pears to decrease the growth length insignificantly except for short axial wavelengths, which are not observed.

In summary, the characteristic features of the diocotron instability correlated well with the experimental observations. It appears that the beam distortion is produced at early time, i.e., before $f > \gamma_B^{-2}$, and when breakdown of the background gas occurs shortly thereafter, it occurs along the distorted path of the beam, and the beam follows that path for the remaining 40 nsec.

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Nonlinear Interaction of Electromagnetic Waves in a Plasma Density Gradient*

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Two intense electromagnetic waves interact strongly where the local plasma frequency equals their difference frequency, resulting in an irreversible transfer of action from the higher-frequency wave to the lower-frequency wave. The amount of transfer depends only on the intensities and the density scale length. Successive transfers among a set of waves may produce efficient plasma heating.

Interest in the nonlinear interaction between coherent electromagnetic waves arises from the possibility of exciting longitudinal plasma modes in an underdense ($\omega_p \ll \omega$) plasma by resonance with the difference frequency of two lasers,[1] thereby heating the plasma upon damping of the longitudinal modes. This process has been studied by Rosenbluth and Liu[2] for an inhomogeneous plasma, but neglecting the reaction of the longitudinal mode on the transverse waves; and by Cohen, Kaufman, and Watson,[3] including the reaction and allowing for a cascade, but for a homogeneous plasma.

The present paper treats the transfer of energy between two transverse waves (of frequencies ω_0, ω_1 with $\omega_0 > \omega_1$) in a plasma density gradient. The mechanism of the transfer is the resonant excitation of an electron longitudinal mode at the beat frequency $\Omega \equiv \omega_0 - \omega_1$ and beat wave number $K \equiv k_0 + k_1$ (for the optimum case of opposed lasers, which we consider for definiteness). The excitation occurs over a zone of thickness $h \sim (\nu/\omega_p)L$ about the surface where $\omega_p = \Omega$; L is the density scale length, and ν the longitudinal damping rate.

We stress two important conclusions[4]: (1) The dominant effect of the process is the transfer of action ΔJ from the higher-frequency (ω_0) wave to the lower-frequency (ω_1) wave, transverse action being conserved. Accordingly, the energy loss $\omega_0 \Delta J$ of the ω_0 wave is partitioned, with $\omega_1 \Delta J$ going to the ω_1 wave, and $\Omega \Delta J$ being irreversibly deposited in the plasma. The maximum heating efficiency is thus Ω/ω_0. (That this ratio is low for an underdense plasma led us in Ref. 3 to suggest cascading; we return to this below.) (2) The total amount of action transfer depends on the input power and on the density scale, but is *independent* of the damping rate[5] ν (as long as WKB conditions are satisfied: $h \gg k^{-1}$). There is thus no need to be concerned with the damping mechanism, be it collisional, Landau, or nonlinear.

Our formulation of the interaction is in terms of the local longitudinal dielectric function, and thus is quite model independent. For simplicity of presentation, we ignore ion dynamics, but its inclusion is straightforward. As a byproduct of the calculation, we obtain the exponential spatial growth of Raman back-scattering instability; our result is identical to that of Liu and Rosenbluth,[6] although our basic assumptions are somewhat antithetical to theirs.

After treating the problem of two opposed lasers, we consider using additional lasers to cascade the action to still lower frequencies, with each step providing an incremental efficiency $\sim\Omega/\omega$. We find that this induced cascading, with alternate laser directions (see Fig. 1), appears feasible, in that the intensities required are below the effective Raman instability threshold, as determined by Mostrom *et al.*[7] On the other hand, self-induced cascading,[3] which requires two equally intense parallel lasers, is effective only for intensities well above this threshold.[8]

