Transition Charge Densities of the 2_1^+ and 2_2^+ States in ^{106}Pd and ^{114}Cd

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We have obtained inelastic electron-scattering 2_1^+ and 2_2^+ form factors in ^{106}Pd and ^{114}Cd in the momentum-transfer range between 0.6 and 1.8 fm^{-1}. The transition charge densities of the 2_1^+ and 2_2^+ levels were extracted from the experimental form factors by assuming a phonon model in which the one- and two-phonon states are mixed. The wave functions (transition charge densities) obtained have been found to describe reasonably the measured 2_1^+ quadrupole moments and the radiative transition rates of the crossover and cascade γ rays.

Both ^{106}Pd and ^{114}Cd are classic examples of vibrational nuclei, having near-harmonic level sequences.[1] From Coulomb excitation experiments, however, the static quadrupole moments of the first excited states have been found to be rather large,[2-5] while the harmonic vibrator model predicts a vanishing quadrupole moment. The deviation from this model is also seen in the transition rates of the crossover and cascade γ rays from the 2_2^+ state.[2,6]

Inelastic electron scattering is a little-explored technique for study of the higher phonon states, but because it gives charge distributions associated with the transitions, it could be an important source of information. We have applied this technique to the study of the 2_1^+ and 2_2^+ states of ^{106}Pd and ^{114}Cd. Then using the deduced transition charge densities in a phonon model in which the one- and two-quadrupole-phonon states are mixed,[7] we have calculated quadrupole moments in excellent agreement with the measured values, while at the same time obtaining values in reasonable agreement with the transition rates for the γ rays from these levels.

The experiment was performed by using the high-resolution equipment at the Tohoku 300-MeV electron linear accelerator laboratory for incident electron energies of 183 and 250 MeV. The differential cross sections were obtained in the angular range between 35° and 90°. A 96.8% enriched ^{106}Pd (48.6 mg/cm^2) target and a 99.2% ^{114}Cd (50.3 mg/cm^2) target were used. The spectrometer and detection apparatus have been described elsewhere.[8] The cross sections were normalized against elastic and inelastic electron-carbon cross sections. The experimental form factors were obtained by dividing the experimental cross section by the Mott cross section.

In the present measurement we have not resolved the three two-phonon levels at 1.127 (2_2^+), 1.133 (0_2^+), and 1.228 (4_1^+) MeV in ^{106}Pd and 1.133 (0_2^+), 1.208 (2_2^+), and 1.283 (4_1^+) MeV in ^{114}Cd. In addition, the 2_2^+ form factor for ^{114}Cd includes the two intrinsic states at 1.305 (0_3^+) and 1.363 (2_3^+) MeV. For the two-phonon triplets (p,p') studies[9,10] have suggested that the admixtures of the one-phonon and two-phonon states are needed to explain the observed cross sections of the 2_2^+ levels, while such mixings are very small for the 0_2^+ and 4_1^+ levels. Thus the contributions of the 0_2^+ and 4_1^+ levels were estimated by assuming them to be pure two-phonon states. The cross sections of the 0_3^+ and 2_3^+ levels in ^{114}Cd are known[11] to be very small relative to that of the 2_2^+ level and can be neglected. The form factors for the 2_1^+ levels (at 0.512 MeV in ^{106}Pd and at 0.558 MeV in ^{114}Cd) and unresolved form factors for the 0_2^+, 2_2^+, and 4_1^+ triplets are displayed in Figs. 1 and 2 for ^{106}Pd and ^{114}Cd, respectively, as functions of the scattering angle. The data at 183 MeV were normalized to those at 250 MeV by taking into account the difference in bombarding energies with the help of a distorted-wave Born approximation (DWBA) calculation.

Lightbody has applied the anharmonic vibrator model, in which a uniform charge density was assumed for the ground state, to the inelastic electron scattering data.[12] Hence the transition charge densities associated with the one- and two-phonon states in his calculation are, respectively, a δ function and its derivative with respect to the radius. A more realistic form of the transition charge density together with a DWBA calculation would be required for the present medium-heavy nuclei.

