tering. These constraints are of no relevance to the asymptotic behavior of the direct-channel physical scattering.

Note added in proof.—After this work had been completed, we received a paper by Branson[5] pointing out some specific mechanisms which allow hard Regge cuts.

We would like to thank J. Bronzan for a helpful conversation.

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[2]I. J. Muzinich, F. E. Paige, T. L. Trueman, and L.-L. Wang, Phys. Rev. Lett. 28, 850 (1972).

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[5]D. Branson, to be published.

Two-Photon Exchange in Electron-Deuteron Scattering*

J. Gunion† and Leo Stodolsky
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
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We show that multiple scattering of the electron (i.e., two-photon exchange) in electron-deuteron elastic scattering leads to a small component in the cross section, which decreases very slowly with momentum transfer and may dominate the cross section at high momentum transfer.

Measurements of electron-deuteron elastic scattering have, over the years, been pushed to higher and higher values of momentum transfer. Such experiments are a means of exploring the small-distance electromagnetic structure of the deuteron, and in the most recent series reached momentum transfers squared of 1.4 GeV^2, or distances of about $\frac{1}{6}$ fm.[1] An essential ingredient in the analysis of such measurements is the assumption that the process is of the lowest relevant order in the electromagnetic interaction, that is, that the matrix element is proportional to a photon propagator times form factors which depend only on the structure of the deuteron. In the case of electron-proton scattering this assumption has been tested by searching for effects that would arise from a two-photon–exchange contribution.[2]

In this note we would like to point out that electron-deuteron elastic scattering offers conditions where two-photon exchange may become significant or perhaps dominant. This possibility is due to the existence at high-momentum transfer of the simple mechanism of successive scatterings on the two nucleons. This mechanism has been observed in pion-deuteron and proton-deuteron elastic scattering and even in ρ photoproduction,[3] and is calculable in terms of known quantities in a relatively simple manner.

In general the amplitude for an elastic-scattering process on the deuteron can be decomposed into two terms, as in the method developed by Glauber.[4] The first, the single-scattering term, is large but drops rapidly with momentum transfer, since it represents coherence over the entire spatial extent of the deuteron. The second, the double-scattering term, drops much more slowly with momentum transfer since the neutron and the proton can each take half the transferred momentum and move off together, recombining to form the deuteron. While the room available for this effect in the n-p phase space will clearly be small, its decrease with t is rather slow, depending only on the structure of the constituents. Thus it can eventually overtake the single-scattering contribution. In electron-deuteron scattering naturally, the double-scattering cross section is intrinsically smaller by a factor of $\alpha^2 = (1/137)^2$. We show below that it may, nevertheless, be comparable to the one-photon exchange at values of momentum transfer t of the order of a few GeV^2.

The characteristic indication of the multiple-scattering effect is its slow decrease at high t. While there exists the possibility of mechanisms (due to internal vector mesons) which could cause the *usual* form factors to fall more slowly than expected from standard nonrelativistic wave functions,[5] the very slow decrease of the cross section we find at high t (see Fig. 1) would be a clear sign of the two-photon effect. Note that since we have a no-parameter calculation, the effect cannot be ignored once the data sink to the levels shown in Fig. 2.

Of course, the observation of an interference between one- and two-photon exchanges, as in recoil-deuteron polarization effects or in a difference between e^--D and e^+-D scattering, would be definitive evidence for the two-photon process. Unfortunately, the e^+-e^- difference only occurs if the two-photon and one-photon contributions are in phase. Since the double scattering is essentially imaginary, the e^+-e^- difference depends upon the small real part of the multiple-scattering amplitude. This can be estimated by a method developed by Gunion and Blankenbecler[6] and is at most of the order of 15% of the double scattering at Stanford Linear Accelerator energies. Its effects are further reduced because the single- and double-scattering amplitudes produce, in general, different final spin states. Recoil polarization effects, on the other hand, may be substantial, in the region where the one- and two-photon contributions are comparable, but would appear more difficult to obtain experimentally.

