ions.

The final average values of all experiments with ^{16}O beams, where the corrections discussed above are small, are $Q_{2^+}(^{150}Sm) = -1.25 \pm 0.2$ b, corrected for a transient field precession of 0.1°, and $Q_{2^+}(^{194}Pt) = 0.77 \pm 0.5$ b, corrected for a transient field precession of 0.05°. These values agree well with the previous results of -1.31 ± 0.19 for Sm and 0.68 ± 0.13 for Pt obtained by the traditional reorientation technique.[7,8]

Unfortunately, experimental errors are still too large to draw conclusions about the sign of the interference term in the latter analysis. The present results support the view that the quadrupole moment of the first 2^+ state of ^{150}Sm is larger than rotational. The magnitude of Q for ^{194}Pt is smaller than rotational, in agreement with the prediction of Kumar and Baranger,[12] who also predict the oblate shape.

We would like to thank J. de Boer, S. Steadman, and A. Winther for helpful discussions. We would like to thank M. Neiman for help in performing the experiment.

*Permanent address: Physics Department, Massachusetts Institute of Technology, Cambridge, Mass. 02139. John Simon Guggenheim Foundation Fellow, 1971–1972.

†Danida Fellow.

[1]J. de Boer and J. Eichler, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1968), Vol. 1.

[2]D. Cline, R. M. Diamond, and O. F. Hauser, in International Conference on Nuclear Moments and Nuclear Structure, Osaka, Japan, September 1972, edited by H. Horie and K. Sugimoto (J. Phys. Soc. Jap., Suppl., to be published).

[3]G. Breit, R. L. Gluckstern, and J. E. Russell, Phys. Rev. 103, 727 (1956).

[4]J. Eichler, Bull. Amer. Phys. Soc. 9, 152 (1964).

[5]J. de Boer and A. Winther, in *Coulomb Excitation*, edited by K. Alder and A. Winther (Academic, New York, 1966).

[6]S. Steadman, thesis, Rutgers, The State University, 1969 (unpublished).

[7]D. Cline, P. Jennens, C. W. Towsley, and H. S. Gertzman, in International Conference on Nuclear Moments and Nuclear Structure, Osaka, Japan, September 1972, edited by H. Horie and K. Sugimoto (J. Phys. Soc. Jap., Suppl., to be published).

[8]J. E. Glenn, R. J. Pryor, and J. X. Saladin, Phys. Rev. 188, 1905 (1969).

[9]L. Grodzins, in International Conference on Nuclear Moments and Nuclear Structure, Osaka, Japan, September 1972, edited by H. Horie and K. Sugimoto (J. Phys. Soc. Jap., Suppl., to be published).

[10]B. Herskind, R. Bauer, A. Gelberg, L. Grodzins, B. Skaali, and D. R. S. Somayajulu, in International Conference on Nuclear Moments and Nuclear Structure, Osaka, Japan, September 1972, edited by H. Horie and K. Sugimoto (J. Phys. Soc. Jap., Suppl., to be published).

[11]U. Smilansky, Phys. Lett. 29B, 15 (1969).

[12]K. Kumar and H. Baranger, Nucl. Phys. A122, 273 (1968).

Radiation Reaction and Radiative Frequency Shifts*

Jay R. Ackerhalt,† Peter L. Knight, and Joseph H. Eberly
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 9 October 1972)

We investigate the question of quantum-electrodynamic radiative corrections to atomic transition frequencies. It is shown that the Heisenberg equations of motion allow a novel and fruitful exploitation of the concept of radiation reaction.

