Diffusion of a Plasma with a Small Dielectric Constant in the dc Octopole*

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A new type of diffusion is observed. The observed dependences and magnitude of the diffusion coefficient are consistent with the theory of "classical" diffusion due to thermal equilibrium fluctuations for closed line configurations. The experimental results with the toroidal field are essentially the same as those without the toroidal field. The shear may be ineffective in modifying the diffusion rate because of the short perpendicular diffusion damping time compared with the parallel phase mixing time.

In the experiments reported previously[1-3] fairly low magnetic fields were employed in order to investigate the classical or neoclassical transport process. Recently, a study of particle transport processes in higher magnetic fields and with low plasma densities was carried out. Because the classical diffusion rate is sufficiently reduced in this regime, a new type of diffusion is observed.

The details of the dc octopole device have been described in earlier publications.[1-4] The device is an axisymmetric octopole with a major radius of 1.43 m. An additional toroidal field up to 400 G on the multipole axis can be applied. Helium plasma is injected from a coaxial plasma gun with an initial density of about 10^{17} m^{-3}. The base pressure of the system is 5×10^{-8} Torr or lower; however, influx of neutral helium from the plasma gun increases the pressure to 1×10^{-5} Torr after plasma injection. A Langmuir probe is used to measure the plasma density, and it is calibrated with a 10-cm microwave interferometer. Electron temperature is determined from He I (5876 Å) light intensity measured with an optical interference filter.[5,6] A microwave system (1 kW, 3 GHz) providing nonresonant heating is used to vary the plasma temperature.

The measurement with the octopole field only will be discussed first. The plasma density decays as 1/(time) initially. As reported previously[1,2] this decay is due to classical diffusion. When the density becomes smaller (electron plasma frequency $\leq$ electron cyclotron frequency), the decay mode changes. In this low-density regime the density is a parabolic function of time at each point on the profile. Figure 1 shows a plot of the square root of the plasma density n as a function of time t. Here, the mean value of the poloidal magnetic field $\langle B_\chi\rangle \equiv \oint(d\chi/B_\chi)/\oint(d\chi/B_\chi^2)$ at the half-flux line is 1.0×10^{-2} Wb/m^2, where χ is the magnetic potential, and the measured electron temperature is 900°K. The ion temperature is estimated from electron-ion and ion-neutral relaxation times to be close to the wall temperature.

The fact that the rate $d\sqrt{n}/dt$ is constant in the low-density regime shows that the diffusion coefficient is proportional to $n^{-1/2}$. This is quite different from the density dependences of classical diffusion and Bohm diffusion.

The dependence of the diffusion coefficient on the parameters other than the density can be studied by measuring the rate $d\sqrt{n}/dt$ under various conditions. In Fig. 2 the rate $d\sqrt{n}/dt$ is plotted as a function of the reciprocal of the averaged poloidal field strength. The result indicates that the diffusion coefficient is a very weak function of the magnetic field strength. Here again the scaling is quite different from either classical or Bohm diffusion scaling.

Figure 3(a) shows the rate $d\sqrt{n}/dt$ as a function of the electron temperature T. The electron tem-

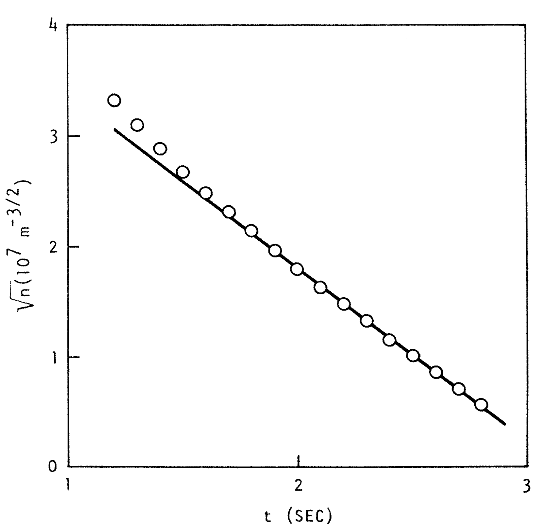

FIG. 1. Square root of the density versus time at the separatrix.