A simple derivation of the double-scattering cross section may be patterned after the usual deuteron multiple-scattering calculation.[3] The double-scattering result is, neglecting spin complications,

$$\frac{d\sigma^{\mathrm{dbl}}}{dt}=\frac{1}{\pi}\langle r^{-2}\rangle^2\frac{d\sigma^n}{dt}\bigg|_{t/4}\frac{d\sigma^p}{dt}\bigg|_{t/4}. \tag{1}$$

The expectation value $\langle r^{-2}\rangle$ results from an integral over the deuteron wave function, representing the effective phase space for the neutron and proton to move off together in a configuration which can form a deuteron. This evaluation involves the large, well-known, parts of the deuteron wave function, and is insensitive to the character of the high-momentum components. The cross sections on the right-hand side are to be evaluated at $t/4$, since each nucleon receives only half of the transferred momentum. This circumstance, implying that the n and p form factors involved are taken at $t/4$, will enhance our effect. If we were now, for a moment, to consider the case of spinless Coulomb scattering[7] at small angles, where the Coulomb cross section on a single particle is $d\sigma^{\mathrm{Coul}}/dt=\alpha^2(4\pi/t^2)$, then Eq. (1) would become

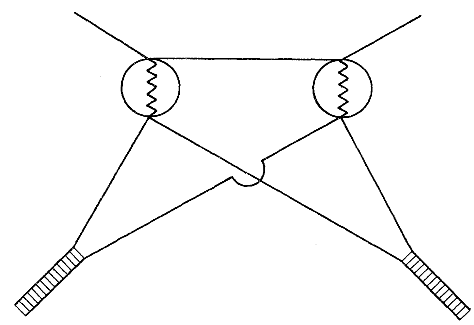

FIG. 1. Experimental and two-photon–effect values for the parameter A in $d\sigma=d\sigma^{\mathrm{Coul}}A(t)$. Some typical errors are shown for the experimental data.

$$\frac{d\sigma^{\mathrm{dbl}}}{dt}=\frac{d\sigma^{\mathrm{Coul}}}{dt}\left[\alpha\langle r^{-2}\rangle\frac{32}{t}G^n(t/4)G^p(t/4)\right]^2, \tag{2}$$

where the G's would be the form factors of the two constituent particles. Observe that the only momentum-transfer dependence other than the Coulomb cross sections would come from the form factors of the constituents, there being no effect due to the spatial extent of the "deuteron."

For the case of the actual deuteron we must take account of the spins of the nucleons. This is most simply done by viewing the process in the Breit frame, the frame in which the deuteron simply reverses direction with no change in energy. Since the deuteron is very loosely bound, the two nucleons must very nearly each carry half the momentum of the deuteron, and must be moving very nearly parallel in both the initial and final states. Thus the Breit frame for the deuteron will be also approximately the Breit frame for the individual nucleons. Since in the Breit frame the Sachs form factors[8] G_E and G_M are, respectively, the coefficients of the spin-independent and spin-flip terms for the nucleon, $f\sim G_E+(\sqrt{t}/M)G_M i\vec{\sigma}\cdot\hat{n}$, we find that the spin structure for the double scattering on the deuteron can be simply expressed in Pauli spinor notation by the operator

$$f^{\mathrm{dbl}}\sim\{G_E{}^p+[(\tfrac{1}{4}t)^{1/2}/M]G_M{}^p i\vec{\sigma}^p\cdot\hat{n}\}\{G_E{}^n+[(\tfrac{1}{4}t)^{1/2}/M]G_M{}^n i\vec{\sigma}^n\cdot\hat{n}\}, \tag{3}$$

evaluated between the triplet state of the deuteron. (It is not necessary to consider the D state of the deuteron.) By choosing the quantization axis along the normal to the scattering plane, $\hat{n}$, which, again, is approximately the normal to the plane of the individual scatterings, the spin sums may be easily performed. With the normalization of the spin-independent $G_E{}^pG_E{}^n$ term fixed by Eq. (2), we find that the cross section for the double scattering is

$$\frac{d\sigma^{\mathrm{dbl}}}{dt}=\frac{d\sigma^{\mathrm{Coul}}}{dt}\left(32\frac{\alpha}{t}\langle r^{-2}\rangle\right)^2\left\{(G_E{}^pG_E{}^n)^2+\left(\frac{1}{16}\right)^2\frac{t^2}{M^4}(G_M{}^pG_M{}^n)^2+\frac{t}{24M^2}\left[(G_E{}^pG_M{}^n)^2+(G_E{}^nG_m{}^p)^2+(G_E{}^pG_M{}^nG_E{}^nG_M{}^p)\right]\right\}\times(1+t/16M^2)^{-2}. \quad (4)$$

All G's are to be evaluated at $t/4$.