For simplicity, we treat the case of one-dimensional spatial variation (density gradient, propagation, and amplitude modulation all along z), polarization of the transverse waves along x, and steady-state amplitudes (corresponding to intensities below the absolute instability threshold[9]). The dimensionless vector potential $a(z,t) \equiv eA_x(z,t)/mc^2$ satisfies the nonlinear wave equation[3]

$$\{\nabla^2 - c^{-2}[\partial^2/\partial t^2 + \omega_p^2(z)]\}a = -a\nabla^2\psi,$$

where the term in the dimensionless scalar potential, $\psi(z,t)\equiv e\varphi(z,t)/mc^2$, is the nonlinear part of the transverse current density. The vector potential is expressed in terms of the amplitudes of the two opposed transverse waves:

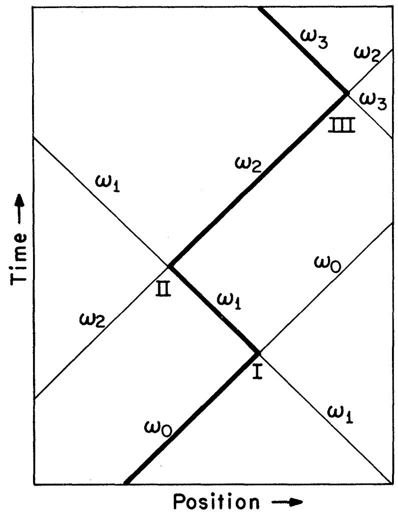

FIG. 1. Schematic space-time plot of a three-step cascade, using four lasers, at frequencies $\omega_0 > \omega_1 > \omega_2 > \omega_3$, propagating in alternate directions. Intensity is represented by line thickness. Each line represents a continuous family of parallel lines, corresponding to steady-state intensities. At each resonance of a difference frequency with the local plasma frequency ($\omega_p{}^{\mathrm{I}} = \omega_0 - \omega_1$, $\omega_p{}^{\mathrm{II}} = \omega_1 - \omega_2$, $\omega_p{}^{\mathrm{III}} = \omega_2 - \omega_3$), most of the action of the higher-frequency wave is transferred to the lower-frequency wave of the interacting pair. A small fraction ($\lesssim \omega_p/\omega$) of the wave energy is deposited locally in the plasma at each transfer.

$$a(z,t) \equiv a_0(z)\exp[-i\omega_0 t + i\int^z k_0(z')\,dz'] + a_1(z)\exp[-i\omega_1 t - i\int^z k_1(z')\,dz'] + \text{c.c.},$$

where $k_l{}^2(z)c^2 \equiv \omega_l{}^2 - \omega_p{}^2(z)$, $l = 0, 1$. Upon substituting into the wave equation, we obtain the coupled set

$$D_0 a_0 \equiv \left[\frac{\partial}{\partial t} + c_0\left(\frac{\partial}{\partial z} + \frac{\partial \ln k_0{}^{1/2}}{\partial z}\right)\right]a_0 = \frac{K^2c^2}{2i\omega_0}a_1\psi_B, \quad D_1 a_1 \equiv \left[\frac{\partial}{\partial t} - c_1\left(\frac{\partial}{\partial z} + \frac{\partial \ln k_1{}^{1/2}}{\partial z}\right)\right]a_1 = \frac{K^2c^2}{2i\omega_1}a_0\psi_B{}^*, \tag{1}$$

where the first two terms are the convective derivatives ($\partial/\partial t$ vanishes here; $c_l \equiv k_l c^2/\omega_l$ is the group velocity), the third term produces the WKB variation $a_l \sim k_l{}^{-1/2}$, and the coupling involves the local Fourier amplitude $\psi_B \equiv \psi(\Omega, K; z)$ of the scalar potential at the beat frequency and wave number. On the left-hand sides of (1), we have neglected second derivatives of a_l; on the right-hand sides we have kept only the potentially resonant terms.

