other words, there may be preferred configurations that characterize collisionless plasmas which are continuously perturbed thermally by bulk flow inhomogeneities. The nature and number of the kinetic processes responsible for the existence of these preferred configurations is at present not known.

One such process is evidently the ion-ion two-stream instability since a marginally stable configuration has been experimentally isolated in the above data by Feldman *et al.*[3] and shown to agree in detail with the two-dimensional numerical simulation by Forslund and Shonk.[9] Here a symmetric distortion (about the bulk flow of the composite plasma) produced by two equal intensity, interpenetrating ion beams, each with the same thermal energy, is initially characterized by $T_{\parallel}/T_{\perp} \gg 1$, but $q_{\parallel} = q_{\perp} = 0$. The results of Forslund and Shonk[9] showed that if the plasma is two-stream unstable, short-wavelength longitudinal electrostatic waves grow in amplitude. The nature of the process is such that the relative streaming energy is fed into thermal energy preferentially perpendicular to the relative streaming direction. The net result is a velocity distribution with $T_{\parallel}/T_{\perp} \simeq 1$ in qualitative agreement with the empirical $q_{\parallel}(T_{\parallel}/T_{\perp})$ and $q_{\perp}(T_{\parallel}/T_{\perp})$ relations (i.e., when $T_{\parallel}/T_{\perp} \simeq 1$, $q_{\parallel}$ and $q_{\perp}$ are small). Such empirical relations, where applicable, can be used to close the Vlasov moment equations.

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[1]G. Chew, M. L. Goldberger, and F. Low, Proc. Roy. Soc., London, Ser. A 236, 112 (1956).

[2]S. J. Bame, J. R. Asbridge, H. E. Felthauser, E. W. Hones, Jr., and I. B. Strong, J. Geophys. Res. 72, 113 (1967).

[3]W. C. Feldman, J. R. Asbridge, S. J. Bame, and M. D. Montgomery, to be published.

[4]M. D. Montgomery, J. R. Asbridge, and S. J. Bame, J. Geophys. Res. 73, 4999 (1968).

[5]K. W. Ogilvie, J. D. Scudder, and M. Sugiura, J. Geophys. Res. 76, 8165 (1971).

[6]L. Spitzer, Jr., and R. Härm, Phys. Rev. 89, 977 (1953).

[7]D. W. Forslund, J. Geophys. Res. 75, 17 (1970).

[8]N. F. Ness, A. J. Hundhausen, and S. J. Bame, J. Geophys. Res. 76, 6643 (1971).

[9]D. W. Forslund and C. R. Shonk, Phys. Rev. Lett. 25, 281 (1970).

Fermi-Liquid Effects in Cyclotron-Phase-Resonance Transmission through Alkali Metals at 116 GHz

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Cyclotron phase resonance (CPR) is observed at 116 GHz in sodium and potassium. The CPR signal shows effects due to electron correlations, including notches in the transmission amplitude, an asymmetry, and possibly a shift in resonance field from the Azbel-Kaner cyclotron-resonance (AKCR) value. A simple model describing these effects appears to indicate that CPR is a $k=0$ resonance in the conductivity, i.e., a uniform resonance whose shift from AKCR should give the Fermi-liquid parameter A_1. Values found for A_1, however, disagree with previous estimates.

Cyclotron-phase-resonance (CPR) experiments have been previously reported for copper and silver at 35 GHz.[1] The detailed explanation of those experiments required knowledge of the rather complicated shapes of the Fermi surfaces. We have extended these measurements to 116 GHz, investigating sodium and potassium, metals whose spherical Fermi surfaces would be expected to yield rather simple spectra, and have observed certain novel phenomena which must be ascribed to electron-electron correlation effects. Such effects are accentuated at high values of $\omega\tau$. These phenomena, observed in the phase-resonance transmission amplitude as a function of magnetic field, include notches in the amplitude, a strong asymmetry between the oscillations on the low-field and high-field sides of the resonance, and a possible shift in the position of the peak from that of Azbel-Kaner cyclotron resonance (AKCR). The analysis of the data in terms of a simple model, which makes use of the Pippard[2] ineffective electron concept, leads to a surprisingly good agreement between experiment and theory. The model also suggests that CPR is equivalent to a uniform excitation experiment involving a $k = 0$ resonance in the conductivity. If true, this would lead to a measurement of the Landau Fermi-liquid parameter A_1,[3-5] for which there is no previous experimental determination.

