185, 1555 (1969).

[17]R. Avida, I. Ben-Zvi, P. Gilad, M. B. Goldberg, G. Goldring, K. H. Speidel, and A. Sprinzak, Nucl. Phys. A147, 200 (1970).

[18]P. M. Berglund, H. K. Collan, G. J. Ehnholm, R. G. Gylling, and O. V. Lounasmaa, J. Low Temp. Phys. 6, 357 (1972). This work discusses problems associated with obtaining saturation of the ferromagnetic host.

[19]Taking the z axis as the $\vec{H}$ direction, a sudden change of quantization axis out of the cone into the z direction following the electron capture results in initial states which are mixtures of I_z values in the $I=\frac{5}{2}$ 136-keV level. A perturbation approximation was used to arrive at these numerical results.

Recoil Effects in Single-Nucleon–Transfer Heavy-Ion Reactions

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Using a distorted-wave Born-approximation treatment which exactly includes recoil for the data from the reaction $^{12}C(^{14}N,^{13}C)^{13}N$ at $E(^{14}N)=78$ MeV, we demonstrate the necessity for including recoil effects in calculations for single-nucleon–transfer reactions with heavy ions.

At sufficiently low energies ($Z_1Z_2e^2/\hbar v \gtrsim 10$), heavy-ion reactions appear to behave semiclassically.[1] Angular distributions for single-nucleon–transfer reactions seem to be well described[2,3] using formalisms[4] which neglect "recoil effects." In Fig. 1 the vector diagram is shown which is relevant to the distorted-wave Born-approximation (DWBA) amplitude used in theoretical calculations for such reactions. The vectors $\vec{r}_a$ and $\vec{r}_b$, which must be integrated over, can be expressed as[4]

$$\vec{r}_a=\vec{r}-(x/a)\vec{r}_{bx}, \quad \vec{r}_b=(A/B)\vec{r}+(x/B)\vec{r}_{bx}.$$

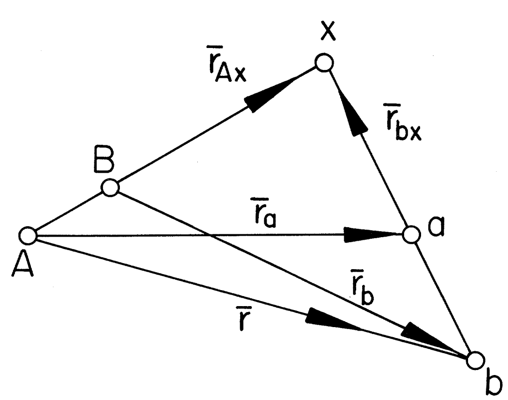

FIG. 1. Vector diagram for the reaction $A(a,b)B$, where $B=A+x$ and $a=b+x$.

Since this involves a six-dimensional integral, the "no-recoil" approximation is introduced which neglects the $\vec{r}_{bx}$ terms, with obvious simplifications. The dangers of this approximation were pointed out long ago.[5]

Recently measurements of heavy-ion single-nucleon–transfer reactions at higher energies[6-8] produced structureless angular distributions in disagreement with recoilless DWBA calculations and semiclassical predictions. The purpose of this paper is to show how the exact inclusion of recoil is necessary to obtain quantitative and qualitative agreement with the data of Ref. 6 for the reaction $^{12}C(^{14}N,^{13}C)^{13}N$ at 78 MeV.

For a reaction $A(a,b)B$, where B (a) consists of a core A (b) with a particle x bound with angular momentum l_1 (l_2), the selection rules are[8,9]

$$\Delta\vec{j}=\vec{J}_A-\vec{J}_B, \quad \Delta\vec{l}=\vec{l}_1-\vec{l}_2,$$
$$\Delta\vec{s}=\vec{S}_a-\vec{S}_b, \quad \Delta\vec{j}=\Delta\vec{l}+\Delta\vec{s}.$$

If we assume that the directions of J_A and S_b do not change (inert core) in the reaction, then

$$\Delta j=j_1,$$

where $\vec{j}_1=\vec{l}_1+\vec{S}_x$, and

$$\Delta s=j_2,$$

where $\vec{j}_2=\vec{l}_2+\vec{S}_x$. Note that if $l_2=0$, the rules become the familiar values of (d,p) etc. reactions.

If l_1 *and* l_2 are nonzero then, as has been noted,[11] there are *no* parity selection rules to limit the possible Δl values, but if $l_1 = 0$ or $l_2 = 0$ then $(-1)^{\Delta l} = \Delta\pi$ (change of parity in the reaction). In nonrecoil DWBA theories, however, the $(-1)^{\Delta l} = \Delta\pi$ rule always holds,[4,10] which is a consequence of the symmetry properties of the nonrecoil DWBA integral. In the case of $^{12}C(^{14}N, ^{13}C)^{13}N$, $|\Delta\vec{1}| = |\vec{1} - \vec{1}| = 0, 1, 2$; however, $\Delta j = \Delta s = \frac{1}{2}$, therefore only $\Delta l = 0, 1$ are physically possible. Since there is no total change in parity in the reaction we have

$\Delta l = 0, 1$ with recoil,

$\Delta l = 0$ without recoil.