Quantum electrodynamics (QED) is usually regarded as the most successful example of a quantum field theory. Its predictions have been verified time after time by experiments of very high precision. Nevertheless, the logical foundations of QED are apparently unsound and encumbered with difficulties which have occupied the attention of large numbers of physicists.[1-6]

It has been pointed out repeatedly[1,3,5] that the conceptual indirectness of QED is one of its primary shortcomings. A simple, intuitive, general explanation for the most elementary radiative corrections to magnetic moments, spontaneous level shifts, and level widths has yet to be given.[7] In connection with atomic-level shifts and widths, renewed attempts[5,6] have been made recently to provide logically consistent alternatives to the usual QED by building on the intuitively clear classical idea of radiation reaction. These attempts have the very attractive feature that the

ordinary time-dependent perturbation theory is not necessary for their calculations. Several authors[2,5] have suggested that a theory which works directly with the dynamical variables of a problem may be superior to one which is phrased in terms of Schrödinger-picture probability amplitudes. A theory based on the Heisenberg picture would, at the least, come much closer in spirit to classical physics (and in particular to the spirit of Lorentz's work) where one's insights and intuitions may be more firmly founded.

We have taken these questions, comments, and suggestions seriously in considering some of the simplest nontrivial problems of electrodynamics. We have found that if QED *itself* is formulated in terms of its dynamical equations of motion, then it is possible to obtain nonperturbative approximate expressions for various radiative corrections, such as atomic frequency shifts and level widths.[8]

In this Letter, we illustrate the basic ideas and utility of our approach to QED by concentrating on the problem of spontaneous emission from a fictitious two-level atom.[8] A two-level model atom has been associated with the problem of natural linewidth from the beginning,[9] and forms the basis of the Lee model used in discussing scattering and decay in quantum field theory.[10] In our approach we solve approximately the Heisenberg equations for the electromagnetic field. The field solution is then substituted into the atomic operator equations. Radiation-reaction effects are evident: The part of the field due to the atomic source current drives the atomic operators. We readily identify the radiative corrections without encountering renormalization divergences.

Since the atom has only two energy levels, there are only four atomic matrix elements, and thus only four independent atomic operators in the problem. One of these may be taken to be the unit operator. The other three are the dynamical variables of the atom: the atom's unperturbed energy, and the absorptive and dispersive parts of its electric dipole moment. Following Dicke's dimensionless notation,[11] these are written R_3, $\frac{1}{2}(R_+ + R_-)$, and $\frac{1}{2}i(R_- - R_+)$, where R_+ and R_- may be identified as the two atomic operators which raise and lower the state of the atom, respectively. They are normalized to obey angular-momentum algebra: $[R_3, R_\pm] = \pm R_\pm$, etc.

In this notation, in the dipole approximation and neglecting the A^2 term, the Hamiltonian for the problem may be written

$$H = \hbar\omega_0 R_3(t) + i(\omega_0 d/c)[R_+(t) - R_-(t)]A_d(0,t) + \textstyle\sum_\lambda \hbar\omega_\lambda a_\lambda^\dagger(t) a_\lambda(t). \tag{1}$$

Here ω_0 is the atom's transition frequency in the absence of interactions; d is the magnitude of the linear electric-dipole matrix element; and $A_d(0,t)$ is the component of the vector-potential operator along the direction of the dipole moment, evaluated at the center of the atom. It is useful to expand $A_d(0,t)$ in terms of the usual photon creation and destruction operators:

$$A_d(0,t) = \textstyle\sum_\lambda g_{\lambda d}[a_\lambda(t) + a_\lambda^\dagger(t)]. \tag{2}$$

Here $g_{\lambda d} \equiv (2\pi\hbar c^2/\omega_\lambda V)^{1/2}\epsilon_{\lambda d}$, V is the volume of the quantization region, and λ indexes both wave vector and polarization. As usual, $[a_\lambda(t), a_{\lambda'}^\dagger(t)] = \delta_{\lambda\lambda'}$.

The Heisenberg equations of motion are

$$dR_3/dt = (\omega_0 d/\hbar c)[R_+(t) + R_-(t)]A_d(0,t), \tag{3}$$

$$dR_\pm/dt = \pm i\omega_0 R_\pm - 2(\omega_0 d/\hbar c)R_3(t)A_d(0,t), \tag{4}$$

$$da_\lambda/dt = -i\omega_\lambda a_\lambda(t) + (\omega_0 d/\hbar c)g_{\lambda d}[R_+(t) - R_-(t)]. \tag{5}$$

The equation for $a_\lambda(t)$ is basically simpler than the atomic equation because it contains no operator products, and it may be integrated formally:

$$a_\lambda(t) = a_\lambda^v(t) + (\omega_0 d/\hbar c)g_{\lambda d}\int_0^t dt'[R_+(t') - R_-(t')]\exp[-i\omega_\lambda(t-t')], \tag{6}$$

where $a_\lambda^v(t) = a_\lambda(0)\exp(-i\omega_\lambda t)$ is the free field or "vacuum part" of the solution.