While values of the parameter A_2 determined from this resonance are in good agreement with experimental values found by other spectroscopic techniques,[6] the values of A_1 are of opposite sign and considerably smaller than accepted estimated values,[7] particularly those obtained by examining the contributions to the AKCR mass value. It is possible that an oversimplification of the macroscopic transport problem leading to the resonance has resulted in an incomplete description of the role of the correlation parameters, in spite of the remarkable agreement between theoretical and experimental spectra. Alternatively, there may be aspects of the microscopic estimates of A_1 which need re-examination, in particular, the size of the band-structure contribution to the AKCR mass.

The experimental apparatus consists of a microwave transmission spectrometer[1] of a homodyne type, with a 100-mW klystron as a source. A high-purity alkali metal sample, protected from corrosion by thin quartz plates, forms the common end wall of two TE_{110} cavities which are tuned to the same frequency. These are mounted in a helium cryostat vertically above one another such that the plane of the sample is *normal* to the field of a high-homogeneity superconducting magnet. Microwave power to the upper (driving) cavity is modulated at 20 kHz, and the signal propagating through the sample is received in the lower cavity where it is fed to a helium-cooled InSb hot-electron bolometer operating in a mixing mode.[1] Unmodulated bias power is incident on the detector by a separate path, and the phase shift between the paths is adjustable. This means that an observed signal is the projection of the weak signal field onto the bias field, and consequently a signal whose microwave phase is varying linearly with magnetic field will appear as sinusoidal in the detector output.

The transmission spectrum as a function of swept magnetic field is shown for potassium in Fig. 1(a). Many interesting features are present, including conduction-electron spin resonance and spin waves,[8] Gantmakher-Kaner oscillations (GKO),[9] and CPR. This Letter will discuss only

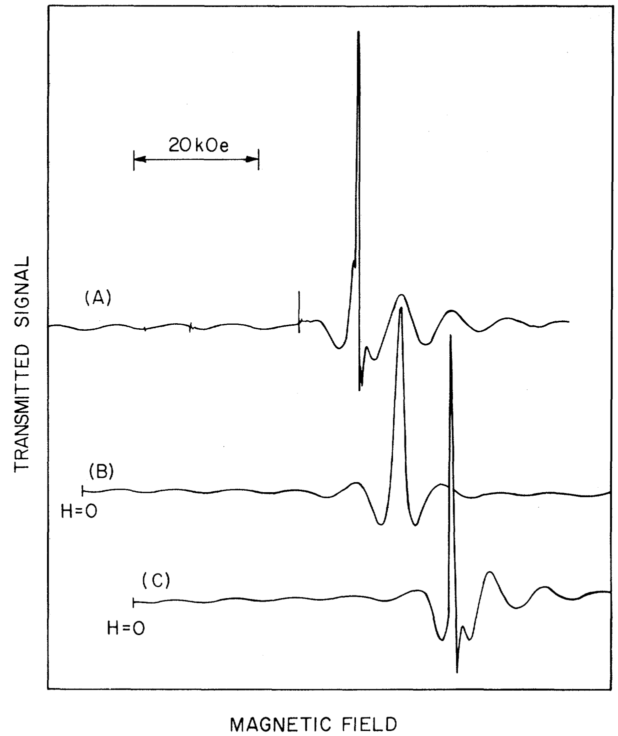

FIG. 1. (a) 116.0-GHz transmission spectrum of a sheet of potassium of thickness 3.25×10^{-3} cm. The magnetic field is normal to the sheet and has a value of 50.93 kOe at CPR. The temperature is 1.3°K. (b) Calculated spectrum using a free-electron model, $\omega_0\tau = 300$. (c) Calculated spectrum, including Fermi-liquid effects, with parameter values $A_1 = -0.015$, $A_2 = -0.03$, $A_{n>2} \equiv 0$, and $\omega_0\tau = 300$.

the GKO and CPR.

