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New Charge-Exchange Polarization Data and πN-Amplitude Analyses at 3.6 and 6.0 GeV/c*

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Using the results of recent πN charge-exchange polarization measurements at Argonne National Laboratory we have determined the amplitudes for πN scattering 3.6 and 6.0 GeV/c. Two different methods, individual t-by-t analysis and t-dependent analysis, have been adopted, and uncertainties of these amplitudes are discussed.

After the parameters R and A for πN scattering were measured at 6 GeV/c, a number of authors[1-3] attempted to extract the πN scattering amplitudes using various methods. We have determined the amplitudes to within an overall t-dependent phase. We have performed the analyses using new charge-exchange polarization data,[4] and have employed two new methods; the uncertainties of the amplitudes are realistically determined by the second method. We only show 6.0-GeV/c results of the analyses I but present both 3.6- and 6.0-GeV/c results of the analyses II.

Amplitude analysis I (individual t analysis).—In this individual t-by-t analysis two features were incorporated to improve upon the early Halzen-Michael work.[1] First we determined the seven amplitudes algebraically from seven measurements [$d\sigma(\pi^+p\to\pi^+p)/dt$, $d\sigma(\pi^-p\to\pi^-p)/dt$, $d\sigma(\pi^-p\to\pi^0n)/dt$, $P(\pi^+p\to\pi^+p)$, $P(\pi^-p\to\pi^-p)$, $P(\pi^-p\to\pi^0n)$, and $R(\pi^-p\to\pi^-p)$, denoted by σ^+, σ^-, σ^0, P^+, P^-, P^0, and R^-, respectively], obtaining the eight solutions at each momentum transfer. These solutions were used as the starting point in a gradient search, including the additional measurements[5] of $R(\pi^+p\to\pi^+p)$ and $A(\pi^-p\to\pi^-p)$, denoted by R^+ and A^-, respectively, in the data-fitting program. The fits of seven amplitudes to nine measurements incorporate more experimental information; the gradient search here was appropriate to find final amplitudes of the same character as the initial algebraic solutions from seven measurements.

The other new feature incorporated was the use of a "shortest-path" approach to determine the smoothest solution passing through each of the eight solutions at individual t. At a given momentum transfer t_2 the distances of each solution from each of the eight at the previous momentum transfer t_1 are calculated from differences in the corresponding amplitudes at t_2 and t_1 with an appropriate metric. We define the distance between solutions as follows: Let $H_1, \ldots, H_7$ be real or imaginary parts of the various helicity amplitudes; $M_{ij}=\partial^2\chi^2/\partial H_i\,\partial H_j$ is the error matrix evaluated at a solution. Then the distance is defined to be

$$d_{12}=\sum_{i=j}^{7}\sum_{j=1}^{7}[H_i(t_2)-H_i(t_1)]M_{ij}[H_j(t_2)-H_j(t_1)].$$

In analogy to "shortest-path" constructions in phase-shift analysis, we find solutions with smoothest dependence on momentum transfer by minimizing the total distance between neighboring solutions.

The new charge-exchange polarization data[4] at 5 GeV/c and the recent σ^+ and σ^- data[6] at 6 GeV/c were used together with those data of σ^+, σ^-, σ^0, P^+, P^-, and R^- used in Ref. 1 and R^+ and A^- as mentioned earlier. The P^0 data of Bonamy *et al.*[7] were not included in the analysis. We note that there exist some differences in σ^+ and σ^- data of Ref. 6 and the interpolated data used in Ref. 1, particularly as illustrated by our calculations of isospin bounds; for the first set of data there is no value of charge-exchange polarization at $t = -0.5$ which is consistent with the isospin bounds of Dass *et al.*[8] We made separate analyses using (i) σ^+ and σ^- data of Ref. 6, and (ii) Ref. 1. The "shortest-path" solutions covering the $|t|$ range from 0 to 0.625 are given in the plots of Fig. 1. The errors are taken from the diagonal elements of the error matrix and are to be used for relative comparison only. Our convention for helicity amplitudes H_{++} and H_{+-} (following Ref. 1) is such that cross section $d\sigma/dt = |H_{++}|^2 + |H_{+-}|^2$, in mb/(GeV)^2.