We start from the general form $\rho(r)$ of the ground-state charge density and as usual replace

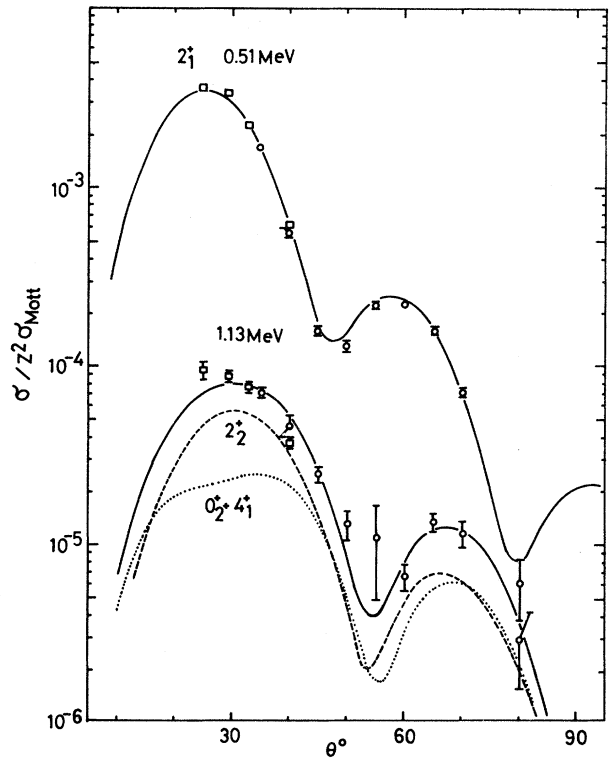

FIG. 1. Inelastic electron scattering form factors for the 2_1^+ and unresolved 0_2^+, 2_2^+, and 4_1^+ levels in ^{106}Pd at an incident energy of 250 MeV. The triplet form factors are compared with the sum of the 0_2^+ and 4_1^+ form factor (dotted line) and 2_2^+ form factor (dashed line).

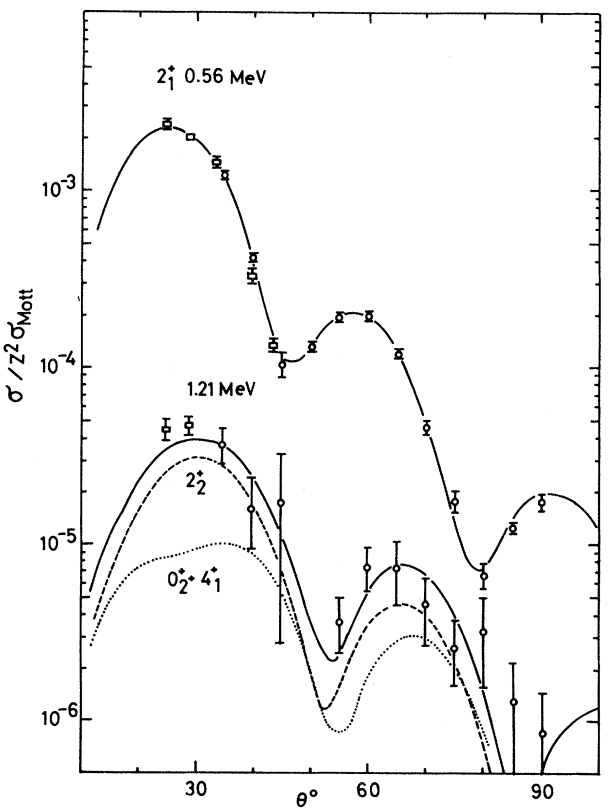

FIG. 2. The same as for Fig. 1 except for ^{114}Cd.

r by

$$r = r_0(1 + \sum_m q_{2m} Y_{2m} - \pi^{-1/2} \sum_m |q_{2m}|^2),$$

where

$$q_{2m} = (\beta/\sqrt{5})[a_{2m}^\dagger + (-)^m a_{2-m}],$$

$a^\dagger$ and a being phonon creation and destruction operators. Introducing the above form of r into $\rho(r)$ and expanding it with respect to β, we obtain the expressions

$$\rho_{\text{tr one-phonon}} = (\beta/\sqrt{5}) r (d/dr) \rho(r)$$

and

$$\rho_{\text{tr two-phonon}} = \left(\frac{5}{28\pi}\right)^{1/2} \left(\frac{\beta}{\sqrt{5}}\right)^2 r^2 \left(\frac{d^2}{dr^2}\right) \rho(r)$$

for the transition charge densities of the one-phonon and two-phonon states. The wave functions of the first and second 2^+ states are taken to be a linear combination of the one-phonon and two-phonon states:

$$\psi_1 = (1 - \alpha^2)^{1/2} |\text{one phonon}\rangle - \alpha |\text{two phonon}\rangle$$

and

$$\psi_2 = \alpha |\text{one phonon}\rangle + (1 - \alpha^2)^{1/2} |\text{two phonon}\rangle.$$