To determine ψ_B, we note[10,3] that the Lorentz force is equivalent to a ponderomotive potential $\psi_{\text{pon}}(z,t) = \frac{1}{2}a^2(z,t)$, so that its Fourier amplitude is $\psi_{\text{pon}}(\Omega, K; z) = a_0(z)a_1{}^*(z)$. The local longitudinal response of the electron plasma to this potential is then given by $\psi_B = \psi_{\text{pon}}(\Omega, K; z)[\epsilon^{-1}(\Omega, K; z) - 1]$, in terms of the local dielectric function. With these expressions substituted into (1), we obtain

$$D_0 a_0 = (K^2c^2/2i\omega_0)|a_1|^2 a_0(\epsilon^{-1} - 1),$$
$$D_1 a_1 = (K^2c^2/2i\omega_1)|a_0|^2 a_1(\epsilon^{-1*} - 1) \tag{2}$$

It is now convenient to introduce the action flux density for each transverse wave. Since the wave energy density is $W_l = \omega_l^2 |a_l|^2 (mc/e)^2/2\pi$, the (absolute) action flux density is $c_l W_l/\omega_l = (k_l/2\pi)|a_l|^2(mc^2/e)^2$. In our natural units we thus define $J_l(z) \equiv (k_l/2\pi)|a_l|^2$, and convert Eqs. (2) to

$$dJ_0/dz = dJ_1/dz = \overline{8}\pi J_0 J_1 \,\mathrm{Im}\epsilon^{-1}, \tag{3}$$

where $\overline{8} \equiv 2K^2/k_0 k_1 \rightarrow 8$ for $\Omega \ll \omega_0$. Noting that $\mathrm{Im}\epsilon^{-1} < 0$, we see that the ω_0 wave loses action flux as it propagates to the right (increasing z), while the ω_1 wave increases action flux as it propagates to the left. The invariance of the signed action flux $\tilde{J} \equiv J_0 - J_1$ represents action conservation. The irreversible dissipation of energy by nonlinear coupling follows from (3): $d(\omega_0 J_0 - \omega_1 J_1)/dz = \overline{8}\pi\Omega J_0 J_1 \,\mathrm{Im}\epsilon^{-1}$, the left-hand side being the divergence of the (signed) energy flux density.

To solve Eq. (3), convert it to $d\ln(J_0/J_1)/dz = -\overline{8}\pi\tilde{J}\,\mathrm{Im}\epsilon^{-1}(\Omega,K;z)$ and integrate across the resonant zone $z \sim z_R$ (where $\epsilon \approx 0$), obtaining

$$\Delta \ln(J_0/J_1) = \overline{8}\pi\tilde{J}\int dz\, \mathrm{Im}\epsilon^{-1}(\Omega,K;z),$$

with $\Delta f \equiv f(z<z_R) - f(z>z_R)$. To evaluate the integral, consider the limit $\mathrm{Im}\epsilon \rightarrow 0^+$ (representing weak damping, $\nu \ll \omega_p$, i.e., a narrow resonance zone, $h \ll L$). Then $\mathrm{Im}\epsilon^{-1} \rightarrow -\pi\delta(\epsilon(\Omega,K;z))$, and

$$\int dz\, \mathrm{Im}\epsilon^{-1} \rightarrow -\pi|\partial\epsilon(\Omega,K;z)/\partial z|^{-1} \equiv -\pi L$$

(defining the scale length L precisely[11]). We finally obtain for the action transfer ΔJ the formula

$$(1-\Delta J/J_0^{\,\mathrm{in}})(1+\Delta J/J_1^{\,\mathrm{in}}) = \exp[\overline{8}\pi^2 L(J_0^{\,\mathrm{in}} - J_1^{\,\mathrm{in}} - \Delta J)], \tag{4}$$

where $J_0^{\,\mathrm{in}} \equiv J_0\ (z<z_R)$ and $J_1^{\,\mathrm{in}} \equiv J_1\ (z>z_R)$ are the input action flux densities. This transcendental relation yields ΔJ as a function of $J_0^{\,\mathrm{in}}$, $J_1^{\,\mathrm{in}}$, and L, and is independent of the dissipative mechanism and magnitude. All that is required of the dissipation is that it be not too large[12] ($\nu \ll \omega_p$) and not too small[13] $[\nu/\omega_p \gg (k_0 L)^{-1}]$.