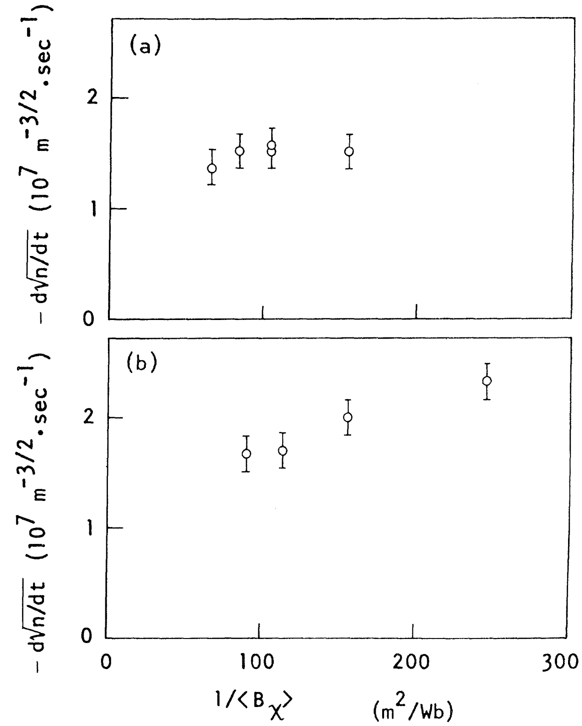

FIG. 2. The rate $d\sqrt{n}/dt$ as a function of reciprocal of the average poloidal field (a) with no toroidal field, and (b) with the toroidal field.

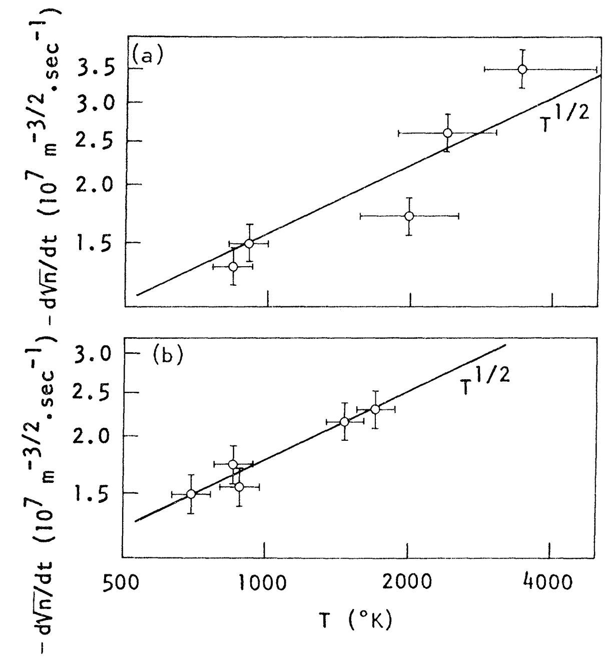

FIG. 3. The rate $d\sqrt{n}/dt$ as a function of temperature (a) with no toroidal field, and (b) with the toroidal field.

perature is varied by use of the nonresonant microwave heating and it is determined by the optical measurement. This result shows the diffusion coefficient has approximately $\sqrt{T}$ dependence.

The mass dependence of the diffusion coefficient is determined by adding a small amount of nitrogen gas to the base pressure which is mostly helium. Then, because of the exothermic charge exchange process,[7] helium ions are replaced by nitrogen ions ($He^+ + N_2 \rightarrow N^+ + N + He$). For example, at the pressure of 2×10^{-8} Torr nitrogen, the charge-exchange time is about 1 sec, so most of the ions are nitrogen ions at the time the diffusion rate is measured. Experimentally, at 2×10^{-8} Torr nitrogen partial pressure the rate $d\sqrt{n}/dt$ becomes smaller by a factor of about 2. Further increase in the nitrogen pressure produces no further change in the rate $d\sqrt{n}/dt$. This suggests that the diffusion coefficient is inversely proportional to the square root of the ion mass. (Note that experimentally the rate has very weak dependence on the base pressure below 5×10^{-8} Torr.) Based upon the observations above, the experimental scaling law of the diffusion coefficient is given by

$$D \propto (T/M)^{1/2} n^{-1/2}. \qquad (1)$$

The observed loss in this regime cannot be explained by the loss due to supports or error magnetic fields because of the following facts.

