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Relation between Regge Trajectory Intercepts and the Asymptotic Behavior of the Multiplicity Moments f_k

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Assuming that the multiplicity moments $f_k(s)$ formed from the topological cross section $\sigma_n(s)$ or from integrated inclusive correlation functions behave for large s as $f_k(s) \sim c_k \ln s$, we discuss a general relation between the c_k and the leading asymptotic behavior of $\sigma_n(s)$. The relation has been given by Harari, but our arguments demonstrate that it is not connected with any hypothesis concerning the dependence of hadronic parameters on some underlying coupling constants.

In an interesting article[1] Harari has suggested that the Pomeranchukon trajectory intercept α_P is exactly equal to 1 for any value of an underlying hadronic coupling constant g. At the same time he argued that the "ordinary" Regge trajectory $\alpha_R(g)$ which, for example, determines the power dependence in s of the n-particle cross section $\sigma_n(s)$, must vary with g and, most strikingly, have the limiting value $\alpha_R(g=0)=1$. Within the context of a rather general form for $\sigma_n(s)$, he then derived a relation between $\alpha_R(g)$ at the physical value of g and the coefficients of a presumed $\ln s$ behavior in the multiplicity moments

$$f_1 = \langle n\rangle,\ f_2 = \langle n(n-1)\rangle - \langle n\rangle^2,\ \text{etc.} \tag{1}$$

formed from $\sigma_n(s)$.

We would like to demonstrate that in fact the relation between the value of $\alpha_R(g)$ and the f_k's is entirely independent of any assumption on $\alpha_P(g)$ of $\alpha_R(0)$ but follows directly from the presumed behavior[2]

$$f_k(s) \underset{s\to\infty}{\simeq} c_k \ln s + d_k. \tag{2}$$

However interesting the idea that $\alpha_P(g)$ is independent of g and $\alpha_R(0)=1$ indicating some underlying "vector" field theory, the testable relations presented by Harari to defend that idea have no bearing on the issue.

To proceed, we form the generating function

$$R(z) = \sum_{n=0}^{\infty} z^n \sigma_{n+2}(s)/\sigma_T(s) \tag{3}$$

$$= \exp\left[\sum_{k=1}^{\infty} (z-1)^k f_k(s)/k!\right]. \tag{4}$$

Here $\sigma_T(s)$ is the total cross section. If $f_k(s)$ behaves as in (2), then

$$F(z) = \ln R(z) \underset{s\to\infty}{\simeq} p(z)\ln s + q(z), \tag{5}$$

where

$$p(z) = \sum_{k=1}^{\infty} (z-1)^k c_k/k!, \tag{6}$$

and

$$q(z) = \sum_{k=1}^{\infty} (z-1)^k d_k/k!. \tag{7}$$

Using the fact that for $n \geq 1$,

$$\frac{n!\,\sigma_{n+2}(s)}{\sigma_T(s)} = e^{F(0)}\left[\left(F' + \frac{d}{dz}\right)^{n-1} F'(z)\right]_{z=0}, \tag{8}$$

we learn that

$$\frac{n!\,\sigma_{n+2}(s)}{\sigma_T(s)} \underset{s\to\infty}{\simeq} s^{p(0)}\,(\text{polynomial in } \ln s). \tag{9}$$

Suppose now with Harari that there is only one "component" to the production mechanism for $\sigma_{n+2}(s)$, and, up to logarithms, that it has the high-energy behavior

$$\sigma_{n+2}(s) \underset{s\to\infty}{\simeq} s^{2\alpha_R - 2} \tag{10}$$

which builds up the behavior of $\sigma_T(s)$ to be

$$\sigma_T(s) \underset{s\to\infty}{\simeq} s^{\alpha_P - 1}. \tag{11}$$

It then follows immediately from (9) that

$$2\alpha_R - \alpha_P - 1 = p(0) = \sum_{k=1}^{\infty} (-1)^k c_k / k!, \tag{12}$$

which is exactly the relation that Harari employs to defend his interesting ideas about α_P and α_R as functions of g. We now see the independence of the relation (12) from any notion of the behavior of Regge trajectory intercepts on hadronic coupling constants.