Of course Eqs. (3), (4), and (6) cannot be solved explicitly. However, as far as the atom is concerned, spontaneous radiative decay is a very slow process, requiring many millions of cycles of dipole oscillation, on the average, before it is complete. Thus we assume that $R_\pm(t)$ may be written $S_\pm(t) \times \exp(\pm i\omega_0 t)$, where $S_\pm(t)$ is an unknown operator whose time variation, compared with $\exp(\pm i\omega_0 t)$, is very slow. Then the second term in (6), which we denote by $a_\lambda^s(t)$ and call the "source part" of the quantized field, may safely be approximated by replacing $S_\pm(t')$ by $S_\pm(t)$ outside the integral. The inte-

gral can then be carried out and, for sufficiently long times t, $a_\lambda^s(t)$ may be expressed in terms of $R_\pm(t)$ and Heitler's ζ function [$i\zeta(x) = \lim_{t\to\infty}\int_0^t d\tau \exp(ix\tau) = iP/x + \pi\delta(x)$]:

$$a_\lambda^s(t) = i(\omega_0 d/\hbar c)g_{\lambda d}[R_-(t)\zeta^*(\omega_\lambda - \omega_0) - R_+(t)\zeta^*(\omega_\lambda + \omega_0)]. \tag{7}$$

{The designation "source part" for $a_\lambda^s(t)$ serves as a reminder that the contribution of the second term in (6) to the vector potential (2) is the analog of the inhomogeneous or source term in the retarded-time solution of the classical wave equation: $\vec{A}(r,t) = c^{-1}\int d^3r' |r-r'|^{-1}[\vec{J}(r',t)]_{\text{ret}}$.}

As time goes on, the source and vacuum parts of the field-mode operator fail to commute with each other: While the vacuum part $a_\lambda^v(t)$ always operates in the Hilbert space of the free field, the source operators $R_\pm(t)$, which make up $a_\lambda^s(t)$, evolve into the joint atom-field space. That this must be the case is already shown by the occurrence of $A_d(0,t)$ in Eqs. (3) and (4). However, it may be shown[8] that both the vacuum part as well as the total-mode operator $a_\lambda(t) = a_\lambda^v(t) + a_\lambda^s(t)$ continue to obey Bose commutation relations for all time. Thus the "fundamental mathematical difficulty" of QED in the Heisenberg picture, discovered by Nesbet,[12] is spurious. It arises from an incomplete treatment of the vacuum part of the field. Other versions of this "difficulty" occur in the work of Series[6] and Bullough[13] and have a similar resolution.

In order to obtain a complete solution to the coupled atom-field spontaneous emission problem, one would have to put (7) into (3) and (4) and integrate the atomic equations. This is not possible to do explicitly,[12] and is fortunately also unnecessary.

For our present purpose it is enough to derive the vacuum expectation value of the dipole operator itself. *After* (6) has been approximately integrated as above and substituted into (4), one finds[14]

$$(d/dt)\langle R_+(t)\rangle - i\omega_0\langle R_+(t)\rangle = (-i\Delta - \tfrac{1}{2}A)\langle R_+(t)\rangle + \text{H.c.} \tag{8}$$

The real and the imaginary parts of the coefficient of $\langle R_+\rangle$ on the right-hand side of (8) act to modify the natural frequency ω_0 and thus impart a width and a shift to the transition. In the continuum limit of the mode sums, the real part becomes one half the usual Einstein A coefficient, as expected:

$$A = \tfrac{4}{3}(\omega_0^2 d^2/\hbar c^3)\int_0^\infty [\delta(\omega_\lambda - \omega_0) + \delta(\omega_\lambda + \omega_0)]\omega_\lambda\, d\omega_\lambda, \tag{9}$$

while the imaginary part, the frequency shift, is

$$\Delta = \frac{8\pi}{3\hbar c}\left(\frac{\omega_0 d}{2\pi c}\right)^2 \int_0^\infty \left[\frac{P}{\omega_\lambda - \omega_0} - \frac{P}{\omega_\lambda + \omega_0}\right]\omega_\lambda\, d\omega_\lambda. \tag{10}$$