(a) The plasma loss directly to the supports is an order of magnitude smaller than the cross-field loss in this experiment. It has been shown that the presence of the support enhances the cross-field loss.[8,9] However, the enhanced loss is only of the same order as the direct loss to the support. Also, the density decays exponentially when the support loss dominates,[8,9] unlike the decay mode in the present experiment. In addition, the decay mode change from the classical to the new diffusion regime is not accompanied by change in the support loss rate.

(b) Measurements with the externally applied error magnetic fields over a density range including the present regime indicate that the density decay due to the error fields is exponential and that the addition of the toroidal field greatly reduces the loss. However, observations in the new diffusion regime are that the density is a parabolic function of time and the addition of the toroidal field does not alter the loss rate, as will be discussed later.

Recently, a number of theoretical works have been published about "classical" diffusion due to thermal equilibrium fluctuations.[10-20] Since there is no detailed calculation for inhomogeneous plasma, we compare the experimental results with the theoretical calculations for uniform plasma by Okuda and Dawson. The theory shows B-independent diffusion between the regions of $1/B^2$ and $1/B$ diffusion.[11-13] The diffusion coefficient D due to the thermal equilibrium fluctuation modes for which $\vec{k}\cdot\vec{B}=0$ is given by

$$D = 2^{3/2}\frac{c}{B}\left[\frac{k_B T \ln(l_\perp k_{max})}{(1+\omega_{pi}^2/\omega_{ci}^2+\omega_{pe}^2/\omega_{ce}^2)\,l_\parallel}\right]^{1/2}, \qquad (2)$$

where ω_p and ω_c are the angular plasma and cyclotron frequencies; the subscripts e and i denote electron and ion quantities, respectively; k_B is Boltzmann's constant; $l_\perp$ and $l_\parallel$ are dimensions perpendicular to and parallel to the magnetic field lines of force, respectively; and k_{max} is the largest allowed wave number perpendicular to the magnetic field lines, which may be given by ρ_i^{-1}, the reciprocal of the ion gyroradius.

In our case the ion dielectric constant is large compared to unity, $\omega_{pi}^2/\omega_{ci}^2 \gg 1$. Therefore, the diffusion coefficient D is reduced to

$$D = \frac{(2k_B T/M)^{1/2}[\ln(l_\perp k_{max})]^{1/2}}{(\pi n l_\parallel)^{1/2}}, \qquad (3)$$

where M is ion mass. The observed scaling law shown in Eq. (1) agrees with all the dependences (n, B, T, M) of the theoretical diffusion coefficient [Eq. (3)]. In the actual plasma both the magnetic field and the plasma density are inhomogeneous. The plasma dielectric constant is modified by the gradients. However, the effect on the diffusion coefficient is only through the logarithmic term and is small. Also the unequal electron and ion temperatures have not been taken into account.

From the diffusion equation in the flux function coordinates, the slope $d\sqrt{n}/dt$ may be given by

$$\frac{d\sqrt{n}}{dt} \approx \frac{Dn^{1/2}\oint R^2\,d\chi}{(\Delta\Psi)^2\oint B_\chi^{-2}\,d\chi}, \qquad (4)$$

where $\Delta\Psi$ is the half-width of the square-root density profile in the flux-function co-ordinate. In the case of Fig. 1, $(2\kappa T/M)^{1/2}$ is 1.9×10^3 m/sec, $l_\perp k_{max}$ is about 50, and $l_\parallel$ is about 1 m; therefore, $Dn^{1/2}$ is 2.7×10^3 m$^{1/2}$ sec^{-1}, and the value of $(\Delta\Psi)^2\oint(d\chi/B_\chi^2)/\oint R^2\,d\chi$ is about 6×10^{-4} m^2. Accordingly, the theoretical value of the slope $d\sqrt{n}/dt$ is 4.5×10^6 m$^{-3/2}$ sec^{-1}, which is smaller than the observed value by a factor of 3.5. This factor may be due to the fact that a theory for uniform plasma is being applied to explain the data.