An approximate view of the physical processes involved here can be obtained by considering a group of free electrons, with velocity V_H along the field, on a spherical Fermi surface. A skin-depth field is set up at the surface ($z = 0$) by the "effective electrons," and extra transverse momentum is given to the ineffective electrons by the microwave field in the skin depth. These electrons continue along a helical path, giving rise to a current in the sample with a q value[1] of $(\omega_c \pm \omega_0)/V_H$ and a phase which differs from that of the reference or bias field by $\varphi_0 + qz$, where ω_c is the cyclotron frequency, ω_0 is the microwave frequency, and φ_0 is a constant. The total current is found by integrating the effect over the Fermi surface, and the phases of the various group of electrons cancel as a result of the various values of V_H. This leaves a weak signal which appears to come from the tip of the Fermi surface, and has a phase at the second surface of the slab ($z=L$) of $(\omega_c \pm \omega_0)L/V_H + \varphi_0$, i.e., it depends linearly on magnetic field. This is the GKO. However, for the negative or resonant sense of polarization, the q value goes to zero at $\omega_c = \omega_0$ and all the cyclotron current has the same phase throughout the sample. The result[1] is a strong symmetric resonant peak in the transmission.

This peak, denoted CPR, is centered at the AKCR field [Fig. 1(b)]. However, Fig. 1(a) for potassium clearly shows a striking asymmetry in the CPR with some extra features (notches) near the center. Close inspection shows that the peak of the resonance is centered at a mass value of $m^*/m = 1.228$ (± 0.002), whereas the AKCR value is 1.21.[10] The errors quoted for this value are $\pm 2\%$; but, subsequently, it has been found that the errors can be reduced to about $\pm \frac{1}{2}\%$.[11]

We now attempt to include correlation effects in the description of the CPR and GKO phenomena. Rephrasing the description of the previous paragraph, we note that the nonlocal conductivity, defined by $j(z) = \int_0^L \sigma_L(z,z')e(z')dz'$, determines the transmitted signal T at the emergent side of the slab via[1] $T = C\sigma_L(L,0)$, where C is some constant independent of field. To the extent that the faces of the slab do not significantly alter the conductivity, we may replace $\sigma_L(z,z')$ by $\sigma_\infty(z-z')$, where σ_∞ is the conductivity in an infinite medium. This latter quantity may be obtained by evaluating

$$\sigma_\infty(|z-z'|) = (2\pi)^{-1}\int dk\, e^{ik(z-z')}\sigma(k,\omega). \tag{1}$$

$\sigma(k,\omega)$ can be calculated by solving the spin-independent part of the Landau equation for the case $\vec{k} \parallel \vec{H} \parallel \vec{z}$

$$-i\omega_0\left(\delta\bar{n} + \frac{\partial n_0}{\partial \epsilon_0}\delta\epsilon\right) + ik_z V_z \delta\bar{n} - \omega_c \frac{\partial}{\partial\varphi}\delta\bar{n} + \frac{\delta\bar{n}}{\tau} = -q\vec{e}\cdot\vec{V}\frac{\partial n_0}{\partial\epsilon}, \tag{2}$$