This simple derivation has glossed over a number of difficult theoretical points. It is not clear at first, for example, that when graphs with intermediate states containing crossed photons, or electron-positron pairs, as required by relativistic considerations, are taken into account, a simple multiple-scattering formula of the type Eq. (1) will result. These and related problems may be handled in the framework of "old fashioned perturbation theory" in which each particular type of intermediate state is handled explicitly. We prefer this to a manifestly covariant treatment, as per Feynman, since here with all virtual particles on mass shell it is possible to give a simple expression to the idea that the neutron and proton, distributed according to a known wave function, are struck successively, each carrying off, in the deuteron's Breit frame, half the momentum transferred. Proceeding in this manner, an analysis of the various intermediate states shows that at high incident electron energy and high momentum transfer all contributions are indeed negligible except those corresponding to the simple multiple scattering of the electron, as shown in Fig. 2. Another interesting problem concerns the effects which arise when the *deuteron* is moving relativistically. It might be thought that the relativistic distortion of the neutron and proton momentum distribution seen in the Breit frame would affect the simple cross-section formula. It is amusing to note that when all effects are taken into account the mechanism of Fig. 2 still leads to Eq. (1), regardless of the velocity of the deuteron in its Breit frame.[9] This conclusion remains true so long as the projectile-nucleon scatterings can be treated as indicated in Fig. 2—that is, as single-time operators.

Finally, we wish to stress that the calculation necessarily involves the approximation of small angles, $\theta^2 \ll 1$ (see Ref. 9). The particularly simple form, Eq. (1), results when, in addition, $\eta \equiv t/8k\gamma \ll 1/\langle r\rangle$, where r is the radius of the deuteron and $\gamma = 1/(1-v^2)^{1/2}$, the Lorentz contraction factor of the deuteron in the Breit frame. When this last inequality is not valid, one should replace $\langle 1/r^2\rangle$, in the amplitude, by the expectation value of $\exp(i\eta r)/r^2$ in the deuteron ground state (see Ref. 6 for a derivation in the nonrelativistic case). The $\exp(i\eta r)$ is responsible for the real part of the double-scattering amplitude mentioned above, typically giving a 15% real part for $\eta \simeq 25$ MeV.

Figure 1 shows an evaluation of $d\sigma/d\sigma^{\mathrm{Coul}}$ according to Eq. (4). We have taken $\langle 1/r^2\rangle = 0.015$ GeV^2,[10] and the "dipole" fit to the form factors:

$$G(t) \simeq (1+t/0.71)^{-2} = G_E{}^p = G_M{}^p/\mu_p = G_M{}^n/\mu_n,$$

$$\mu_p = 2.79, \quad \mu_n = -1.91.$$

The electric form factor of the neutron $G_E{}^n$ has been set equal to zero since it appears to be quite small wherever it is known. If it should turn out to be substantial in the several-GeV range, its effects should be taken into account. At high t, in any case, the curve is dominated by the "magnetic-magnetic" term in Eq. (4), which accounts for the very slow dropoff in this region. We also show in Fig. 1 an interpolation of the data quoted in Ref. 1. [Only A in the usual formula $d\sigma/d\sigma^{\mathrm{Coul}} = A + B\tan^2(\theta/2)$ is relevant since we always as-

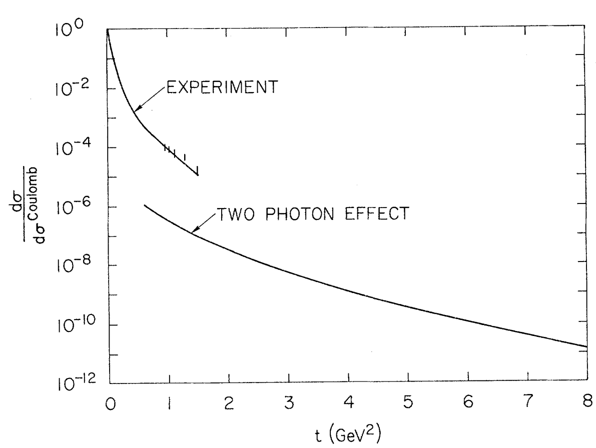

FIG. 2. Double-scattering graph.

sume $\theta^2 \ll 1$.] It appears that if the data were to continue its trend, the two-photon effect would come into play in the region 3 to 5 GeV^2. If indeed this occurs, it would have two interesting implications: (1) Straightforward extraction of form factors from the data would no longer be possible; (2) a two-photon effect would have been directly observed. This latter point may be taken as another instance where extreme values of energies or momentum transfers mean that simple counting of α's does not determine the order of electrodynamic processes.

