[2]R. J. Crewther, Phys. Rev. Lett. 28, 1421 (1972).
[3]M. Gell-Mann, CERN Report No. TH1543, 1972 (to be published).
[4]For example, G. Preparata, "VMD vs. PCAC in $\pi^- \rightarrow \gamma\gamma$" (unpublished); see also S. D. Drell, SLAC Report No. SLAC-PUB-1158 (to be published).
[5]In writing this equation I have used the SU(3) value for the ratio of isoscalar to isovector photon-induced cross sections.
[6]J. D. Bjorken, Phys. Rev. 148, 1467 (1966), and 179, 1547 (1969).
[7]S. L. Adler *et al.*, Phys. Rev. D 6, 2982 (1972).
[8]D. J. Gross, Phys. Rev. D 4, 1130 (1971).
[9]For example, J. Kogut, Phys. Rev. D 2, 1152 (1971), particularly the note added in proof; also J. D. Bjorken, private communication. If one assumes that b_γ (the "photon" radius squared) vanishes when $q^2 \rightarrow -\infty$, then $b_0 \sim b_\pi + b_\gamma \sim 3$.
[10]Recall that the restriction to $J=1$ states only implies that $\rho(s)$ cannot fall slower than $1/s$, so, provided the first assumption is valid, K is finite.
[11]N. Christ *et al.* Phys. Rev. D 6, 3543 (1972).
[12]H. Terazawa, Phys. Rev. D 6, 2530 (1972).

SU(4) ⊗ SU(4) Lepton-Hadron Symmetry, and the Existence of Heavy Charmed Particles

P. Dittner* and S. Eliezer
Department of Physics, Imperial College, Prince Consort Road, London SW7, England

and

T. K. Kuo†
Purdue University, West Lafayette, Indiana 47907, and Department of Physics, Imperial College, Prince Consort Road, London SW7, England

(Received 2 April 1973)

We suggest a lepton-hadron analogy based on the spontaneously broken SU(4)⊗SU(4) symmetry. This leads to the existence of heavy charmed particles ($\gtrsim 5$ GeV), which may explain the rise in the total cross section of pp scattering at $E_{\text{c.m.}} \gtrsim 15$ GeV.

In this paper we wish to investigate an application of the group SU(4) in an attempt to understand the fundamental similarities between the hadrons and the leptons.[1-3] For the hadrons, it is well known[2] that an SU(4) symmetry scheme offers a natural explanation of the absence of strangeness-changing neutral currents. On the other hand, a straightforward application of SU(4) would predict[1-3] a hadron spectrum which, in addition to including a large number of low-lying "charmed" particles, disturbs the known "good" sum rules of SU(3). As has been suggested recently,[4] these bad features may be removed in a theory in which SU(4)⊗SU(4) is realized by Goldstone bosons, with the vacuum invariant only under SU(3). The hadron spectrum then exhibits SU(3) multiplets. The Goldstone bosons consist of seven scalar mesons and fifteen pseudoscalar mesons. The first eight and the fifteenth pseudoscalar mesons we identify as π, K, η, and $\eta'(X)$, while the remaining six are a triplet and antitriplet of charmed particles. Using a lepton-hadron analogy we shall estimate the masses of these charmed pseudoscalars.

As far as the hadrons are concerned, the SU(4) ⊗ SU(4)-symmetry-breaking term is usually assumed to transform[1-4] predominantly as a sum of terms from the representation $(\underline{4}, \underline{4}^*) \oplus (\underline{4}^*, \underline{4})$. The symmetry-breaking term in the Hamiltonian will consequently be

$$\mathcal{H}_{SB} = u_0 + cu_8 + du_{15} + \text{"higher representations,"} \quad (1)$$

which leaves isospin, hypercharge (Y), and the fifteenth generator $[(\frac{2}{3})^{1/2}Z]$ exactly conserved. To introduce the hadron-lepton analogy, we express $\mathcal{H}_{SB}$ in terms of the hadronic quarks $q=(\mathcal{P}, \mathcal{N}, \lambda, \mathcal{P}')$ which transform as the basic representation of SU(4). Only the $(\underline{4}, \underline{4}^*) \oplus (\underline{4}^*, \underline{4})$ terms in $\mathcal{H}_{SB}$ give rise to "mass terms" for the quarks, so that $\mathcal{H}_{SB}$ is written as

$$\mathcal{H}_{SB} = m_0 \bar{q} M q + \text{"higher order in quark fields,"} \quad (2)$$

and

$$M \equiv \begin{bmatrix} \frac{1}{\sqrt{2}}+\frac{c}{\sqrt{3}}+\frac{d}{\sqrt{6}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}+\frac{c}{\sqrt{3}}+\frac{d}{\sqrt{6}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}-\frac{2c}{\sqrt{3}}+\frac{d}{\sqrt{6}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}}-\frac{3d}{\sqrt{6}} \end{bmatrix}. \tag{3}$$