where n_0 and ϵ_0 are the single-particle distribution function and energy, $\varphi = \tan^{-1}(k_y/k_x)$, $\delta\epsilon$ is the extra energy associated with correlations and depends on the Fermi-liquid parameters A_n,[12] $\vec{e}$ is the electric field, $\delta\bar{n} = \delta n_0 - (\partial n_0/\partial\epsilon_0)\delta\epsilon$, and τ is a phenomenological relaxation time. We manipulate this equation, assuming for simplicity that $A_n \equiv 0$ for $n > 2$, and obtain

$$\sigma(k,\omega) \propto k^{-1}[aK(x)+b]/[c-dK(x)], \tag{3}$$

$$a = 1 - 5x\lambda_2, \quad b = 5\lambda_2, \quad c = 1 + 5\lambda_2(x-\lambda_1), \quad d = \lambda_1 + 5x\lambda_2(x-\lambda_1), \quad \lambda_n \equiv (\omega/kV_F)A_n/(1+A_n),$$
$$x = (1-\omega_c/\omega + i/\omega\tau)\omega/kV_F, \quad K(x) \equiv \tfrac{3}{4}\{2x + (1-x^2)\ln[(x+1)/(x-1)]\}. \tag{4}$$

The $k=0$ limit of (3) is $[(1+A_1)^{-1} - \omega_c/\omega + i/\omega\tau]^{-1}$ which exhibits a resonance at a field shifted by a factor $(1+A_1)^{-1}$ from the AKCR field.[4,5] The values of $L = z - z'$ involved in evaluating (1) cause the $k=0$ component of the conductivity to dominate the integral. The $k=0$ component has two distinctive features. The first is the resonance we have mentioned at $\omega_c/\omega = (1+A_1)^{-1}$. The second is that $\sigma(k=0,\omega) = 0$ at $\omega_c/\omega = (1+A_n)^{-1}$, $n>1$. These zeros should diminish the transmitted current, putting a notch in the transmitted field strength, at these values of magnetic field.[13] Our calculation here, which retains the A_n only up to $n=2$, does indeed exhibit the notch at $\omega_c/\omega = (1+A_2)^{-1}$.

In spite of the simplifying assumptions, it has been found possible to obtain a reasonable line-shape fit to the data. The variables are the parameters A_1, A_2 and the scattering time τ. The fit is shown in Fig. 1(c) for potassium and was made in the following way. A_1 was fixed at -0.015 by fitting the position of the resonance peak. A_2 was fixed at -0.03 by fitting the position of the notch to the feature in the data at about 1.03 of the AKCR field. Of course, this value of A_2 was suggested by the previous determination[6] of A_2 and it is satisfactory that the same value can be used here. τ was fixed by fitting the ratio of the CPR amplitude to the GKO amplitude. It may be observed that the spectrum found with the values of the parameters fixed in this way gives the correct sign and approximately the correct magnitude of the asymmetry of the signal.

Figure 2 gives a more detailed view of the spectrum near $\omega_c/\omega = 1$ ($m^*/m = 1.21$). It is clear that the theory has failed to account for some features of the experiment, most notably the two extra notches close to the center of the resonance. Both of these represent reductions in the transmission just as in the case of the A_2 notch. It seems likely that these are due to other A_n parameters which have been neglected in the calculation. If so, one is of the order -0.01 and

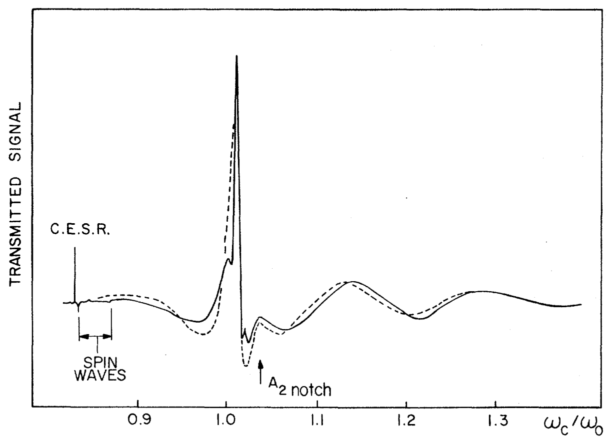

FIG. 2. Expanded view of CPR. Solid line, experiment; dashed line, theory for the parameter values of Fig. 1(c).

