(E.S.) wishes to thank Professor A. Lagarrigue, Directeur du Laboratoire de l'Accélérateur Linéaire, for his hospitality, and the Foundacão de Amparo a Pesquisa do Estado de São Paulo for travel grants.

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Zeros of the Vector - Meson Production Amplitude and Their Implications

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(Received 11 January 1973)

In studying the single-pion production amplitudes in the vector-meson mass range (ρ and K^*) we observed zeros and structures in $\mathrm{Re}\rho_{10}^{11}$ and ρ_{1-1}^{11} density matrix elements. The zero of $\mathrm{Re}\rho_{10}^{11}$ is observed to be approximately independent of beam momentum, charge state, or strangeness of the vector meson and is invariant under s-t crossing at $\Delta^2 \approx m_v^2 - m_{ps}^2$. This gives a strong support to absorptive models. Our analysis shows inconsistencies in the recent amplitude analysis.

In order to give a quantitative description of the experimentally observed characteristics of the single-pion production amplitudes, one has to consider both t-channel exchanges and s-channel absorption effects. With a sufficiently complicated parametrization one is able to fit many reactions by emphasizing the absorptive[1] or Regge aspects[2] of the reactions. Instead we have looked for the existence of dramatic variations of the vector-meson density matrix elements which are expected on general theoretical grounds at a wide variety of incident momenta and reactions. We show that zeros of the amplitudes provide nontrivial consistency checks on analyses like that of Estabrooks and Martin[3] (EM).

The starting point of our analysis is the set of equations[4]

$$3R^{11}_{00} = |\langle 10+|T|+\rangle|^2 + |\langle 10+|T|-\rangle|^2, \tag{1}$$

$$6R^{11}_{10} = (\langle 11+|T|+\rangle - \langle 1\,-1+|T|+\rangle)\langle 10+|T|+\rangle^*, + (\langle 11+|T|-\rangle - \langle 1\,-1+|T|-\rangle)\langle 10+|T|-\rangle^*, \tag{2}$$

$$6R^{11}_{11} = |\langle 11+|T|+\rangle|^2 + |\langle 1\,-1+|T|+\rangle|^2 + |\langle 11+|T|-\rangle|^2 + |\langle 1\,-1+|T|-\rangle|^2, \tag{3}$$

$$3R^{11}_{1-1} = \mathrm{Re}(\langle 11+|T|+\rangle\langle 1\,-1+|T|+\rangle^* + \langle 11+|T|-\rangle\langle 1\,-1+|T|-\rangle^*), \tag{4}$$

where $\langle l\lambda\lambda_n|T|\lambda_p\rangle$ denotes the helicity amplitude of a di-pion system with orbital angular momentum l, helicity λ, and with nucleon helicities λ_n and λ_p. The ${}^cR^{l'l}_{m'm}$ are the unnormalized density matrix elements to describe the $\pi\pi$ system with orbital angular momenta (projections) l, l' (m, m') in the c channel. All other density matrices are presented in Ref. 4. The natural- and unnatural-parity contributions to the t-channel exchanges are $d\sigma^\pm/dt = R^{11}_{11} \pm R^{11}_{1-1}$.

The most general way absorption effects manifest themselves is the dip structure of the net helicity-flip $n=0, 1$ amplitudes.[1,5] The unnormalized density matrices $\mathrm{Re}R^{10}_{10}$ and $\mathrm{Re}R^{11}_{10}$ are proportional to net helicity-flip-1 amplitudes and thus will have zeros at the same value of Δ^2. The invariant momentum transfer squared is denoted by $t = -\Delta^2$.

In Fig. 1 we plotted $\mathrm{Re}({}^tR^{11}_{10})$ for $\pi^+\pi^-$ (2–4, 17.2 GeV/c)[6-9] and for $\pi^+\pi^0$ (1.5, 2–4 GeV/c).[6-8,10] Inspection of Fig. 1 reveals that $\mathrm{Re}({}^tR^{11}_{10})$ *vanishes and that the locations of the zeros at* $\Delta^2 \approx 0.6$ *GeV*^{2} *are in an excellent agreement over a decade*