The effects of including recoil, therefore, are twofold: The radial integral is more complicated, and additional angular-momentum transfers are possible. Even if recoil is approximately treated,[6,12] the additional l transfers are still forbidden. It should be noted that there has been no clear demonstration of the violation of this nonrecoil parity rule.

The program LOLA[13] is written in a formalism[10] which exactly includes recoil to calculate both $\Delta l = 0$ and $\Delta l = 1$ cross sections for the above reaction, which, in the absence of spin-orbit coupling, are added incoherently.[10] The nucleon is bound to the core in a Woods-Saxon potential with $r = 1.25$ fm and $a = 0.65$ fm in both bound states. The same optical-potential parameters were used in the incoming and outgoing channels; the values were those which gave a good fit to the elastic scattering in the incident channel.[14] Typical computing time for the program on the Saclay IBM 360/91 was about 4 min for 55 partial waves.

The results are shown in Fig. 2. As was found in the original study,[6] the $\Delta l = 0$ nonrecoil DWBA cross section oscillates rapidly, in complete disagreement with the experimental data, even though this theory seems to work at lower energies.[3,4,9] The exact recoil DWBA is in good agreement with the data. There are two reasons why: Note that the oscillations of the $\Delta l = 0$ exact recoil DWBA are less those than for the nonrecoil DWBA, in agreement with approximate recoil calculations in Ref. 7. More importantly, the $\Delta l = 1$ component is of the same order of magnitude as, and oscillates *out of phase* with, the $\Delta l = 0$ component. The product of the two spectroscopic factors is found to be 0.52, in good agreement with the theoretical prediction of Cohen

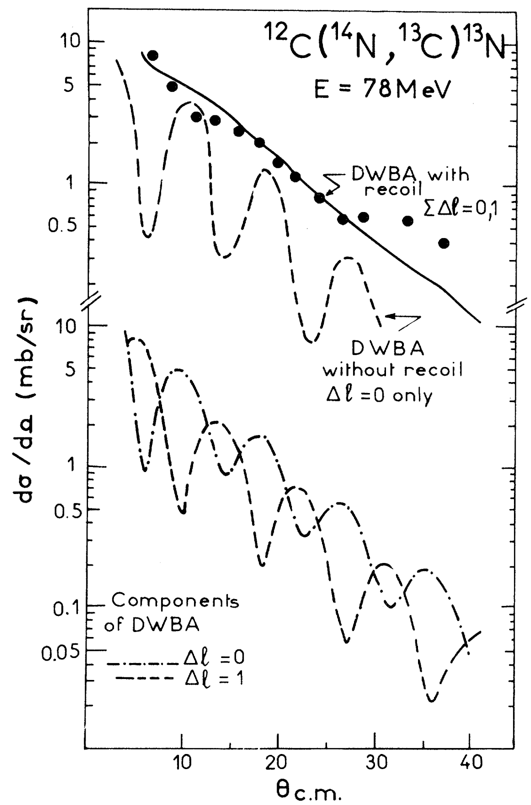

FIG. 2. DWBA calculations for the experimental data from Ref. 6. The exact-recoil DWBA represents the sum of the two contributions shown in the lower part of the figure. The nonrecoil DWBA curve is from Ref. 6.

and Kurath[15] of 0.42.

Very similar results are found for the data for the reaction $^{12}C(^{14}N, ^{13}N)^{13}C$ shown in Fig. 3. Since the two reactions contribute at complementary angles, one should coherently add them. However, the predictions for both reactions are very small at large angles; therefore we have neglected the interference.

In both reactions there is also the possibility of transferring a $p_{3/2}$ nucleon whereby $\Delta s = \frac{3}{2}$ allows $\Delta l = 2$ contributions. These cross sections also oscillate; but, when multiplied by the very small Cohen-Kurath spectroscopic factor, they give a cross section 60 times smaller than the experimental data and are therefore ignored.