Amplitude analysis II (t-dependent analysis). —There are several drawbacks in the previous analysis I: (1) Errors were not realistically calculated. (2) All the data available were not used; in fact, some of them were interpolated at certain values of $|t|$. (3) The analysis can be done more efficiently by using an accelerated convergence expansion.[2]

We have done the continuous-t searches to enforce continuity by fitting to a particular analytic form. Our analysis differs from Ref. 2 in that we determine amplitudes up to an overall t-dependent phase. Our view is that, since the measured quantities at a given energy are unchanged if every helicity amplitude is multiplied by this phase, it is impossible to determine this phase from the data.[9]

If the true helicity amplitudes $F_{ij}^{\,k}(t)$ (k is isospin index; i and j are helicity indices) are analytic in the cut t plane and if a certain linear combination of the amplitudes, $G(t)$, has zeros within the cut plane either at real t or in conjugate pairs, then $\varphi = i[G(t)G^*(t^*)]^{1/2}/G(t)$ is analytic in the cut plane. Thus, $H_{ij}^{\,k}(t) = \varphi(t)F_{ij}^{\,k}(t)$ (i) would also be analytic in the cut plane, (ii) would give the same two-body measurements as $\{F_{ij}^{\,k}(t)\}$, and (iii) would have one fixed t-dependent phase.

In the various amplitude analyses $F_{++}^{\,0}(t)$ comes out to be diffractive and structureless; it certainly has no unpaired complex zeros near the physical data. We have consequently chosen $G(t) = iF_{++}^{\,0}(t)$ so that $H_{++}^{\,0}(t)$ is purely imaginary by convention. We have found satisfactory fits with this choice of phase.

The right- and left-hand cuts are mapped (we call the mapping function ω) onto the edge of a unifocal ellipse in the $\cos\theta_{\text{c.m.}}$ plane with $\cos\theta_{\text{c.m.}} = \pm 1$ as fixed points. We write the helicity amplitudes as products of diffractive terms and sixth-order polynomials in ω:

$$H_j = g_j \exp\{-a_j[(4m_\pi^2 - t)^{1/2} - 2m_\pi]\} \times\left[1 + \sum_{n=1}^{6} b_n^{\,j}(\omega - 1)^n\right],$$

where g_j, a_j, and $b_n^{\,j}$ are the varied parameters and m_π is the pion mass.

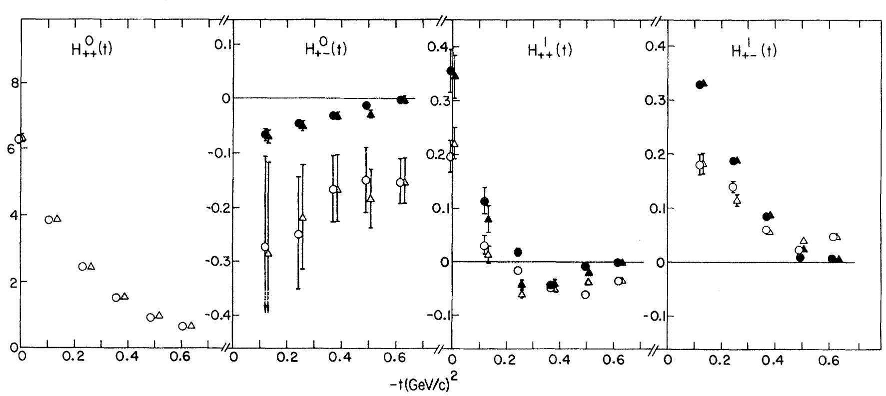

FIG. 1. πN amplitudes at 6.0 GeV/c. The filled and hollow symbols represent real and imaginary parts of the amplitude, respectively. The triangles and circles are obtained by using σ^+ and σ^- data of Refs. 1 and 7, respectively. Data points are at $-t = 0.0$, 0.125, 0.250, 0.375, 0.500, and 0.625 (GeV/c)^2.