The addition of the toroidal magnetic field does not alter the diffusion in this regime. The dependences on density, temperature, magnetic field, and ion mass are virtually unaffected. A density decay very similar to Fig. 1 is observed in a plot of $\sqrt{n}$ versus time. The magnetic field dependence of the diffusion coefficient is very weak as shown in Fig. 2(b). Here the magnitude of the magnetic field is varied maintaining the same ratio between the toroidal and poloidal magnetic fields. The toroidal field is 1.4×10^{-2} Wb/m^2 on the multipole axis corresponding to the average poloidal field $\langle B_\chi\rangle$ of 8.8×10^{-3} Wb/m^2. The temperature dependence of the diffusion coefficient is shown in Fig. 3(b). The measurements with the addition of nitrogen gas indicate the mass dependence of $M^{-1/2}$. In short, the cases with and without the toroidal field are experimentally indistinguishable.

The octopole configuration with the toroidal field has sizable magnetic shear; therefore, the flutelike wave $k_\parallel = 0$ is not possible. The theoretical calculations[18,19] for sheared configurations do not suggest diffusion coefficients which fit these experimental results. These theories may be modified to include the effects of diffusion itself on the dielectric constant.

The fluctuations are constantly being generated and damped. The lifetime of a fluctuation is determined by the perpendicular diffusion damping time $(k_\perp^2 D)^{-1}$ and parallel phase mixing time $(k_\parallel v_i)^{-1}$. For the fluctuations with $k_\perp^2 D \gg k_\parallel v_i$, the parallel mixing is unimportant. Hence, the fluctuations with short perpendicular wavelength are not affected by the finite parallel wavelengths, thus not by the magnetic shear. (The value of $k_{\perp\,max}^2 D$ is an order of magnitude bigger than $k_{\parallel\,min} v_i$ under typical experimental conditions. Therefore, modes with $k_\perp^2 D \gg k_\parallel v_i$ can be excited.) The dielectric constant is given by

$$\epsilon \approx 1 + \omega_{pi}^2/\omega_{ci}^2 + (k\lambda_{D_e})^{-2}. \qquad (5)$$

The diffusion coefficient may be calculated by following the method used by Okuda and Dawson and it is given by

$$D \approx 2^{3/2}\frac{c}{B}\left\{\frac{k_B T \ln[(T_e/T_i)+1]^{1/2}}{(1+\omega_{pi}^2/\omega_{ci}^2)\,l_\parallel}\right\}^{1/2}. \qquad (6)$$

This is the same as Eq. (2) except for the minor difference in the logarithmic factor.

In conclusion, the Okuda-Dawson theory of "classical" diffusion due to thermal equilibrium fluctuations provides a good fit to the experimental results. The observed dependences of the diffusion coefficient on density, magnetic field, temperature, and ion mass are consistent with those of the theoretical diffusion coefficient. The magnitude of the observed diffusion coefficient is larger by a factor of 3.5 than the theoretical value. The results with the toroidal field may be explained by considering that the perpendicular diffusion damping time is shorter than the parallel phase mixing time.

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A Fundamental Property of Quantum-Mechanical Entropy

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We have proved the strong subadditivity of quantum-mechanical entropy and the Wigner-Yanase-Dyson conjecture.

There are some properties of entropy, such as concavity and subadditivity, that are known to hold (in classical and in quantum mechanics) irrespective of any assumptions on the detailed dynamics of a system. These properties are consequences of the definition of entropy as

$$S(\rho) = -\operatorname{Tr}\rho\ln\rho \text{ (quantum)}, \tag{1a}$$

$$S(\rho) = -\int\rho\ln\rho \text{ (classical continuous)}, \tag{1b}$$

$$S(\rho) = -\sum\rho_i\ln\rho_i \text{ (classical discrete)}, \tag{1c}$$