the other so small that its sign cannot be determined due to the uncertainty in the AKCR mass. In the case of solium a similar analysis gives $A_1 = -0.01$ and $A_2 = -0.045$. For all A_n the errors are largely determined by the uncertainty in the AKCR mass, i.e., ± 0.005 for potassium[11] and ± 0.02 for sodium.[10]

We must now question the validity of the apparent agreement between experiment and calculation, since the model contains two simplifying physical assumptions which cannot be rigorously justified at this time. The assumptions are (a) that the fields associated with the currents in the interior of the slab play a minor role in the transmission as compared with that played by the strong skin-effect launching field, and (b) that the presence of the slab boundaries does not affect the conductivity. Although there are difficulties associated with providing firm theoretical support for these assumptions, their use does lead to a model which reproduces the data remarkably well. However, the values of A_1 deduced from the data fit are small and lead to difficulties in explaining the magnitude of the AKCR mass.[7] Attempts to solve the problem without making use of the above assumptions are being pursued vigorously, but so far they have not been encouraging. The only mathematically tractable calculation is that found for specular reflection boundary conditions, which predicts that the transmitted field is no longer proportional to σ, but is approximately given by $1/\sigma$ which shows no resonance at all and cannot possibly describe the data.

In summary, our simple model suggests that the experiment is a uniform cyclotron current excitation ($k = 0$), and that the shift of the resonance from the AKCR (high k) value is a measure of the parameter A_1. The physical picture is appealing, particularly in view of the correct description of the distinctive line-shape features of the resonance and the reasonable values found for the parameter A_2, but the unexpectedly small values of A_1 found in this way force us to consider these as tentative until rigorous justification for the approximations can be found.

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[1]T. G. Phillips, G. A. Baraff, and P. H. Schmidt, Phys. Rev. B 5, 1283 (1972).

[2]A. B. Pippard, Proc. Roy. Soc., London, Ser. A 191, 385 (1947).

[3]P. Nozières, in *Polarisation, Matière et Rayonnement, Livre de Jubilé à l'Honneur du Professeur A. Kastler*, edited by The French Physical Society (Presses Universitaires de France, Paris, 1969),

[4]R. E. Prange and A. Sachs, Phys. Rev. 158, 672 (1967).

[5]P. M. Platzman and K. C. Jacobs, Phys. Rev. 134, A974 (1964).

[6]P. M. Platzman, W. M. Walsh, Jr., and E-Ni Foo, Phys. Rev. 172, 689 (1968).

[7]T. M. Rice, Phys. Rev. 175, 858 (1968).

[8]S. Schultz and G. L. Dunifer, Phys. Rev. Lett. 18, 283 (1967).

[9]V. F. Gantmakher and E. A. Kaner, Zh. Eksp. Teor. Fiz. 56, 1223 (1969) [Sov. Phys. JETP 29, 658 (1969)].

[10]C. C. Grimes and A. F. Kip, Phys. Rev. 132, 1991 (1962).

[11]G. L. Dunifer, unpublished.

[12]V. P. Silin, Zh. Eksp. Teor. Fiz. 35, 1243 (1958) [Sov. Phys. JETP 35, 870 (1959)]; D. Pines and P. Nozières, *Theory of Quantum Liquids* (Benjamin, New York, 1966). $A_n = F_n^S/(2n+1)$, where F_n^S is defined by Pines and Nozières.

[13]These notches in σ are vestigially related to the modes discussed by Silin, Ref. 12, and by Y. C. Cheng, J. S. Clark, and N. D. Mermin, Phys. Rev. Lett. 20, 839 (1968); and observed by G. A. Baraff, C. C. Grimes, and P. M. Platzman, Phys. Rev. Lett. 22, 12 (1969).