$(N_p + N_a) = 0.5$ at $\theta = 67°$ and $E_p = 8.5$ MeV. In addition, it provided a further check on the method.

The method we have applied is an independent way of measuring multipole mixing, with a quite wide range of applications, especially as reasonably intense polarized beams can now be obtained from accelerators. The multipole mixing can be extracted unambiguously from a measurement of a single asymmetry coefficient.

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Intermediate Structure in Isospin-Forbidden Reactions

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A model is proposed which leads to intermediate structure in isospin-forbidden reactions. It is shown that the effects predicted by the model occur in the recent data of Jolivette for $^{16}O(d,\alpha_1)^{14}N(2.31)$.

Intermediate structure in resonant reactions has been associated with doorway states, that is, states of relatively simple structure which serve to couple continuum states to the more complicated structures of the compound nucleus.[1] When the doorway point of view is meaningful, energy averages of the S-matrix elements should give a resonant structure associated with the energies and widths of the doorway states.

I propose a model for isospin-forbidden resonant reactions in which bridge states play a role similar to doorway states.[2] Let us suppose that, in the absence of isospin mixing, we have, in any energy region, two disjoint sets of channels, e.g., one with $T = 0$ and one with $T = 1$. These sets will include both open (P) and closed (Q) channels in the sense of the unified reaction theory.[3] I propose that in the presence of a weak isospin-mixing force (which couples $T = 0$ and $T = 1$ channels) the system may be described schematically by the block diagram in Fig. 1, where the boxes represent channels and the connecting lines represent couplings. The channels Q (Q') are composed of the channels q (q') and b (b'). The channels indicated by b and b' represent pairs of states, one of each isospin for which the coupling $T = 0$ to $T = 1$ is especially strong. These states provide the access, or bridge, between the different isospin channels, in a manner analogous to that by which doorway states link open channels to compound states.

Next consider the formal structure for the T-matrix elements under the assumption that the bridge mechanism, given in Fig. 1, is dominant. For simplicity we first ignore the q and q' channels and assume only one b and one b' channel.

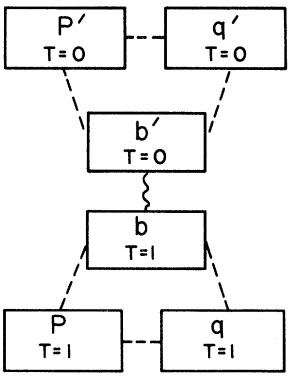

FIG. 1. Schematic diagram of the bridge mechanism.

Then

$$T_{if}(E)=\frac{\langle\chi_f^{(-)}|H|b\rangle\langle b|H_c|b'\rangle\langle b'|H|\chi_i^{(+)}\rangle}{(E-E_b+\frac{1}{2}i\Gamma_b)(E-E_{b'}+\frac{1}{2}i\Gamma_{b'})}, \tag{1a}$$

where

$$(E-H_{PP})\chi_f^{(+)}=0, \tag{1b}$$

$$E_b-i\Gamma_b/2\equiv\epsilon_b=\langle b|H_{QP}+H_{QP}(E^+-H_{PP})^{-1}H_{PQ}|b\rangle, \tag{1c}$$

and H_c represents the isospin-nonconserving part of the Hamiltonian. When the q and q' channels are included, it is convenient to consider the full compound nuclear states of Q (Q') which are composed of a mixture of components from b (b') and q (q'). If we assume one dominant pair of bridge states, b and b', then the T matrix has the form

$$T_{if}=\sum_{ss'}\frac{\langle\chi_f^{(-)}|H|s\rangle\langle s|b\rangle\langle b|H_c|b'\rangle\langle b'|s'\rangle\langle s'|H|\chi_i^{(+)}\rangle}{(E-\epsilon_s)(E-\epsilon_{s'})}. \tag{2}$$