If we pursue this line of thought further and imagine that there are both "multiperipheral" and "diffractive" contributions to production amplitudes, each of which builds up the same power behaviors in σ_T, then we would write for $\sigma_{n+2}(s)$

$$\sigma_{n+2}(s) \underset{s\to\infty}{\simeq} h_n^M(s) s^{2\alpha_R - 2} + h_n^{MD}(s) s^{\alpha_P + \alpha_R - 2} + h_n^D(s) s^{2\alpha_P - 2}, \tag{13}$$

where the multiperipheral piece is taken to generate α_R and the diffractive piece involves α_P itself. Now the full $f_k(s)$ no longer behaves as $\ln s$, as is well known,[3] but grows as $(\ln s)^k$, when the f_k of each component behaves as $\ln s$. Indeed, if we imagine that each component of $\sigma_{n+2}(s)$ produces an f_k which grows like $\ln s$:

$$f_k \underset{s\to\infty}{\simeq} c_k^M \ln s + d_k^M \tag{14}$$

and similarly for f_k^{MD} and f_k^D, then we find three relations like (12):

$$2\alpha_R - \alpha_P - 1 = \sum_{k=1}^{\infty} (-1)^k c_k^M / k!, \tag{15}$$

$$\alpha_R - 1 = \sum_{k=1}^{\infty} (-1)^k c_k^{MD} / k!, \tag{16}$$

and

$$\alpha_P - 1 = \sum_{k=1}^{\infty} c_k^D (-1)^k / k!. \tag{17}$$

Of these, the last is self-inconsistent, if $\alpha_P = 1$. Note that

$$R^D(z) = \sum_{n=0}^{\infty} z^n \sigma_{n+2}^D(s) \Big/ \sum_{n=0}^{\infty} \sigma_{n+2}^D(s) \tag{18}$$

$$= \exp\left[\sum_{k=1}^{\infty} (z-1)^k f_k^D(s)/k!\right] \tag{19}$$

has the property $R^D(0) < 1$ because of the positivity of the σ_n, so

$$\sum_{k=1}^{\infty} (-1)^k f_k^D / k! < 0,$$

which implies via (17) that $\alpha_P < 1$. This is a restatement of the result of LeBellac[4] and in the present context implies that $f_k^D(s)$ probably behaves as $(\ln s)^k$.

The only ingredient in the key assumption that $f_k(s) \sim \ln s$ is that the singularities in the complex j plane at $t = 0$ be isolated and factorizable. Since this is true in the form of model that Harari discusses, it is not surprising that he finds (12). Unfortunately it has no bearing on his other intriguing ideas about the Pomeranchukon intercept being fixed at 1.

We wish to thank Adam Schwimmer and Bill Frazer for helpful discussions. Further, we acknowledge a very informative correspondence with Haim Harari on this subject.

[1]H. Harari, Phys. Rev. Lett. 29, 1708 (1972).

[2]The arguments we are about to present can probably be found in the notebooks of many of the readers. One of us (H.D.I.A.) derived the main result (12) with G. Farrar and L. M. Saunders some time ago, and we learned from W. Frazer that it has also been known to him for quite a while. This relation has also been derived in terms of cluster decomposition properties of φ^3 Feynman graphs by S. J. Chang, T. M. Yan, and Y. P. Yao, Phys. Rev. D 5, 271 (1972) and earlier papers referred to there. We also note that the technique of using generating functions such as our Eqs. (3) and (4) was first used in multiparticle hadronic physics by A. H. Mueller, Phys. Rev. D 4, 150 (1972), and Eq. (12) can, naturally enough, be read from his formula there.

[3]This is discussed by A. Białas, K. Fialkowski, and K. Zalewski, Jagellonian University Report No. TPJU-5/72 (to be published), and by W. R. Frazer, R. D. Peccei, S. Pinsky, and C.-I Tan, University of California, San Diego, Report No. 10P10-113 (to be published).

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