This expression for the shift is remarkable on several counts. In the first place it does not agree with the standard expression for the Weisskopf-Wigner frequency shift.[9] This is because of the term $1/(\omega_\lambda + \omega_0)$ in the integrand. In the second place, again because of the same term, the leading divergence of Δ is not linear, but merely logarithmic, as it must be in a correct nonrelativistic calculation.[15] And, thirdly, Δ is not merely similar to, but exactly the same as, the completely mass-renormalized frequency shift which one could calculate in second-order nonrelativistic perturbation theory (however, see Ref. 15).

A number of conclusions follow from these results. The simplest is that the usual treatments[9] of the Weisskopf-Wigner problem involve an important oversight. Even Källén's careful discussion[9] misses the point that if the essential states assumption is relaxed enough to admit sum-frequency terms [such as $R_+(t)$ in Eq. (7)], the problem can still be solved almost exactly, and for the frequency shift one then finds the equivalent of Bethe's result.[15]

Furthermore, some very recent work on many-atom emission phenomena is flawed by the same oversight, but in a more serious way. Although the original Weisskopf-Wigner solution was not claimed to provide insight into the frequency-shift problem, the essential states method has been adopted widely to attack problems in correlated many-atom systems. Where questions of frequency shifts are raised,[12] this work must be re-examined.

In addition, our results show that the conventional "explanation" of spontaneous decay, that "the vacuum fluctuations 'stimulate' the atom to emit spontaneously,"[16] need not be adopted. Our work shows, in fact, that the vacuum part of the field, $a_\lambda^v(t)$, plays essentially no role in determining either the frequency shift Δ or the decay rate A.

It seems to us that a much more natural inter-

pretation of our (quantum electrodynamic) results, based on the old idea of radiation reaction, is to be preferred. One sees in the transition from Eq. (4) to Eq. (8) that the vacuum part of the field drops away, and the total Lamb shift and decay rate come from the source part of the field in interaction with the atom. That is, in very classical language, it is not the presence of vacuum fields but of the dipole's own radiation field, the source field, that modifies the atom's characteristics in such a way as to produce a finite decay rate, and a shift of the noninteracting natural transition frequency.

We speculate that our method is more general than the simple model presented in this Letter would suggest, and can provide an interpretation of radiative corrections in other situations. In particular we have in mind the real many-level-atom Lamb shift,[8] many-atom emission phenomena, resonance fluorescence, and the anomalous moment of the electron.[17]

We are grateful to Professor F. Rohrlich for a careful reading of an earlier version of the paper, and for critical suggestions. Two of us (P.K. and J.E.) thank Professor S. D. Drell and Professor A. L. Schawlow for their hospitality during 1971–1972 at Stanford, where part of this work was done.

---

*Research partially supported by the U. S. Army Research Office (Durham) and by the U. S. Atomic Energy Commission.

†National Science Foundation Predoctoral Trainee.

[1]R. P. Feynman, in *The Quantum Theory of Fields* (Interscience, New York, 1961).

[2]P. A. M. Dirac, *Lectures on Quantum Field Theory* (Belfer Graduate School of Science, Yeshiva University, New York, 1966). See especially the first lecture.

[3]V. F. Weisskopf, *Physics in the Twentieth Century* (Massachusetts Institute of Technology Press, Cambridge, 1972); see especially pp. 97–111 and 120–127.

[4]W. Heitler, *The Quantum Theory of Radiation* (Oxford Univ. Press, London, England, 1954), 3rd ed., p. 354.