Here s refers to those closed-channel states of isospin $T=1$ which include b as a component, and s' refers to those closed-channel states of $T=0$ which include b'. Equation (2) may be written as a sum of simple poles,

$$T_{if}=\sum_{ss'}N_{ss'}(E)\left(\frac{1}{E-\epsilon_s}-\frac{1}{E-\epsilon_{s'}}\right), \tag{3a}$$

with

$$N_{ss'}(E)=\frac{\langle\chi_f^{(-)}|H|s\rangle\langle s|b\rangle\langle b|H_c|b'\rangle\langle b'|s'\rangle\langle s'|H|\chi_i^{(+)}\rangle}{\epsilon_s-\epsilon_{s'}}. \tag{3b}$$

Approximating T_{if} by a sum of Breit-Wigner terms, one obtains

$$T_{if}=\sum_s\frac{C_s}{E-\epsilon_s}+\sum_{s'}\frac{C_{s'}}{E-\epsilon_{s'}}, \tag{4a}$$

where

$$C_s=\sum_{s'(b')}N_{ss'}(E_s);\quad C_{s'}=-\sum_{s(b)}N_{ss'}(E_s). \tag{4b}$$

It is clear that if $N_{s's}(E)$ were independent of energy, the numerator parameters would satisfy the sum rule

$$\textstyle\sum_s C_s+\sum_{s'}C_{s'}=0. \tag{5}$$

We next consider the energy variation of $N_{s's}(E)$. This comes from the energy dependence of the states in the open channels. If we assume that $|\langle s|H|\hat{\chi}_i(E)\rangle_l|^2$ is composed of a penetration factor $\mathcal{P}(E)$ and an energy-independent factor, then we may write $N_{s's}(E)$ from Eq. (3b) as

$$(N_{ss'})_{l_f l_i}=\exp[i\delta_{l_f}(E)+i\delta_{l_i}(E)][\mathcal{P}_{l_f}(E)\mathcal{P}_{l_i}(E)]^{1/2}\hat{N}_{ss'}, \tag{6}$$

where $\hat{N}_{ss'}$ is energy independent. We define a reduced T matrix

$$\hat{T}=\sum_{ss'}\hat{N}_{s's}\left(\frac{1}{E-\epsilon_s}-\frac{1}{E-\epsilon_{s'}}\right). \tag{7}$$

If $\hat{T}$ is approximated by a sum of Breit-Wigner terms, then

$$\hat{T}=\sum\frac{\hat{C}_s}{E-\epsilon_s}+\sum\frac{\hat{C}_{s'}}{E-\epsilon_{s'}}, \tag{8}$$

where the parameters $\hat{C}_s$ and $\hat{C}_{s'}$ are defined by equations analogous to Eq. (4b) and satisfy the sum

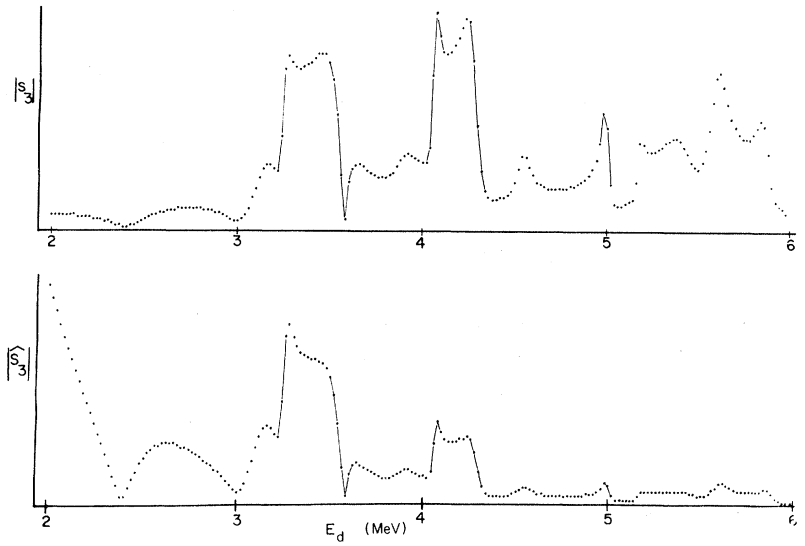

FIG. 2. The magnitude of the $^{16}O(d,\alpha_1)^{14}N(2.31)$ S_3 matrix and $\hat{S}_3$ matrix as a function of energy.

rule given in Eq. (5).

