi, \qquad (5)$$

$$\partial y/\partial t=-n^{5/2}\gamma\cos\Phi, \qquad (6)$$

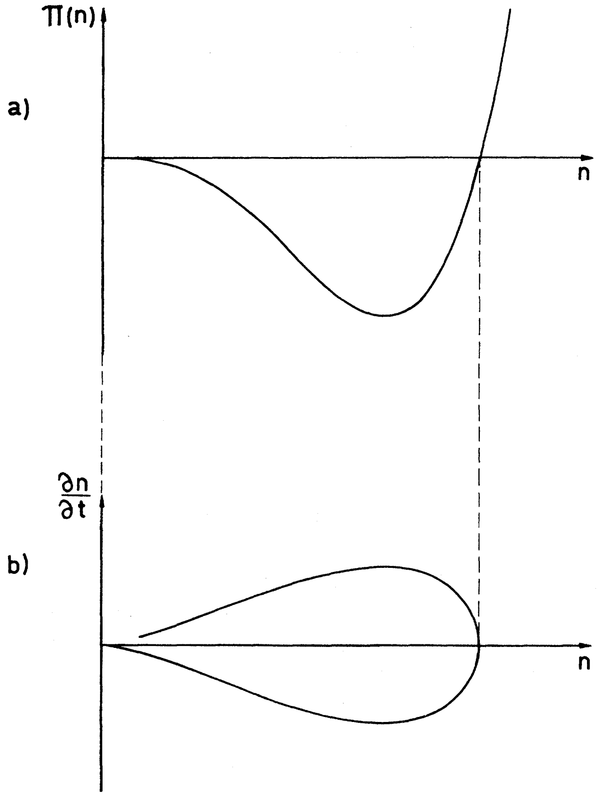

FIG. 1. (a) The function $\pi(n)$. (b) The phase plane.

where

$$y = n^{3/2}\sin\Phi, \tag{7}$$

$$\gamma = \sum \beta_i. \tag{6a}$$

From Eqs. (5) and (6) we find that

$$\partial y/\partial n = -\gamma/2Vn, \tag{8}$$

with the solution

$$y = -\gamma/4Vn^2 + \Gamma, \tag{9}$$

where Γ is a constant determined by the initial conditions. For simplicity we also choose $\Gamma = 0$, in which case we have, from Eqs. (5), (7), and (9),

$$\partial n/\partial t = \pm 2V(-\eta n^4 + n^3)^{1/2}, \tag{10}$$

where

$$\eta = \gamma^2/16V^2.$$

Equation (10) can be written in the form

$$\tfrac{1}{2}(\partial n/\partial t)^2 + \pi(n) = 0, \tag{11}$$

where

$$\pi(n) \equiv 2V^2[\eta n^4 - n^3]. \tag{11a}$$

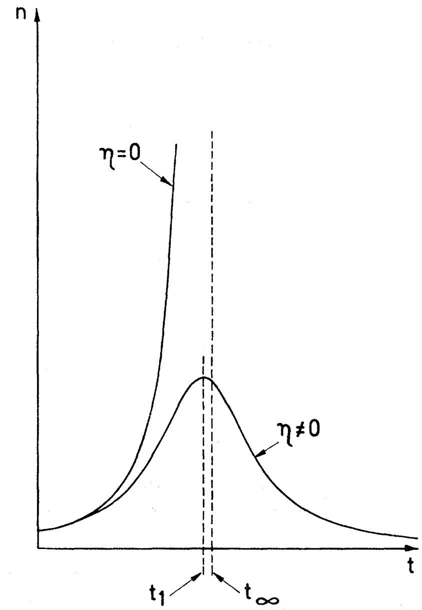

FIG. 2. Qualitative plot of $n(t)$.

We notice that Eq. (11) corresponds to the energy-conservation relation for the motion of material particles in a potential. The potential function (11a) has the form indicated in Fig. 1. The phase-plane description in Fig. 1(b) corresponds to a soliton solution[5,22] in time. The solution to Eq. (10) has the form

$$n(t) = [\eta + V^2(t_1 - t)^2]^{-1}, \tag{12}$$

where

$$t_1 = V^{-1}[n(0)]^{-1/2}[1 - \eta n(0)]^{1/2} \tag{13}$$

and

$$n_{\max} = n(t_1) = \eta^{-1} = 16V^2/\gamma^2, \tag{14}$$

which value for $n_{\max}$ agrees with the limit for the amplitudes obtained by previous authors.[16] For $\eta = 0$ we have the explosion time $t_1 = t_\infty = V^{-1}[n(0)]^{-1/2}$. The time evolution of the quantity $n(t)$ is depicted in Fig. 2. In fact, n stays finite for $\eta \neq 0$ independent of how the constants of motion are chosen. Therefore the amplitudes are generally limited for physically realistic situations and the singular solution corresponds to a limiting case where η tends to zero.