[5]E. T. Jaynes, in Proceedings of the Third Conference on Coherence and Quantum Optics, Rochester New York, June 1972, edited by L. Mandel and E. Wolf (Plenum, to be published). See also R. K. Nesbet, Phys. Rev. Lett. 27, 553 (1971).

[6]G. W. Series, in *Optical Pumping and Atomic Line Shape*, edited by T. Skalinski (Państwowe Wdawnictwo Naukowe, Warszawa, 1969), p. 25.

[7]The Lamb-shift estimate made by Welton 25 years ago [T. A. Welton, Phys. Rev. 74, 1157 (1948)] on the basis of a quasiclassical argument incorporating vacuum fluctuations remains the best-known attempt to supply such an explanation. However, as Weisskopf points out (Ref. 3, p. 127), the Welton explanation seems useful only for atomic level shifts.

[8]J. R. Ackerhalt and J. H. Eberly (unpublished) apply the method discussed here to a real many-level atom; see also J. R. Ackerhalt, Bull. Amer. Phys. Soc. 18, 27 (1973).

[9]V. Weisskopf and E. Wigner, Z. Phys. 63, 54 (1930). See also discussions by G. Källén, in *Encyclopedia of Physics*, edited by S. Flügge (Springer, Berlin, 1958), Vol. 1, pp. 274–279; W. Heitler, Ref. 4, p. 183; W. H. Louisell, *Radiation and Noise in Quantum Electronics* (McGraw-Hill, New York, 1964), p. 191.

[10]See, for instance E. M. Henley and W. Thirring, *Elementary Quantum Field Theory*, (McGraw-Hill, New York, 1962), p. 126.

[11]R. H. Dicke, Phys. Rev. 93, 99 (1954). See also, R. P. Feynman, F. L. Vernon, Jr., and R. W. Hellwarth, J. Appl. Phys. 28, 49 (1957).

[12]The approximate integration of the "source part" of the Heisenberg field operator has played a role in recent work on many-atom cooperative emission effects; M. Dillard and H. Robl, Phys. Rev. 184, 312 (1969); R. H. Lehmberg, Phys. Rev. A 2, 883 (1970); J. H. Eberly and N. E. Rehler, *ibid.* 2, 1607 (1970); N. E. Rehler and J. H. Eberly, *ibid.* 3, 1735 (1971). In the context of single-atom emission, see Nesbet, Ref. 5. Nesbet's suggestion that the fermion algebra of the source will cause the field to depart from its initially pure boson character is incorrect.

[13]See, for example, V. M. Faĭn, Zh. Eksp. Teor. Fiz. 36, 798 (1959) [Sov. Phys. JETP 9, 562 (1959)]; R. H. Lehmberg, Phys. Rev. 181, 32 (1969); D. Dialetis, Phys. Rev. A 2, 599 (1970); G. S. Agarwal, Phys. Rev. A 4, 1778 (1971); Y. C. Lee and D. L. Lin, *ibid.* 6, 388 (1972); R. K. Bullough, in Proceedings of the Third Conference on Coherence and Quantum Optics, Rochester, New York, June 1972, edited by L. Mandel and E. Wolf (Plenum, to be published).

[14]In Eq. (8) the angular brackets denote an expectation value in any state appropriate to spontaneous emission. That is, the source part of the density matrix is arbitrary and the free-field part is the vacuum projector. We have adopted the convention of normal ordering for convenience (see Ref. 8). The Hermitian conjugate terms, indicated by "H.c." in Eq. (8), contribute so-called "counter-rotating" terms, which are completely negligible. However, the retention of counter-rotating terms in the *original* Eqs. (3) and (4) is essential. They make a major contribution to the Lamb shift. We have discussed this point elsewhere; J. R. Ackerhalt, J. H. Eberly, and P. L. Knight, in Proceedings of the Third Conference on Coherence and Quantum Optics, Rochester, New York, June 1972, edited by L. Mandel and E. Wolf (Plenum, to be published); see also I. R. Senitzky, *ibid.*; G. S. Agarwal, unpublished addendum to Agarwal, Ref. 13.