We would like to thank C. L. Jordan for stimulating our interest in this problem and our colleagues E. Lehman, R. Blankenbecler, and S. Brodsky for useful advice and discussions.

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†Present address: Massachusetts Institute of Technology, Cambridge, Mass. 02139.

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[2]A discussion of nucleon form factors and of attempts to see two-photon effects is included in J. G. Rutherglen, in *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, September 1969,* edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).

[3]A review of many aspects of scattering on the deuteron is contained in L. Bertocchi's course at the Herceg-Novi School (1969): *Methods in Subnuclear Physics,* edited by M. Nikolic (Gordon and Breach, New York, to be published), Vol. V. The photorho production is reported in R. L. Anderson *et al.*, Phys. Rev. D 4, 3245 (1971).

[4]R. J. Glauber, in *Lectures in Theoretical Physics* (Interscience, New York, 1959), Vol. I.

[5]R. Blankenbecler and J. Gunion, Phys. Rev. D 4, 718 (1971).

[6]J. Gunion and R. Blankenbecler, Phys. Rev. D 3, 2125 (1971); see also K. Gottfried, Ann. Phys. (New York) 66, 868 (1971).

[7]Strictly speaking the usual approximations leading to Eq. (1) involve the assumption that the interactions are short ranged and so do not apply to the case of Coulomb scattering. One can see that Eq. (1) is still essentially correct, however, by imagining a moderate mass for the photon, which at large t has no effect on the Coulomb amplitude but makes the force short ranged compared to the deuteron size. As this mass goes to zero, there are effects corresponding to small Coulomb corrections to the amplitudes which are unimportant for our considerations. In other words at large t one may divide the intermediate integration into two regions, the first of which is well away from the photon propagator singularities and gives rise to Eq. (1), and the second of which probes those singularities giving rise to the well-known Coulomb phase corrections to the one-photon exchange calculation. [See R. H. Dalitz, Proc. Roy. Soc., London, Ser. A 206, 509 (1951); and D. R. Yennie, S. G. Frautschi, and H. Suura, Ann. Phys. (New York) 13, 379 (1961).] Equation (1) should not, of course, be applied at small t in the situation under consideration.

[8]G. Källén, *Elementary Particle Physics* (Addison-Wesley, Reading, Mass., 1964), p. 224.

[9]One derivation of this last conclusion makes essential use of time-ordered perturbation-theory techniques and eikonal expansions of the relevant energy denominators. The only time-ordered graph of importance is that illustrated in Fig. 2, so long as $\epsilon/\sqrt{-t} \ll 1$, $\epsilon/M \ll 1$, and $t/q_k \sim O((M\epsilon)^{1/2})$ for $t \geq 1$ only the last constraint is significant. For $t=4$ it requires $k \geq 10$ GeV/c, an energy quite accessible at the Stanford Linear Accelerator. The only subtle ingredient then necessary to the calculation of this particular diagram is the wave function describing the breakup of the moving deuteron in the Breit frame (we chose this frame as it treats the initial and final deuteron states in a completely symmetric fashion). The neutron and proton are moving nonrelativistically in relation to one another, are on mass shell, and very nearly on the energy shell, corrections being of $O((\epsilon/M)^{1/2})$. Ignoring such corrections, the required wave function is merely a Lorentz-contracted rest-frame wave function and the resulting eikonal integrals yield a result $\propto \langle 1/r^2 \rangle$. Thus the weakly bound nature of the deuteron plays an essential role. In processes which probe special regions of the wave function or involve more tightly bound systems, one should perhaps turn to the more general technique of H. Cheng and T. T. Wu, Phys. Rev. D 6, 2637 (1972).

[10]See Ref. 3 or, for example, F. Bradamonte *et al.*, Nucl. Phys. B33, 165 (1971) for a discussion of the value of $\langle 1/r^2 \rangle$.