Finally, we consider the results of an energy average of the $\hat{T}$ matrix. Following the techniques used in examining doorway structure,[1] one finds (for an energy averaging width larger than the spacing of E_s but smaller than the spacing of E_b)

$$\langle \hat{T}_l(E)\rangle \approx \frac{\gamma_{fb_l}\langle b|H_c|b'\rangle\gamma_{ib_l'}}{[E-E_b'+\frac{1}{2}i(\Gamma_b^{\uparrow}+\Gamma_b^{\downarrow})][E-E_{b'}'+\frac{1}{2}(\Gamma_{b'}^{\uparrow}+\Gamma_{b'}^{\downarrow})]}, \tag{9a}$$

where

$$\sqrt{\mathcal{P}_{lf}}\gamma_{fb_l} = \left({}_l\langle\hat{\chi}_f|H|b\rangle + \sum_q \frac{{}_l\langle\hat{\chi}_f|H|q\rangle\langle q|H|b\rangle}{E-\epsilon_q+il/2}\right), \tag{9b}$$

$$\sqrt{\mathcal{P}_{ib'}}\gamma_{ib_l'} = \left(\langle b'|H|\hat{\chi}_i\rangle_l + \sum_{q'} \frac{\langle b'|H|q'\rangle\langle q'|H|\hat{\chi}_i\rangle_l}{E-\epsilon_{q'}+iI/2}\right). \tag{9c}$$

I have applied this theory to the reaction $^{16}O(d,\alpha_1)^{14}N(2.31)$ which has been carefully analyzed by Jolivette.[4] He has developed a fitting scheme for determining the partial-wave S-matrix elements. I chose the $l=3$ partial wave for special attention since, in the low-energy region, his fit for this wave was the least ambiguous.[4]

In Fig. 2 is shown the magnitude of the $l=3$ S-matrix element, $|S_3|$, as a function of energy, obtained by Jolivette from his data. Also presented is $|\hat{S}_3|$, obtained by dividing S_3 by a factor proportional to the geometric mean of the entrance- and exit-channel Coulomb penetration factors. The penetration effects are significant for the energies included in Fig. 2.

The magnitude of $\hat{S}_3$ was fitted by a sum of Breit-Wigner terms, and the numerators for the cluster of nine levels between 3 and 4.5 MeV were found to have essentially the same phase (except for sign) and to sum as

$$\begin{aligned}\textstyle\sum \mathrm{Re}\hat{C}_j &= 2.85-2.46=0.39,\\ \textstyle\sum \mathrm{Im}\hat{C}_j &= 1.00-1.08=-0.08.\end{aligned} \tag{10}$$

The normalization of $\hat{C}_j$ is arbitrary, so the sum of positive and negative values individually are indicated. These results can be compared with similar sums for the numerators which Jolivette obtained for the unreduced S_3,

$$\begin{aligned}\textstyle\sum \mathrm{Re}C_j &= 2.85-1.63=1.22,\\ \textstyle\sum \mathrm{Im}C_j &= 1.34-1.51=-0.17.\end{aligned} \tag{11}$$

(The normalization is arbitrary but chosen to be comparable to that of $|\hat{S}_3|$.) Improvement in satisfying Eq. (5) is obtained when penetration factors are removed, as should be the case.

