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Origin of Magnetic Fields in the Early Universe

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(Received 4 December 1972)

Primordial turbulence in the radiation era generates a weak seed magnetic field on all scales of the turbulence. This field is stochastically amplified by turbulence motions, and it is possible for magnetic and small-scale kinetic energies to attain equipartition within an expansion time. Field generation ceases at the epoch of equal radiation and matter densities, when the field strength is of the order of 1 G. It is estimated that the present intergalactic magnetic field has an intensity of ~10^{-8} G and a scale-length of ~1 Mparsec.

An initially chaotic universe possesses many attractive features.[1] In particular, the chaotic inhomogeneities that survive in the extreme early universe will, quite probably, generate later a state of primordial turbulence in the radiation era. The significance and importance of primordial turbulence in galaxy formation, first stressed by von Weizsäcker[2] and Gamow,[3] has been investigated and discussed by many authors.[4] It has also been proposed that turbulence generates relatively intense magnetic fields during the radiation era of the early universe.[5]

The radiation era, in which blackbody radiation is more dense than matter, extends from $t\sim 10$ sec to $t\sim 10^5$ yr, where t is the Friedmann age. It is shown[6] that uniform rotation in this era generates the weak magnetic field

$$\vec{B}=-\alpha\vec{\xi}=-2\times10^{-4}\vec{\xi}\text{ G}, \tag{1}$$

where $\vec{\xi}$ is the vorticity, and $\alpha=e/m_{\rm H}c$ for a hydrogen plasma. From the ion equation of motion we obtain[7]

$$(d/dt)[R^2(\vec{\xi}_i+\alpha\vec{B})]=R^2(\vec{\xi}_i+\alpha\vec{B})\cdot\nabla\vec{v}_i, \tag{2}$$

in which $d/dt=\partial/\partial t+(\vec{u}+\vec{v})\cdot\nabla$ follows the motion; $\nabla\times\vec{v}=\vec{\xi}$, where $\vec{v}$ is the subsonic turbulence velocity; and $\nabla\cdot\vec{u}=3\dot{R}/R$, where $\vec{u}$ is the expansion velocity and $R(t)$ is the scaling variable. Hence, if the rotation were uniform, then

$$\vec{\xi}_i+\alpha\vec{B}=\vec{\xi}_1(R_1/R)^2, \tag{3}$$

where subscript 1 denotes initial values and $\vec{B}_1=0$. In a radiation-dominated plasma the electron and photon gases are tightly coupled by Thomson scattering and their vorticity $\vec{\xi}$ varies as R^{-1}. The magnetic field of (3) is therefore generated in order to maintain $\vec{\xi}_i\simeq\vec{\xi}$ thus giving (1) when $R\gg R_1$.

The term on the right-hand side of (2) is responsible for stretching and winding field lines when rotation is nonuniform. Consequently, in a turbulent radiation era, a seed magnetic field on the order of magnitude of (1) is generated on all scales of the turbulence, and is then acted on and amplified stochastically by turbulence motions.

The ultimate state of an initially weak magnetic field in a turbulent and conducting fluid is still an unresolved problem.[8] Batchelor[9] suggests that the magnetic field, like the vorticity field, has maximum itensity on the smallest scale of the turbulence and there is equipartition of the magnetic and small-scale kinetic energies. This argument implies that

$$B^2 = 4\pi(\tfrac{4}{3}\rho_\gamma + \rho_m)v_L{}^2 \mathfrak{R}^{-1/2}, \tag{4}$$

where $\rho_\gamma \propto R^{-4}$ is the radiation density, $\rho_m \propto R^{-3}$ is the matter (i.e., ion) density, v_L is the turbulence velocity on the largest scale, and $\mathfrak{R}$ is the Reynolds number. Biermann and Schlüter,[10] on the other hand, propose that equipartition of magnetic and kinetic energies is attained on all scale lengths, thus giving

$$B^2 = 4\pi(\tfrac{4}{3}\rho_\gamma + \rho_m)v_L{}^2. \tag{5}$$

A question of immediate concern is whether the seed field of (1) can be amplified to the intensity of (4) or (5). To resolve this matter it is first necessary to consider some elementary aspects of turbulence in the radiation era.

We assume, following previous work,[4] that the turbulence is homogeneous and isotropic.[11] Energy is transported down the turbulence spectrum, from eddies of largest scale length L where most kinetic energy resides, to eddies of smallest scale length l where it is viscously dissipated. The lifetime of a turbulence cell of scale length $L > \lambda > l$ and velocity v_λ is typically a turnover time $t_\lambda \sim \zeta_\lambda{}^{-1} \sim \lambda/v_\lambda$, and the energy flow per unit mass down the spectrum is of order $v_\lambda{}^2/t_\lambda \sim v_\lambda{}^3/\lambda$, thus giving the Kolmogorov spectrum $v_\lambda \propto \lambda^{1/3}$. The largest scale of the turbulence motions is $L = v_L \tau$, where $\tau = R/\dot{R}$ is the expansion time. The chaotic inhomogeneities of $\lambda > L$ have time scales greater than τ and are, in effect, "frozen" and not part of the turbulence spectrum. If we assume R is normalized to unity at the epoch of equal radiation and matter densities, then $v_L = \text{const}$ when $R \ll 1$ and the cosmic medium is radiation dominated, and $v_L \propto R^{-1}$ when $R \gg 1$ and the medium is matter dominated. More specifically, $v_L = \beta c(1 + \tfrac{3}{4}R)^{-1}$ from conservation of angular momentum, where β is a turbulence parameter (which is assumed to be of order 0.1). The Reynolds number is therefore[12]