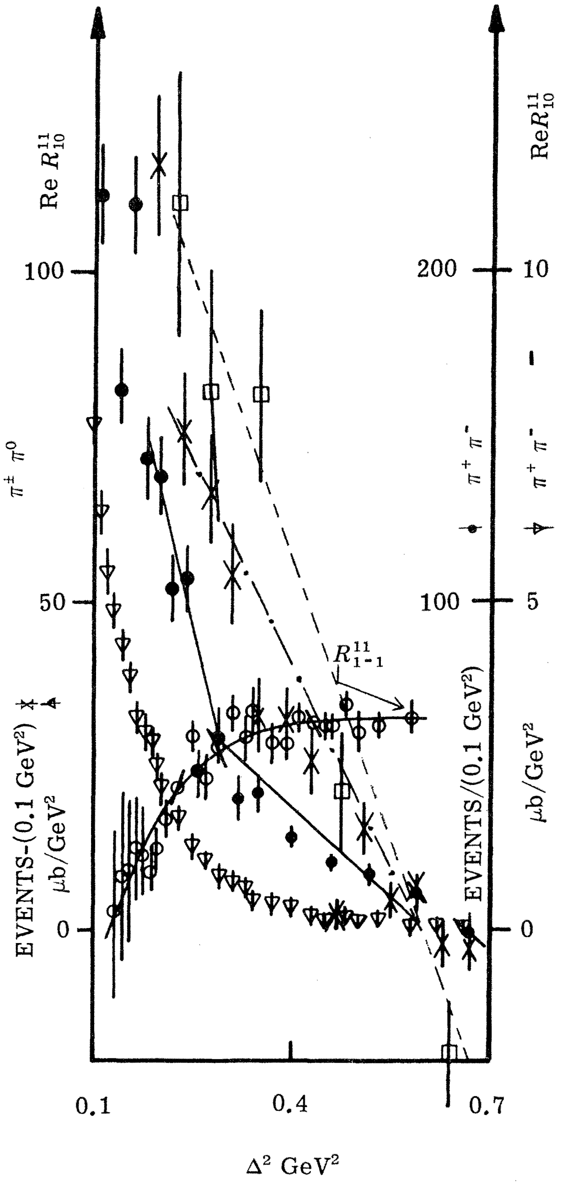

FIG. 1. The invariant momentum-transfer ($-\Delta^2$) dependence of the ρ-decay density matrix element $-\mathrm{Re}({}^t\rho^{11}_{10})$ multiplied by either the number of events or the differential cross section. Squares, 1.55 GeV/c $\pi^+\pi^0$; crosses 2–4 GeV/c $\pi^-\pi^0$; filled circles, 2–4 GeV/c $\pi^+\pi^-$; triangles, 17.2 GeV/c $\pi^+\pi^-$. The open circles represent the neutral ρ-decay density matrix element ${}^t\rho^{11}_{1-1}$ multiplied by the ρ-production differential cross section. The units of the y axis are (left-hand side), for crosses, events/(0.1 GeV^2); for squares, μb/GeV^2; (right-hand side), closed circles, events/(0.1 GeV^2); open circles and triangles, μb/GeV^2. The four curves are drawn on the data points to guide the eye, they do not represent a fit. Note that the zeros of these curves are invariant under the scale of the y axis.

of incident beam momenta, from the ρ-meson threshold[10] *up to 17.2 GeV/c.* Recent publication of a high-statistics K^-p experiment[11] makes it possible to look for this effect in the $K^-\pi^+$ and $K^0\pi^-$ final state in the $K^*(890)$ region. Again we find that the location of the $\mathrm{Re}({}^tR^{11}_{10})$ zeros are in good agreement with that of the $\pi\pi$ system. A very interesting point is that $\mathrm{Re}({}^sR^{11}_{10})=0$ at about the same value of Δ^2. From the equation

$$\mathrm{Re}({}^s\rho^{11}_{10})=({}^t\rho^{11}_{00}-{}^t\rho^{11}_{11}+{}^t\rho^{11}_{1-1})(\sqrt{2}/4)\sin 2\chi + \mathrm{Re}({}^t\rho^{11}_{10})\cos 2\chi, \tag{5}$$

it is clear that this can happen if χ (rotation angle between the s- and t-channel helicity frames) $\approx\pi/2$. To a leading order in $1/s$ this requires that $\Delta^2=m_v{}^2-m_{ps}{}^2$ which is approximately the location of the zeros.

The zero of $\mathrm{Re}R^{11}_{10}$ cannot come from the simultaneous zeros of either $\langle 10+|T|\pm\rangle$ or $\langle 11+|T|\pm\rangle - \langle 1\,-1+|T|\pm\rangle$, because in the former case ${}^sR^{11}_{00}$ would vanish and in the latter case $d\sigma^-/dt$ would vanish, in disagreement with the experimental data. Thus these zeros cannot be explained by a simple Regge-pole model with wrong-signature zeros in the unnatural-parity amplitudes. On the other hand, our results give strong support to absorptive models[1,5] and also support the low-energy results of Williamson *et al.*[10] as shown in Fig. 1.