Using the above wave functions together with the transition charge densities of the one- and two-phonon states, we calculated the differential cross sections leading to the first and second 2^+ states of ^{106}Pd and ^{114}Cd by the DWBA.[13] Here, the ground-state charge density $\rho(r)$ was assumed to be given by a two-parameter Fermi distribution,

$$\rho(r) = \frac{\rho_0}{1 + \exp[4.4(r - c)/t]}.$$

The 2_1^+, 0_2^+, 2_2^+, and 4_1^+ form factors were fitted simultaneously by adjusting the parameters c, t, β, and α; however, since the mixing parameter α is small, c, t, and β were determined from the 2_1^+ form factor only. The deduced c and t were slightly different from those of the ground-state charge density. By adjusting α the 2_2^+ form factors were determined so that the total sum of the 0_2^+, 2_2^+, and 4_1^+ form factors reproduced the experimental form factors. As

TABLE I. Values of c, t, β, and α (the parameters of ρ_{tr}), $B(E2)$ in single-particle units, and the 2_1^+ state static quadrupole moment.

					Present		Coulomb excitation	
	c	t	β	α	$\frac{B(E2)}{B_{sp}(E2)}$	Q_2 (b)	$\frac{B(E2)}{B_{sp}(E2)}$	Q_2 (b)
^{106}Pd	$0.97c_0$ [a]	$1.05t_0$ [b]	0.23	0.23	50 ± 5	-0.51 ± 0.07	48 ± 3 [c]	
							42 ± 3 [d]	-0.458 ± 0.059 [d]
^{114}Cd	$0.95c_0$ [e]	$t = t_0$ [f]	0.18	0.19	29 ± 3	-0.36 ± 0.07	31 ± 2 [g]	-0.32 ± 0.08 [g]
							34 [h]	-0.38 [h]

[a] $c_0 = 5.14$ fm. [c] Ref. 2. [e] $c_0 = 5.43$ fm. [g] Ref. 3.
[b] $t_0 = 2.59$ fm. [d] Ref. 5. [f] $t_0 = 2.50$ fm. [h] Ref. 4.

mentioned before the 0_2^+ and 4_1^+ form factors were calculated by assuming that these states are entirely two phonon. The calculated results are presented in Figs. 1 and 2. The parameters and the $B(E2)$ values in single-particle units are tabulated in Table I. The transition charge densities of the 2_1^+ and 2_2^+ states obtained are displayed in Fig. 3. For the purpose of comparison, the transition charge density of the pure two-phonon state is also shown. Relative to the peak of ρ_{tr} for the 2_1^+ state, the peak of the 2_2^+ state lies inward, indicating that the sign of α is positive. Accordingly the maximum (or minimum) of the 2_2^+ form factor appears at higher q that that of the 2_1^+ form factor.

By using the values of the parameters α, β, c, and t which were determined from the best fit to the form factors, we calculated the value of the static quadrupole moment of the first excited 2^+ state. The results are contained in Table I together with the Coulomb-excitation results. The radiative transition rates corresponding to $B(E2, 2_2^+ \rightarrow 2_1^+)/B(E2, 2_1^2 \rightarrow 0_1^+)$ and $B(E2, 2_2^+ \rightarrow 0_1^+)/B(E2, 2_1^+ \rightarrow 0_1^+)$ were also calculated with the same parameters. The results are tabulated in Table II. The nuclear wave functions (transition charge densities) extracted from the inelastic electron scattering form factors reproduce excellently the measured 2^+ quadrupole moment and significantly improve the predictions of the transition rates over those given by the harmon-