With the use of the same parameters, the predicted cross section as a function of incident energy is shown in Fig. 4. As the energy decreases, so does the relative importance of the $\Delta l = 1$ component, in qualitative agreement with Refs. 16 and 17. This may be understood in the semiclassical theory of Brink,[18] where it is sug-

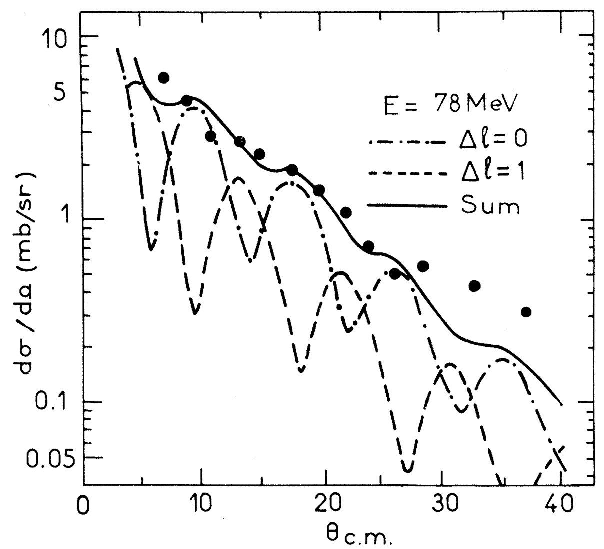

FIG. 3. Exact-recoil DWBA calculations for the data for the reaction $^{12}C(^{14}N, ^{13}N)^{13}C$ of Ref. 6.

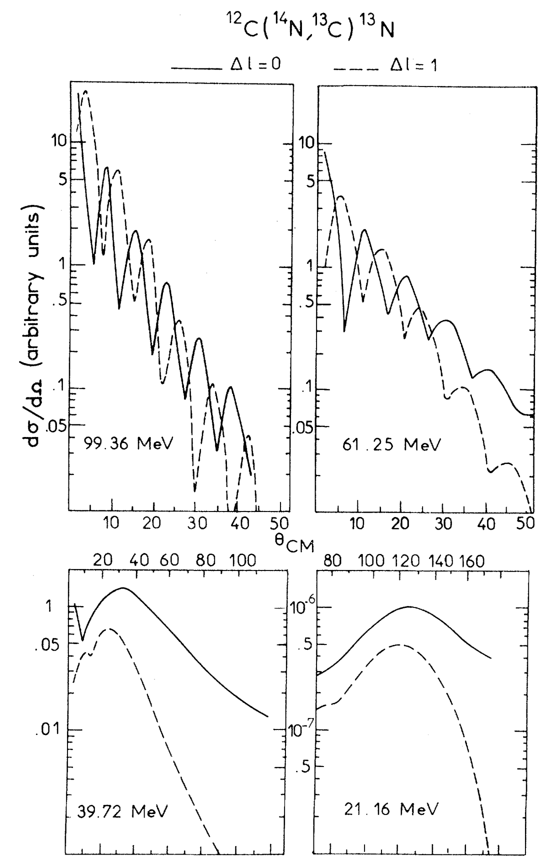

FIG. 4. DWBA predictions as a function of incident energy using the same parameters as used in Figs. 1 and 2. The arbitrary units are mb/sr but should not be taken seriously since the same 78-MeV optical potential has been used for all the calculations.

gested that the transfer is most probable in the plane defined by the motion of the cores. This assumption leads to a suppression of the $\Delta l = 1$ component in this case. At sufficiently low bombarding energies, where the two cores never touch (classically), this seems a good approximation and indeed the DWBA predicts relatively small $\Delta l = 1$ cross sections. But at 78 MeV incident energy, the distance of closest approach (classically at $\theta_{\mathrm{c.m.}} = 20^\circ$) is only 4.4 fm, compared with a core-core radius $[R = 1.4(14^{1/3} + 12^{1/3})]$ of 6.6 fm. Thus we would not expect any particular plane to be favored, allowing $\Delta l = 1$ components.

It is important to note that, even at low relative energies, the recoil $\Delta l = 1$ component, although reduced, cannot be neglected. Buttle and Goldfarb[12] have shown that recoil effects will be even *more* important on heavier targets although the effects are obscured by the featureless angular distributions obtained at energies near the Coulomb barrier. Recent measurements of $(^{16}O, ^{15}N)$ reactions[19] seem qualitatively to bear this out.

Work in progress indicates excellent preliminary quantitative agreement with the experimental data of Ref. 7. These results indicate, therefore, that recoil affects the predicted angular distributions of the "allowed" transfers *and* introduces additional l-transfer cross sections in single-nucleon-transfer reactions. Since recoil effects are proportional to the mass ratio x/a, multinucleon-transfer reactions are expected to be affected even more strongly.[20]

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[1]For a review of the status of heavy-ion reactions, see G. C. Morrison, in Proceedings of the European Conference on Nuclear Physics, Aix-en-Provence, France, June 1972 (unpublished).

[2]For example, G. C. Morrison, H. J. Korner, L. R. Greenwood, and R. H. Siemssen, Phys. Rev. Lett. 28, 1662 (1972).

[3]U. Schlotthauer-Voos, H. G. Bohlen, W. Von Oertzen, and R. Bock, Nucl. Phys. A180, 385 (1972).