The case we have investigated is the simplest possible one with regard to constants of integration. It elucidates the stabilization effect and carries the solution beyond the time of explosion. The solutions for more general constants of integration can be found in terms of elliptic functions, which naturally include the possibilities of periodic solutions in time.

If dissipative effects are included, an oscillating structure of the solution for $n(t)$ will be expected to appear instead of the described solution. Besides, dissipation will affect the time scale of the problem so as to delay[12,15] the phenomenon and contribute to the saturation.

The authors would like to express their gratitude to professors B. B. Kadomtsev and A. G. Sitenko for discussions during the September 1972 meeting of the international working group on plasma theory in the Institute for Theoretical Physics, Academy of Sciences of the Ukrainian SSR, Kiev, U. S. S. R.

[1]B. B. Kadomtsev, A. B. Mikhailovskii, and A. V. Timofeev, Zh. Eksp. Teor. Fiz. 47, 2266 (1964) [Sov. Phys. JETP 20, 1517 (1965)].

[2]V. M. Dikasov, I. I. Rudakov, and D. D. Ryutov, Zh. Eksp. Teor. Fiz. 48, 913 (1965) [Sov. Phys. JETP 21, 608 (1965)].

[3]P. A. Sturrock, Phys. Rev. Lett. 16, 270 (1966).

[4]V. N. Tsytovich, Zh. Eksp. Teor. Fiz. 51, 1385 (1966) [Sov. Phys. JETP 24, 934 (1967)].

[5]R. L. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, New York, 1969).

[6]E. G. Harris, in *Advances in Plasma Physics*, edited by A. Simon and W. B. Thomson (Interscience, New York 1969), Vol. 3, p. 157.

[7]V. N. Tsytovich, *Nonlinear Effects in Plasma* (Plenum, New York, 1970).

[8]R. C. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972).

[9]B. Coppi, M. N. Rosenbluth, and R. N. Sudan, Ann. Phys. (New York) 55, 207 (1969).

[10]F. Engelmann and H. Wilhelmsson, Z. Naturforsch. 24a, 206 (1969).

[11]L. Stenflo, Plasma Phys. 12, 509 (1969), and Phys. Scr. 2, 50 (1970).

[12]H. Wilhelmsson, L. Stenflo, and F. Engelmann, J. Math. Phys. 11, 1738 (1970).

[13]H. Wilhelmsson and K. Östberg, Phys. Scr. 1, 267 (1970).

[14]K. S. Karplyuk, V. N. Oraevskii, and V. P. Pavlenko, Ukr. Phys. J. 15, 859 (1970).

[15]H. Wilhelmsson, Phys. Rev. A 6, 1973 (1972).

[16]J. Fukai, S. Krishan, and E. G. Harris, Phys. Rev. Lett. 23, 910 (1969).

[17]C. T. Dum and R. N. Sudan, Phys. Rev. Lett. 23, 1149 (1969).

[18]S. Hamasaki and N. Krall, Phys. Fluids 14, 1441 (1971).

[19]M. Caponi and R. C. Davidson, Phys. Fluids 14, 1463 (1971).

[20]A. G. Bonch-Osmolovsky, V. G. Makhankov, V. N. Tsytovich, and B. G. Shchinov, to be published.

[21]H. Wilhelmsson, Phys. Fluids 4, 335 (1961).

[22]V. I. Karpman, *Nonlinear Waves in Dispersive Media* (Novosibirsk, U. S. S. R., 1968).

[23]P. K. C. Wang, J. Math. Phys. 13, 943 (1972).

Absolute Cross Section for Producing ^{11}C from Carbon by 270-MeV/Nucleon ^{14}N Ions*

L. Skoski,† M. Merker, and B. S. P. Shen
Astrophysics Laboratory, Department of Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 30 October 1972)

The cross section for producing ^{11}C from carbon irradiated by 270-MeV/nucleon ^{14}N has been found to be 90.0 ± 4.6 mb. The measurement was made using internal counting in plastic scintillators of three different thicknesses; the nitrogen beam was monitored by nuclear emulsions.

The recent successful acceleration of nitrogen ions to several hundred MeV/nucleon at both the Princeton Particle Accelerator (PPA)[1] and the Berkeley Bevatron[2] has ushered in a new era of research using these high-energy heavy ions. In this paper, we report an absolute determination of the cross section for producing ^{11}C from carbon irradiated by 270-MeV/nucleon ^{14}N ions. The process under study is an interesting one not only because it illustrates the collision between two typical medium nuclei but also because it will very likely be used as a convenient primary monitor for the intensity of nitrogen beams.

The experimental technique is similar to that used by Cumming and co-workers,[3] Poskanzer *et al.*,[4] and Radin.[5] Briefly, a packet consisting of a plastic scintillator and a pellicle of nuclear emulsion was exposed to the PPA nitrogen beam, with the beam axis perpendicular to the emulsion surface. The pellicle monitored the nitrogen