Equation (4) can be converted to the formula

$$\tilde{J}_0 = (1-R-\rho)^{-1}\ln[(1-R)(\rho+R)/\rho] \tag{5}$$

for the dimensionless input action $\tilde{J}_0 \equiv \overline{8}\pi^2 L J_0^{\,\mathrm{in}}$ needed to produce a relative action transfer $R \equiv \Delta J/J_0^{\,\mathrm{in}}$ for given input ratio $\rho \equiv J_1^{\,\mathrm{in}}/J_0^{\,\mathrm{in}}$. This relation is plotted in Fig. 2. The relation between $\tilde{J}_0$ and input power density (in units of 10^{12} W/cm^2) is $P_0^{\,\mathrm{in}} = \frac{2}{3}\tilde{J}_0 L_{\mathrm{cm}}^{-1}(\omega_0/\overline{\omega})$, where $\overline{\omega} \equiv 1.8 \times 10^{14}\ \mathrm{sec}^{-1}$ is the frequency of a CO_2 laser.

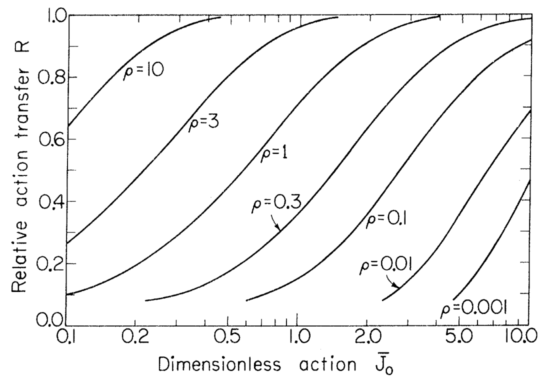

FIG. 2. Relative action transfer $R \equiv \Delta J/J_0^{\,\mathrm{in}}$ as a function of $\tilde{J}_0$, the dimensionless action input in the ω_0 wave, for representative values of input ratio $\rho \equiv J_1^{\,\mathrm{in}}/J_0^{\,\mathrm{in}}$.

To illustrate the use of Fig. 2, we see that with $\tilde{J}_0 \approx 6$, $L = 10$ cm (so that $P_0^{\,\mathrm{in}} \approx 4\times10^{11}$ W/cm^2), and $\rho = 0.1$ (so that $P_1^{\,\mathrm{in}} \approx 4\times10^{10}$ W/cm^2), a fraction $R \approx 0.8$ of the action of the ω_0 wave is transferred to the ω_1 wave. The heating efficiency is $\approx (\Omega/\omega_0)R \approx 8\%$ for $\Omega/\omega_0 \approx 0.1$. A number of special cases are of interest:

(i) For $\rho \ll 1$, $R \ll 1$, Eq. (4) yields $J_1^{\,\mathrm{out}} = J_1^{\,\mathrm{in}} \times \exp(\tilde{J}_0)$, corresponding to the exponential growth of a small-amplitude wave in the Raman backscattering instability. The exponent is $\tilde{J}_0 = \frac{3}{2}P_0^{\,\mathrm{in}} \times (10^{12}\ \mathrm{W/cm^2})L_{\mathrm{cm}}$ for $\omega_0 = \overline{\omega}$, in exact agreement with Liu and Rosenbluth.[6] While those authors neglect dissipation but include convection of the longitudinal mode, our approach ignores convection relative to dissipation. A more general study of the instability by DuBois and Williams,[9] including both dissipation and convection parameters for the longitudinal mode, again yields this result, now independent of both parameters.

(ii) For $\rho \gg 1$, Eq. (4) yields $J_0^{\,\mathrm{out}} = J_0^{\,\mathrm{in}}\exp(-\tilde{J}_1)$. Here the ω_1 wave produces an exponential attenuation of the ω_0 wave, just the opposite of case (i).

(iii) For $\rho \sim O(1)$, $\tilde{J}_0 \ll 1$, Eq. (5) yields $R = \tilde{J}_1$.