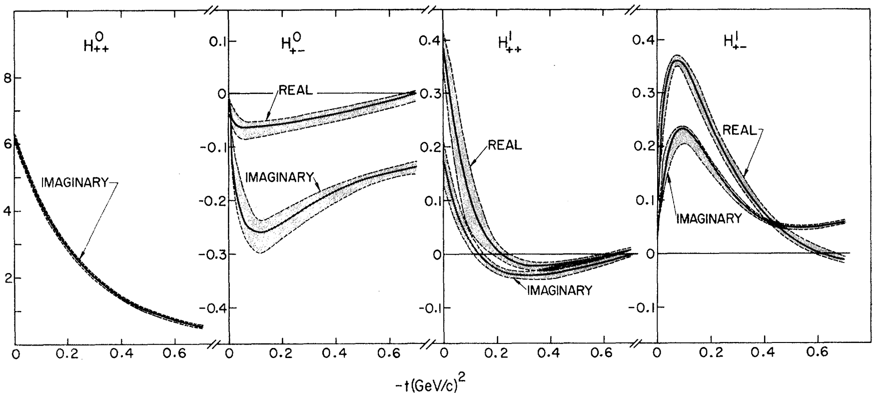

FIG. 2. πN amplitudes at 6.0 GeV/c as determined from t-dependent fit shown by dark central lines. The error bands (determined by the envelope of fits to randomized data) are represented by shaded regions.

The amplitudes at $P_{\rm lab} = 6.0$ GeV/c were obtained by using all the data points (σ^+ and σ^- from Refs. 1 and 6, σ^0 from the CERN compilation,[10] P^+ and P^- from Borghini *et al.*,[11] P^0 from Ref. 4 and Drobnis *et al.*,[12] and R^+, R^-, and A^- from Ref. 5). Those at $P_{\rm lab} \approx 3.6$ GeV/c were determined from all the available 201 data points (σ^+ and σ^- from Ref. 6 and Coffin *et al.*,[13] σ^0 from Ref. 10, P^+ and P^- from Scheid *et al.*,[14] P^0 from Refs. 4 and 12, and R^-). For R^- we used the data at 6.0 GeV/c with uncertainties enlarged by 50%. This may be a safe extrapolation (at least within the large error bars on R^- at 6 GeV/c) because comparison of all the polarization data (P^-, P^+, P^0) from 3.6 to 6 GeV/c and higher shows a relatively slow change with respect to energy at fixed momentum transfer. A variable metric routine[15] incorporating an analytic gradient with respect to search parameters was used to obtain fits to the data.

Figures 2 and 3 show the results at 6.0 and 3.6 GeV/c, respectively. The error bands shown in these figures about the best-fit values were obtained by shifting each data point in random manner about its measured value, weighting this shift by a Gaussian of width given by the experimental error, and repeating the fitting procedure for each set of such random shifts. The fits obtained are found to be unique within the stated errors.

Conclusions.—The amplitudes are determined with appropriate uncertainties only up to an overall t-dependent phase $\theta(t)$ which can be determined[16] experimentally only near $t=0$; we shall not attempt to extrapolate it but will discuss our amplitudes assuming that it is relatively smooth. The amplitudes obtained at 3.6 and 6.0 GeV/c are qualitatively similar. We will discuss their features at 6.0 GeV/c and similar conclusions apply at the lower energy.