The same reasoning can be applied to $\mathrm{Re}({}^sR^{10}_{10})$ and $\mathrm{Re}({}^sR^{10}_{00})$ with slight modifications. Thus we can say that $\mathrm{Re}({}^sR^{10}_{10})$ and $\mathrm{Re}({}^sR^{10}_{00})$ must be zero at least at $\Delta^2\approx 0.6$ GeV^2, in agreement with the experimental data both at 2–4 and at 17.2 GeV/c.

In their analysis EM assumed that[12] $\langle 10+|T|+\rangle = r\langle 10+|T|-\rangle$ which implies by Eq. (1) and the absorption zero of $\langle 10+|T|-\rangle$ that $R^{11}_{00}(\Delta^2\approx 0.6)=0$, in contrast to the experimental data used by EM. The lower the momentum the worse the EM approximation becomes. In view of these zeros the Cho-Sakurai[13] relation $({}^sR^{11}_{11}-{}^sR^{11}_{1-1}){}^sR^{11}_{00}=2[\mathrm{Re}({}^sR^{11}_{10})]^2$ is not satisfied at low energies.

Next we studied the density matrix element R^{11}_{1-1}. Inspecting the data used by EM for their analysis, we find that both $d\sigma^\pm/dt$ are large. However, the comparison of ${}^t\rho^{11}_{1-1}$ at 2.7, 4.1, 7.0, 8.0, 11.2, 15.0, and 17.2 GeV/c showed the *striking fact*[6-8,14-17,9] *that* ${}^t\rho^{11}_{1-1}\approx 0$ *for* $\Delta^2\leqslant 0.13$ *GeV*^{2} *for all momenta and a change of sign (or at least a maximum-minimum structure) at all the above momenta in* ${}^s\rho^{11}_{1-1}$. At low momenta the zero of ${}^t\rho^{11}_{1-1}$ extends out to $\Delta^2=0.4$ GeV^2, beyond which it becomes slightly positive. With increasing beam momentum the Δ^2 range of the zero shrinks to 0.13 GeV^2 at 17.2 GeV/c. The ${}^s\rho^{11}_{1-1}$ starts from zero, reaches a positive maximum, and dips towards zero at $\Delta^2\approx 0.1$ GeV^2 at all momenta. Below 7 GeV/c ${}^s\rho^{11}_{1-1}$ becomes definitely negative for $\Delta^2>0.1$ GeV^2.

This strange behavior of ${}^s\rho^{11}_{1-1}$ can be readily explained by

$${}^s\rho^{11}_{1-1}=\tfrac{1}{2}({}^t\rho^{11}_{11}-{}^t\rho^{11}_{00})\sin^2\chi-\tfrac{1}{2}\sqrt{2}\,\mathrm{Re}({}^t\rho^{11}_{10})\sin 2\chi + \tfrac{1}{2}{}^t\rho^{11}_{1-1}(1+\cos^2\chi) \tag{6}$$

or by sufficiently complicated parametrization of the π-A_2 residue structure as is evident from the analysis of EM.[3] In carrying out their analysis[3] EM argued that the t-channel helicity amplitudes are drastically changed by absorption, and hence they proceeded with their analysis in the s-channel helicity frame. The presently existing (admittedly meager) experimental results[18-20] and theoretical calculations[20] indicate that at least in the case of ω exchange, absorption does not seem to affect strongly the density matrix element ${}^t\rho^{11}_{1-1}$, but ρ^{11}_{1-1} is affected violently by crossing from t channel to s channel. While ${}^t\rho^{11}_{1-1}\approx 0$, it has a dramatic variation in the s channel.

Since the difference between natural and unnatural contributions to ρ^0 production in $\lambda=\pm 1$ states is proportional to ρ^{11}_{1-1}, in the t channel one finds that the natural and unnatural trajectories (N and U) are degenerate for a range of Δ^2, while EM found that the intercept of N and U are separated by ≈ 0.5. Since ${}^t\rho^{11}_{1-1}$ is zero for almost a decade of incident beam momenta and a finite range of low momentum transfer, it is more natural to explain this in terms of equality of the t-channel N and U contributions coming from the absorptive corrections to the dominant pion pole. The feasibility of the latter interpretation has been shown recently by Kimel and Reya.[21] EM claim further support for the existence of A_2 exchange by citing good agreement with the pion photoproduction data. To test this claim directly, in Fig. 2 we compared ${}^s\rho^{11}_{1-1}/{}^s\rho^{11}_{11}$ obtained in the 17.2-GeV/c experiment with the asymmetry parameter measured in polarized photoproduction experiments.[22] There is an obvious disagreement for $\Delta^2>0.1$ GeV^2, as has been seen earlier in low-energy experiments.[23] From this and other comparisons[17] it is clear that it is precisely the natural-parity contribution which strongly disagrees with the predictions of the vector dominance model.