[4]P. J. A. Buttle and L. J. B. Goldfarb, Nucl. Phys. 78, 409 (1966); F. Schmittroth, W. Tobocman, and A. A. Golestaneh, Phys. Rev. C 1, 377 (1970).

[5]L. R. Dodd and K. R. Greider, Phys. Rev. Lett. 14,

959 (1965).

[6]W. Von Oertzen, M. Liu, C. Caverzasio, J. C. Jacmart, F. Pougheon, M. Riou, J. C. Roynette, and C. Stephan, Nucl. Phys. A143, 34 (1970).

[7]M. Liu, W. Von Oertzen, J. C. Jacmart, F. Pougheon, M. Riou, J. C. Roynette, and C. Stephan, Nucl. Phys. A165, 118 (1971).

[8]D. K. Scott, in Proceedings of the European Conference on Nuclear Physics, Aix-en-Provence, France June 1972 (unpublished).

[9]R. H. Bassel, R. M. Drisko, and G. R. Satchler, ORNL Report No. ORNL-3240, 1962 (unpublished).

[10]N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler, Phys. Rev. 133, B3 (1964).

[11]See Austern *et al.*, Ref. 10; R. M. DeVries, Ph. D. thesis, University of California at Los Angeles, 1971 (unpublished), and to be published; J. L. Perrenoud and R. M. DeVries, Phys. Lett. B36, 18 (1971). Also R. Bock and H. Yoshida, Nucl. Phys A189, 177 (1972). Or, for example, G. R. Satchler, Nucl. Phys. 55, 1 (1964); R. C. Johnson and F. D. Santos, Particles Nucl. 2, 285 (1971); K. I. Kubo and M. Hirata, Nucl. Phys. A187, 186 (1972).

[12]P. J. A. Buttle and L. J. B. Goldfarb, Nucl. Phys. A176, 299 (1971).

[13]DeVries, Ref. 11, and Perrenoud and DeVries, Ref. 11.

[14]Von Oertzen, Ref. 6, p. 42, parameter set 3; parameter sets 1 and 2 gave essentially the same shape, and magnitudes less than 10% different.

[15]S. Cohen and D. Kurath, Nucl. Phys. A101, 1 (1967).

[16]Bock and Yoshida, Ref. 11.

[17]M. A. Nagarajan, Nucl. Phys. A196, 34 (1972).

[18]D. M. Brink, Phys. Lett. 40B, 37 (1972).

[19]D. G. Kovar, F. D. Becchetti, B. G. Harvey, F. Puhlhofer, J. Mahoney, D. W. Miller, and M. S. Zisman, Phys. Rev. Lett. 29, 1023 (1972).

[20]R. M. DeVries, to be published.

Experimental Verification of the Kramers-Kronig Relation at High Energy

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The real part of the forward amplitude for Compton scattering on protons was measured through the interference between the Compton and Bethe-Heithler amplitudes by detecting the zero-degree electron pairs asymmetrically. The measurement was made at an average photon energy of $\langle k\rangle = 2.2$ GeV, and an average momentum transfer to the recoil proton $\langle t\rangle = -0.027$ $(\mathrm{GeV}/c)^2$. The result confirms the prediction of the Kramers-Kronig relation.

The Kramers-Kronig relation[1] was first derived more than forty years ago from the causality principle. Gell-Mann, Goldberger, and Thirring[2] obtained the same result from field-theoretical considerations. It relates the real part of the forward Compton scattering amplitude, $\mathrm{Re}f_1$, to the total hadronic photon nucleon cross section, σ_T, via the relation

$$\mathrm{Re}f_1(k) = \frac{-\alpha}{M} + \frac{k^2}{2\pi^2}\mathrm{P}\int_{k_\pi}^{\infty}\frac{dk'}{k'^2-k^2}\sigma_T(k'), \tag{1}$$

where P denotes the Cauchy principal value of the integral, k_π is the one-pion threshold energy, α is the fine-structure constant, and M is the mass of the proton. The explicit evaluation of Eq. (1) has been carried out by Damashek and Gilman[3] using the known total photon-nucleon cross section σ_T. The purpose of this experiment is to measure $\mathrm{Re}f_1$ directly and to compare it with the prediction of Eq. (1), and thereby to check the validity of dispersion relations for photons.

The classical way to study the phase of the amplitude of πp and pp scattering[4] is to measure the interference between the elastic-scattering amplitude and the Coulomb amplitude. For Compton scattering the scattered photon, being a neutral particle, does not interfere with the Coulomb field. To study the Compton amplitude we consider the case where the scattered photon is "almost real," i.e., we study the asymmetric pair distribution from the reaction

$$\gamma + p \to p + \gamma(\text{virtual}), \quad \gamma(\text{virtual}) \to e^+ + e^-, \tag{2}$$

where the invariant mass of the pair is almost zero.