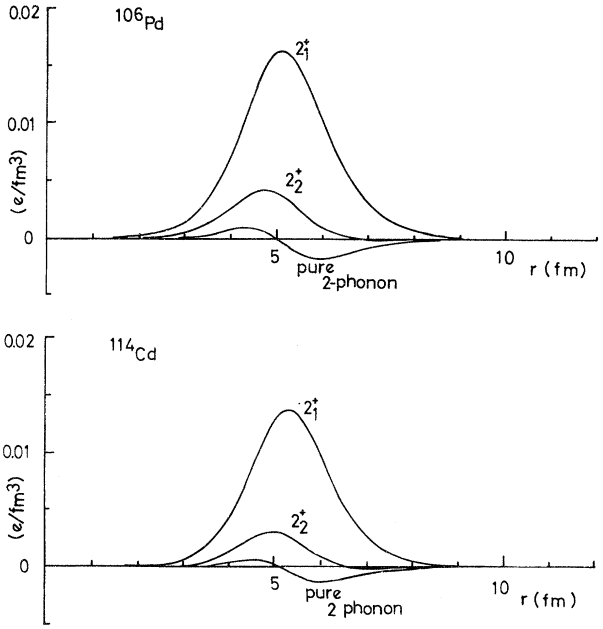

FIG. 3. The transition charge densities of the 2_1^+ and 2_2^+ levels determined from the experimental form factors. The transition charge densities of the pure two-phonon states are shown for comparison.

TABLE II. Values of the radiative transition rates for the crossover and cascade transitions from the 2_2^+ state.

	^{106}Pd		^{114}Cd	
	Present	Experimental	Present	Experimental
$\frac{B(E2, 2_2^+ \rightarrow 2_1^+)}{B(E2, 2_1^+ \rightarrow 0^+)}$	$1.39^{+0.06}_{-0.14}$	0.96 ± 0.13 [a]	$1.56^{+0.08}_{-0.13}$	0.75 ± 0.20 [b]
$\frac{B(E2, 2_2^+ \rightarrow 0^+)}{B(E2, 2_1^+ \rightarrow 0^+)}$	$(0.82^{+0.92}_{-0.72}) \times 10^{-2}$	$(2.51^{+0.43}_{-0.33}) \times 10^{-2}$ [a]	$0.75^{+1.05}_{-0.59}$	$(1.6 \pm 0.3) \times 10^{-2}$ [b]

[a] Ref. 2. [b] Ref. 6.

ic vibrator model. The transition charge density extracted from the fits to the experimental form factor may not be affected very much by which nuclear model is used. A microscopic description of the vibrator nuclei would be required to explain this transition charge density as well as the other quantities.

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Measurement of Multipole Mixing in Nuclear Radiation by a New Method*

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The multipole mixing in the $3^+(5.22\ \text{MeV}) \rightarrow 2^+(1.369\ \text{MeV})$ transition in ^{24}Mg has been determined by a method based on observing the asymmetry in the radiation emitted from a state which is oriented by a reaction initiated with a vector polarized beam. The particular reaction employed was $^{24}\text{Mg}(p,p')^{24}\text{Mg}(3^+)$.

A standard method for determining the multipole mixing present in the radiation emitted when a nucleus undergoes a transition from one state to another is to measure the anisotropy of the radiation that arises when the initial nuclear state is suitably oriented. The production and orientation of a state usually requires a reaction process, and, in such cases, it is usual to exploit the simplification brought about by making the reaction system axially symmetric. This requires that the initial particles are unpolarized and the ingoing and outgoing particle directions are fixed and collinear, or one of them is fixed while the other is isotropic, unobserved, or absent as in capture and decay. The oriented state then has a single axis of symmetry about which the radiation is emitted with cylindrical symmetry.

In this Letter we report the determination of multipole mixing based on the orientation produced by a reaction process that is initiated by a polarized beam. When the beam is polarized transversely to its direction of motion it is no longer possible to produce an oriented state with a single axis of symmetry, even if the particle motions are axially symmetric. However, there is still an advantage in having a single axis for the particle motions, for then the radiation exhibits a simply interpreted left-right asymmetry with respect to this axis.

The $E2/M1$ mixing in the $3^+(5.22\ \text{MeV}) \rightarrow 2^+(1.369\ \text{MeV})$ transition in ^{24}Mg has been determined by measuring such an asymmetry. The 3^+ state was oriented by initiating the reaction $^{24}\text{Mg}(p, p')^{24}\text{Mg}(3^+)$ with a transversely polarized incident proton beam. The collinear condition for the particle motions was satisfied by detecting the inelastic protons at 180° relative to the incident beam direction. The γ rays were detected in coincidence with the inelastic protons corresponding to