Finally, energy averages of the $\hat{S}$-matrix elements were taken, using a Lorenzian weighting factor with width I. This gave

$$\langle\hat{S}\rangle=\sum\hat{C}_j[E-E_j+\tfrac{1}{2}i(\Gamma_j+I)]^{-1}. \quad (12)$$

An energy average, with $I=250$ keV, for the nine levels clustered between 3 and 4.5 MeV (deuteron energy) is shown in Fig. 3. This average was fitted by a sum of two Breit-Wigner terms which would be predicted by the bridge mechanism (one pole for the state b and one for the state b'). The formalism would predict numerators which are equal in magnitude and differ in phase by 180°. For the best fit, resonant energies were obtained about 500 keV apart with $\Gamma_1=286$ keV and $\Gamma_2=470$ keV. The numerators have magnitudes of 2.80 and 2.35 and differ in phase by 204°. I interpret this result as supporting the postulated bridge structure.

Let us next consider the nature of the pair of bridge states involved in the above example. These should be states with $T=0$ and $T=1$ whose mutual spatial overlap is great and whose coupling to the $(^{16}O+d)$ and $(^{14}N^*+\alpha)$ channels is also great. These considerations favor weak coupling states roughly of the form

$$[(^{16}O^*)^J_{T=0}\times(np)_{T=1}]^{\mathcal{J}=J}_{T=1}$$

for b, and

$$[(^{16}O^*)^J_{T=0}\times(np)_{T=0}]^{\mathcal{J}=J}_{T=0}$$

for b', where the states of ^{16}O are those with large components of $(C^{12}+\alpha)$. A determination of states in ^{16}O by $(C^{12}+\alpha)$ elastic scattering[5] shows a $3^{(-)}$ state at 11.60 MeV excitation with a width of 830 keV, and $\Gamma_\alpha/\Gamma\approx1$. (The cluster of nine levels in F^{18}, considered above, lies in the range of excitation energies 10.3–11.5 MeV.) We thus propose that the structure in the $3^{(-)}$ bridge states is that associated with states of the form given above, where $^{16}O^*$ is $^{16}O(11.60)^{J=3}_{T=0}$.

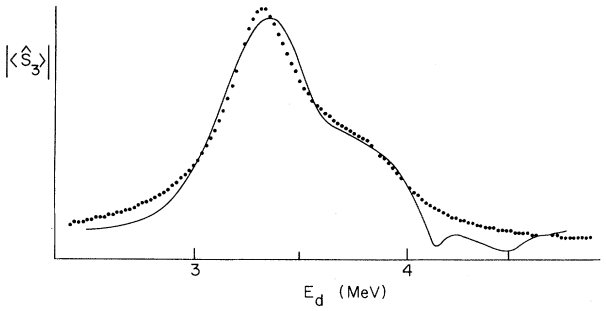

FIG. 3. Calculated $|\langle\hat{S}_3\rangle|$ (solid curve) and fit (dotted curve) for levels clustered between 3.0 and 4.5 MeV.

In the spirit of weak coupling, one may imagine a set of bridge pairs whose energies are primarily given by the energies of the ^{16}O component, with an energy splitting between members of each pair of about 500–600 keV, as is found for the $3^{(-)}$ levels discussed above. Preliminary study of the energy average of the $5^{(-)}$ S-matrix elements between 6.4 and 7.5 MeV (deuteron energy) indicates a structure similar to that in the $3^{(-)}$ case. I would propose that this structure is associated with the $5^{(-)}$ state, $^{16}O(14.82)$, seen in $^{12}C+\alpha$ scattering. (The difference in excitation energy between the $3^{(-)}$ and $5^{(-)}$ clusters corresponds closely to the difference in energy between the two states in ^{16}O.) Furthermore, with the $3^{(-)}$-state identification given above, correlations are found in energy and spin between gross features (large level density spanning about 1 MeV in energy) of the $^{16}O(d,\alpha_1)^{14}N$ data and each of the twelve $(T=0)$ $(^{12}C+\alpha)$ resonances found between 11.50 and 16.50 MeV excitation in ^{16}O. I suggest, therefore, that each of the $^{12}C+\alpha$ resonances may be associated with a pair of bridge states for the isospin-forbidden reaction.

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