Our $H_{++}{}^0$, by convention purely imaginary, is structureless and diffractive. $\mathrm{Im}H_{+-}{}^0$ has a sim-

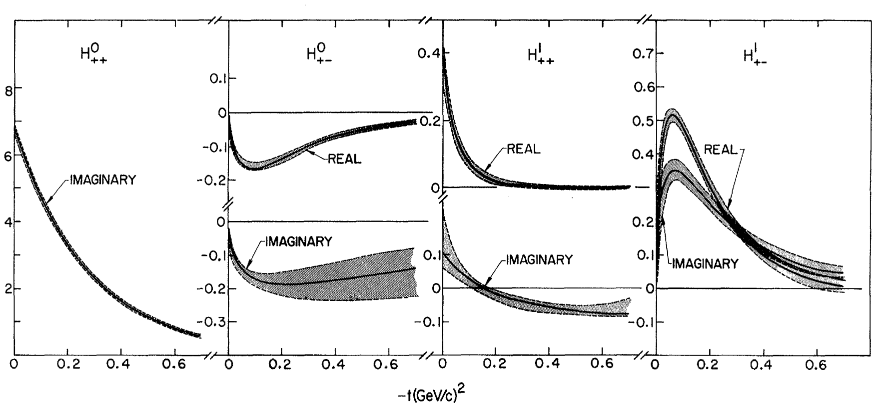

FIG. 3. πN amplitudes at 3.6 GeV/c as determined from t-dependent fit shown by dark central lines. The error bands are represented by shaded regions.

ilar diffractive shape except for the kinematical zero, whereas $\mathrm{Re}H_{+-}{}^0$ is becoming small near $t=-0.6$. Unless $\theta(-0.6)$ is near a multiple of $\pi/2$ the real or imaginary part of the true amplitude, $F_{+-}{}^0$, will not have a zero; we know of no dynamical origin for such a zero.

The amplitude $H_{++}{}^1$ has zeros in both its real and imaginary parts near $t=-0.2$ and -0.6. The true amplitude $F_{++}{}^1$ must be *small* there also; the precise shape of $\mathrm{Re}F_{++}{}^1$ and $\mathrm{Im}F_{++}{}^1$ crucially depend on $\theta(t)$. For example, one *need not* have both functions change sign near $t=-0.2$ or -0.6.

The true flip amplitude $F_{+-}{}^1$, by contrast, *cannot* have zeros in its real and imaginary parts neat $t=-0.6$ no matter what $\theta(-0.6)$ is. Of course, if $\theta(-0.6)$ is small $\mathrm{Im}F_{+-}{}^1$ has a zero and $\mathrm{Re}F_{+-}{}^1$ has a minimum near $t=-0.6$.

Compared with previous analyses[1-3] our results show much smaller difference between zero-crossing points of $\mathrm{Re}H_{++}{}^1$ and $\mathrm{Im}H_{++}{}^1$ and smaller magnitude in $|H_{++}{}^1|$ near $t=-0.5$. These differences are primarily caused by polarization data.[4] Zero-crossing points, as shown in Fig. 2, are determined mainly by the data of Ref. 6.

These amplitudes have been obtained from data at a given energy in a relatively model-independent and numerically stable manner. We will not examine their detailed agreement with various intermediate-energy models of πN amplitudes.[17]

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Unitarity Lower Bound on the Longitudinal Contribution to Inelastic Electron-Proton Scattering*

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The diffractive electroproduction of longitudinal ρ mesons sets, via unitarity and the Schwarz inequality, a lower bound to the longitudinal cross section in inelastic electron-proton scattering. Recent ρ electroproduction data may be used to infer that the longitudinal-to-transverse ratio R in total electroproduction at $\omega\simeq 16$, $q^2\simeq 1$ GeV^2 is an order of magnitude greater than is predicted by models yielding $R=q^2/\nu^2=(4m_N{}^2/q^2)\omega^{-2}$.

A precise determination of the longitudinal-to-transverse ratio,

$$R\equiv\sigma_S/\sigma_T\equiv\frac{(1+\nu^2/q^2)W_2-W_1}{W_1},\tag{1}$$

in inelastic electron-proton (or neutrino-nucleon) scattering is of fundamental importance in high-energy physics today. The single-arm experiment of the famous Stanford Linear Accelerator