Finally we consider the behavior of R^{11}_{1-1} in the

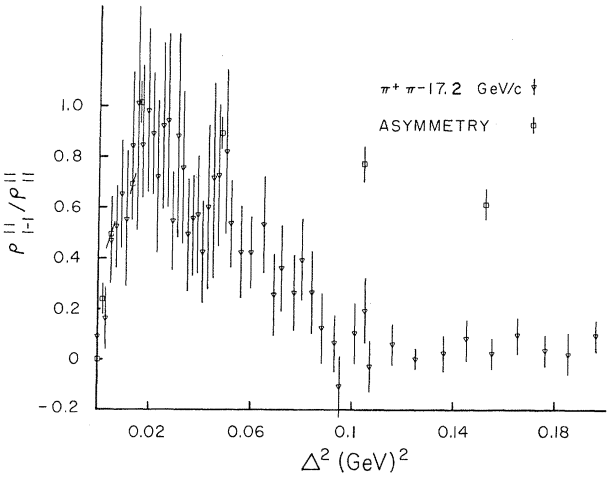

FIG. 2. The invariant momentum-transfer ($-\Delta^2$) dependence of the ratio of the difference and the sum of the natural- and unnatural-parity exchange contribution to the helicity-1 neutral ρ-production cross section; $(\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-) = \rho^{11}_{1-1}/\rho^{11}_{11}$ in the s channel (triangles). Squares, the asymmetry ratio for charged pion photoproduction using polarized photons.

intermediate ($0.13 \leqslant \Delta^2 \leqslant 0.6$ GeV^2) momentum-transfer region at high energies. We have plotted ${}^tR^{11}_{1-1}$ in Fig. 1 for $\Delta^2 > 0.13$ GeV^2 using the 17.2-GeV/c data. In contrast to the 2-4-GeV/c data there is a clear enhancement in the $0.3 < \Delta^2 < 0.6$ GeV^2 region, in agreement with the results of Carroll *et al.*[14] It clearly indicates that the approximate equality of the N and U contributions to ${}^tR^{11}_{1-1}$ found at low energies for any Δ^2 and at high energies for small Δ^2 is destroyed. This suggests that in ${}^t\rho^{11}_{1-1}$ the N and U absorptive pion contributions are still approximately equal and cancel each other. The nonzero contribution would then arise from A_2 exchange, since it does not contribute at the 2–4 GeV/c region, in contrast to ω exchange.[18-20] If A_2 couples dominantly to net helicity-flip–1 amplitude, its contribution to ${}^tR^{11}_{1-1}$ is proportional to[1] $\approx s^{-2\Delta^2\alpha'}(\Delta^2)^2 s^{2\alpha_0}$. Thus at sufficiently high center-of-mass energy s the A_2 exchange will dominate in the intermediate–momentum-transfer region.

We would like to thank our colleagues at the Universities of Colorado, Pennsylvania, Toronto, Wisconsin, Notre Dame, and at the Stanford Linear Accelerator Center for participating in the data compilation; L. Balazs, J. Gaidos, and A. Hirata for reading the manuscript; and J. H. Scharenguivel for participating in the initial phase of this analysis. After completion of this work, R. Fields and G. Kane[24] made two observations relevant to this Letter.

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*Work supported by the U. S. Atomic Energy Commission.

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[24] G. Kane pointed out that the model of Ref. 1 predicts zeros in both the real and imaginary parts of the helicity-flip amplitude, while Ref. 5 does not require a zero in the real part. We cannot distinguish experimentally between the two alternatives. R. Field noted that $\mathrm{Re}R^{11}_{10}$ in Eq. (2) can become zero without having a zero in both terms. If the factor $\langle 11+|T|+\rangle - \langle 1-1+|T|+\rangle$ is small and the second term in Eq. (2) changes sign at $\Delta^2 \approx 0.6$ GeV^2, then $\mathrm{Re}R^{11}_{10}$ will have a zero near $\Delta^2 \approx 0.6$ GeV^2. Note that in the absence of A_1 exchange, the first term in Eq. (2) is down by a factor $1/s$ from the second and hence at large s it will become negligible compared to the second term.