(iv) For $\rho < 1$, $R = 1-\rho$, (5) yields $\tilde{J}_0 = \rho^{-1} - 1$. In this special case $J_1^{\,\mathrm{out}} = J_0^{\,\mathrm{in}}$ and $J_0^{\,\mathrm{out}} = J_1^{\,\mathrm{in}}$; i.e., there is an exchange of actions. For example, choose $\rho = 0.1$; $\tilde{J}_0 = 9$ and $\tilde{J}_1 = 0.9$ are the inputs, while 0.9 and 9 are the respective outputs.

The last example (iv) is typical of useful orders of magnitude for a study of a cascade arrangement (Fig. 1). Suppose we have available four lasers with $\omega_0 \approx 1.8\times10^{14}$ (CO_2), $\omega_1 \approx 1.6 \times 10^{14}$, $\omega_2 \approx 1.5\times10^{14}$, $\omega_3 \approx 1.2\times10^{14}$, so that the successive beat frequencies are $\Omega \approx 2\times10^{13}$, 1×10^{13}, 3×10^{13}. With the parameters of example

(iv) for the first two lasers, and $L \sim 10$ cm, we need $P_0^{\text{in}} \sim 6\times 10^{11}$ W/cm, $P_1^{\text{in}} \sim 6\times 10^{10}$ W/cm^2. These waves interact in the zone at $\omega_p \approx 2\times 10^{13}$ sec^{-1}, whereupon now $P_0 \sim 6\times 10^{10}$, $P_1 \sim 6\times 10^{11}$. The exchange is repeated between the ω_1 and ω_2 lasers at $\omega_p \approx 1\times 10^{13}$ sec^{-1}; choosing $P_2^{\text{in}} \sim 6\times 10^{10}$, the ω_2 wave extracts most of the power from the ω_1 wave, producing $P_2 \sim 6\times 10^{11}$, $P_1 \sim 6\times 10^{10}$. Thus the ω_1 wave has acted as a catalyst for transferring action from ω_0 to ω_2. The process can be repeated with ω_3 in an obvious way. The heating efficiencies of the successive steps are roughly 9%, 5%, 16%, with a total efficiency of about 30%.

The study of Mostrom, Kaufman, and Nicolson[7] has shown that nonlinear attenuation due to Raman side- and back-scattering is effective over a distance $< L$ when $\bar{J}_0 > 35$. The parameters chosen here are below this threshold, which is extremely sharp. The reason is that the transfer mechanism for the instability is identical to that for coherent interaction, but the former starts from small-amplitude noise.

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[11]For a cold-plasma model $[\epsilon = 1 - \omega_p^2(z)/\omega^2]$, $L = |d \ln n / dz|^{-1}$.

[12]Model calculations with $\nu \sim \omega_p$ indicate that (4) retains qualitative validity, within a factor of 2 in the exponent.

[13]C. Oberman brought the need for such a condition to our attention.

Optical Quenching of Photoluminescence in Chalcogenide Glasses; Evidence for States in Midgap

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Optical quenching of photoluminescence by radiation with photon energy less than the band-gap energy has been observed in glassy As_2Se_3 and $As_2Se_3 \cdot As_2Te_3$. The quenching spectra provide evidence of optical absorption by a narrow band of localized states in the gap which are involved in the radiative recombination process and which are attributable to defects or impurities rather than to tails in the density of states extending into the gap.

Perhaps the most striking feature of photoluminescence (PL) spectra in chalcogenide glasses is the fact that in all the glasses studied thus far[1-4] the luminescence maxima occur at energies well below the band-gap energy in a spectral range of high transparency. Kolomiets and co-workers first observed such spectra in bulk samples of glassy $As_2Se_3 \cdot AsTe_3$,[1] As_2Se_3,[2,4] and As_2S_3,[2] and attributed them to transitions involving impurity-like levels lying deep within the band gap. Davis and Mott[5] regarded those results as evidence for the existence of a high density of defect centers near the center of the forbidden gap in some amorphous semiconductors. However, Fischer