$$\mathfrak{R} = \frac{L v_L}{\nu} = \frac{2.6 \times 10^5 \hat{\Omega}^2 \beta^2}{R(1+R)^{1/2}(\tfrac{4}{3}+R)}, \tag{6}$$

where ν is the radiative kinematic viscosity.[13] The smallest scale of the turbulence is $l \sim (\nu t_l)^{1/2}$, and it follows that

$$l = L\mathfrak{R}^{-3/4}, \quad t_l = \tau \mathfrak{R}^{-1/2}. \tag{7}$$

It is evident that LR^{-1} has a maximum at $R \simeq 1$, and therefore at the epoch of equal densities (i.e., $R = 1$) the energy input from chaotic inhomogeneities of $\lambda \geq L$ ceases and the turbulence commences to decay.

On the smallest scale of the turbulence $\zeta_l = t_l{}^{-1}$, and the seed field (1) is therefore

$$B = \frac{2 \times 10^{-13} \hat{\Omega}^3 \beta (1+R)^{1/4}}{R^{5/2}(\tfrac{4}{3}+R)^{1/2}}\ \text{G}, \tag{8}$$

or approximately $2 \times 10^{-13} \hat{\Omega}^3 \beta$ G at $R = 1$, when turbulence commences to decay. The rate at which the magnetic field grows owing to turbulence motions on the smallest scale is

$$dB/dt \sim |\nabla \times (\vec{v} \times \vec{B})| \sim B/t_l, \tag{9}$$

and the time required to attain the field strength of (4), starting from an initial seed field (1) of $(\alpha t_l)^{-1}$, is

$$t_B \sim \frac{\tau}{2\mathfrak{R}^{1/2}} \ln\left(\frac{\beta^2}{\mathfrak{R}^{3/2}} \frac{e^2}{G m_{\text{H}}{}^2}\right). \tag{10}$$

In many cases of interest this is less than the active lifetime $l/\dot{l} \sim \tau$, from (7), of the short-scale end of the turbulence spectrum. Within the available time it is therefore possible, in principle at least, to attain the field strength of (4) by stochastic amplification. The growth time on scales much larger exceeds τ and the greater field strength of (5) apparently cannot be realized.

From (4) we obtain

$$B = \frac{15 \hat{\Omega}^{3/2} \beta^{1/2} (1+R)^{1/8}}{R^{7/4}(\tfrac{4}{3}+R)^{1/4}}\ \text{G}, \tag{11}$$

and at the epoch of equal densities, when turbulence commences to decay, the field strength

$$B_{\text{eq}} = 10 \hat{\Omega}^{3/2} \beta^{1/2}\ \text{G} \tag{12}$$

is roughly of order 1 G for reasonable values of the Hubble-density parameter $\hat{\Omega}$ and the turbulence parameter β. Field generation now ceases, because kinetic energy decays at least as fast as R^{-5}, whereas the magnetic energy diminishes as R^{-4}. Let us assume the intergalactic magnetic field is primordial in origin and has an intensity of B_{eq}/R^2. The present intensity of the intergalactic field is then

$$B_{\text{eq}}(1 + z_{\text{eq}})^{-2} = 1 \times 10^{-7} \hat{\Omega}^{-1/2} \beta^{1/2}\ \text{G}, \tag{13}$$

thus giving $3 \times 10^{-8} \hat{\Omega}^{-1/2}$ G and a scale length ($\propto \beta^{-1/2}$) of 1 Mparsec for $\beta = 0.1$. This result is not in conflict with intergalactic Faraday rotation measurements.[14]

The existence of a moderately intense primordial magnetic field puts a new face on the problem of galaxy formation. Turbulence motions are supersonic after matter and radiation decouple, and hitherto a strong objection to turbulence cosmogonies has been that the universe fragments prematurely into condensations of excessive density.[15] But magnetic stresses now inhibit the formation of high-density condensations in shocked regions, and quite possibly are responsible for determining the magnitude of galactic masses. The Jeans length (or radius determined from the virial theorem), taking account of magnetic stresses only, is $\lambda_J \sim (v_A^2/G\rho_m)^{1/2}$, where $v_A = B/(4\pi\rho_m)^{1/2}$ is the Alfvén speed. After field generation ceases, the Jeans mass $M_J \sim \rho_m \lambda_J^3$ remains constant during expansion and is

$$M_J \sim 3\times 10^{12} \hat{\Omega}^{-7/2} \beta^{3/2} M_\odot. \qquad (14)$$

Thus if $\beta \sim 0.1$, then $M_J \sim 10^{11}\hat{\Omega}^{-7/2}$ solar masses, which is a characteristic galactic mass for $\hat{\Omega} \sim 1$.

I am indebted to B. Jones, M. Rees, and W. Saslaw for informative discussions, and to D. De Young for stimulating and helpful discussions concerning this paper. This research was supported in part by the National Science Foundation and was performed at the Aspen Center for Physics.

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