[15]H. A. Bethe, Phys. Rev. 72, 339 (1947). Each of the energy-level shifts calculated in perturbation theory diverges linearly and requires Bethe's mass renormalization for proper interpretation. In the perturbative

frequency shift calculation these linear divergences exactly cancel for a two-level atom. In a many-level atom it is the contributions of the remaining levels to the mass term which require renormalization (see Ref. 8).

[16]For an entertaining outline of the conventional explanation, see M. O. Scully and M. Sargent, Phys. Today 25, No. 3, 38 (1972).

[17]P. L. Knight, S. B. Lai, and J. H. Eberly, Bull. Amer. Phys. Soc. 18, 27 (1973).

Scaling Relation for a Quantity Related to Particle Production Multiplicities

Myron Bander*
National Accelerator Laboratory, Batavia, Illinois 60510
(Received 20 November 1972)

A scaling relation, $d\ln[\sigma_n(s)]/d\ln s \to p(n/\ln s)$, for high-energy production of n particles is proposed. This relation is supposed to be valid for large n and large s. An extension to present energies is suggested and compared with experiment.

Various models of high-energy production yield limiting (in energy) relations for certain experimental quantities. It is a hope that deviations from these relations are small at present large but finite energies and that we may confront these theoretical ideas with current data. In this note I present a scaling relation for a quantity related to σ_n, the cross section for producing n particles of a certain type. The index n may refer to charged particles, negative particles, pions, etc. We consider a process where these n particles are produced by an incident state whose center-of-mass energy is $\sqrt{s}$. Let

$$Y = a\ln(s/s_0); \tag{1}$$

s_0 and a are at present arbitrary but finite. The limiting relation we propose is

$$\lim_{n, Y\to\infty; n/Y=\rho} \frac{\partial}{\partial Y}[\ln\sigma_n(Y)] = p(\rho). \tag{2}$$

If this relation is true then the left-hand side, which is *a priori* a function of two variables, n and Y, approaches at high energies a nontrivial function of their ratio.

The assumptions necessary to establish (2) are the following:

(1) Correlations in inclusive production are taken to be of short range.[1] More generally we assume the existence of the thermodynamic limit of the Feynman fluid analog[2] to multiparticle production. Specifically if

$$Q(z,Y) = \sum z^n \sigma_n(Y), \tag{3}$$

we assume that the following limit exists:

$$\lim_{Y\to\infty} \frac{\ln Q(z,Y)}{Y} = p(z), \tag{4}$$

with $p(z)$ some finite function of the parameter z.

(2) We need an assumption about the rate of decrease of $\sigma_n(Y)$ with Y fixed and n increasing. The simplest assumption is that $\sigma_n = 0$ for $n > N(Y)$ where $N(Y)$ is bounded by a power of Y. (The kinematic limit $N = \sqrt{s}/m$ is not sufficient.) The stringent requirement that $\sigma_n \equiv 0$ for $n > N$ could be relaxed to a smooth but rapid decrease. Not wishing to get involved in delicate details we use the above simple assumption.[3]

In the Feynman fluid analog $Q(z,Y)$ corresponds to the grand canonical partition function, and $p(z)$ to the pressure as a function of the fugacity z.[4] In this framework $\sigma_n(Y)$ is the analog of the partition function in the canonical ensemble and (2) is just the relation between this partition function and the pressure which is a function of the density ρ. The derivation of the equivalence of the two ways of obtaining the pressure is identical to that in statistical mechanics[5] and will not be reproduced here.

As mentioned previously (2) is to hold for large n and Y. If we wish to test it with presently available data we must decide on what value to assign to s_0 in (1). For present energies the value of s_0 may be crucial for the test of (2). An appealing suggestion comes from the fluid analog itself. Y is related to the length of the plateau in one-particle inclusive production, and the average inelastic multiplicity, $\langle n\rangle$, is in this analog directly proportional to Y. Thus, it is plausible that a proper continuation of (2) to present energies is to replace Y by $\langle n\rangle$. The scaling hypothesis we propose to test is

$$\frac{\partial}{\partial\langle n\rangle} \ln[\sigma_n(\langle n\rangle)] = p\left(\frac{n}{\langle n\rangle}\right). \tag{5}$$