As was discussed elsewhere, $\mathcal{H}_{SB}$, which conserves isospin and parity, ought to satisfy another condition[5,6]:

$$[W, \mathcal{H}_{SB}] = 0, \tag{4}$$

where the finite chiral rotation W is given by

$$W = \exp[i\pi(Q_3{}^V - Q_3{}^A)], \tag{5}$$

with $Q_i{}^V$ and $Q_i{}^A$ being the vector and axial-vector generators of SU(4)⊗SU(4). Condition (4) gives the relation

$$1/\sqrt{2} + c/\sqrt{3} + d/\sqrt{6} = 0, \tag{6}$$

which gives SU(2)⊗SU(2) invariance[7] for the $(\underline{4}, \underline{4}^*) \oplus (\underline{4}^*, \underline{4})$ part of $\mathcal{H}_{SB}$ and massless $\mathcal{P}$ and $\mathfrak{N}$ quarks. Thus, in this model the higher representations in $\mathcal{H}_{SB}$ [such as (15, 15), etc.] are necessary in order to reproduce a nonvanishing pion mass. However, since m_π is indeed very small on the hadronic scale, we shall keep only the $(\underline{4}, \underline{4}^*) \oplus (\underline{4}^*, \underline{4})$ term in the Hamiltonian. Thus using Eq. (6) we can write the hadronic symmetry-breaking Hamiltonian as

$$\mathcal{H}_{SB} = -\sqrt{3}m_0 c\bar{\lambda}\lambda + m_0(2\sqrt{2}+\sqrt{3}c)\bar{\mathcal{P}}'\mathcal{P}'. \tag{7}$$

We will now estimate the parameter c in Eq. (7) by invoking the lepton-hadron analogy. The mass term in the lepton Hamiltonian is

$$\mathcal{H}_l = m_e\bar{e}e + m_\mu\bar{\mu}\mu. \tag{8}$$

Now we assume that the leptons (μ, e, ν_μ, ν_e) (in some order) transform as the basic representation of *the same SU(4) as the hadrons*, and because of the experimental fact that the muon is much more massive than the electron, we place the electron in the SU(3) triplet with the neutrinos and leave the muon as the singlet. Assuming that a similar mechanism (from the symmetry point of view) is responsible for the masses of both the hadron quarks and the leptons, we choose

$$-\sqrt{3}c/(2\sqrt{2}+\sqrt{3}c) = m_e/m_\mu \simeq \tfrac{1}{200}, \tag{9}$$

and make the analogy $\mathcal{P} \leftrightarrow \nu_1$, $\mathfrak{N} \leftrightarrow \nu_2$, $\lambda \leftrightarrow e$, and $\mathcal{P}' \leftrightarrow \mu$, where ν_1 and ν_2 are mixtures of ν_μ and ν_e, since the neutrinos are indistinguishable as far as strong and electromagnetic interactions are concerned. Equations (6) and (9) give

$$c = -0.008, \quad d = -1.720. \tag{10}$$

Thus within this scheme both chiral SU(2)⊗SU(2) and SU(3) are good symmetries. To obtain the masses of the pseudoscalar mesons in terms of the parameters c and d, we use the formula[8]

$$f^2 m_{ij}{}^2 = -\langle 0|[Q_i{}^A, [Q_j{}^A, \mathcal{H}_{SB}]]|0\rangle, \tag{11}$$

and remember that u_{15} as well as u_0 has a nonvanishing vacuum expectation value in the chiral limit, since the vacuum is invariant only under SU(3). Taking the masses of K and η as input we predict a mass of 910 MeV for the $\eta'(X)$ particle, while the mass of the charmed triplet is around 5 GeV! Using the appropriate analog to Eq. (11) for the charmed scalar Goldstone bosons (and assuming roughly the same magnitude for the decay constants f), we obtain a similar value, of the order of 5 GeV, for the scalar charmed particles. All of these heavy charmed mesons can of course interact with uncharmed particles to form massive charmed states. One might therefore expect to see a vast number of heavy, charmed states beyond energies of the order of 5 GeV. That the masses of the charmed particles can differ so significantly from those of the ordinary particles is just a further consequence of realizing SU(4) with Goldstone bosons.

Experimentally, the most direct test of our ideas would be the search for these charmed particles—massive states with only weak decay modes into ordinary, uncharmed particles. Less directly, we may observe pair production of these heavy particles which represents the onset of many new channels when the center-of-mass energy reaches a value of the order of 10 GeV.

Since the total cross sections of uncharmed-particle scattering have reached a plateau below this energy, it is conceivable that a rise in the total cross section should start for center-of-mass energies around 10 GeV, signifying the opening up of new processes. Recently, the intersecting-storage-rings experiments at CERN seem to indicate an upturn of the proton-proton total cross section at about 15 GeV.[9] Needless to say, the copious pair production of charmed particles would fit these experiments rather nicely. Quantitatively, we may expect that the production cross section of charmed particles is comparable with that of strange particles. At $E_{\text{c.m.}} \simeq 4$ GeV, which is only about 1.5 GeV above the strange-particle threshold, the total cross section of pp into strange particles $\simeq 2$ mb.[10] A similar situation for charmed-particle production would give a nice explanation of the data for pp total cross sections above 15 GeV.

We proceed further with the lepton-hadron symmetry by investigating the electromagnetic and weak currents. In order to implement the analogy we shall require that the hadronic quarks have the same charge spectrum as the leptons, namely 0, 0, -1, and -1. The physical mesons and baryons we take to be $q\bar{q}$ and $qq\bar{q}$, respectively, which implies that the $\mathcal{P}'$ quark is uncharged and that the hadronic quarks have unit baryon number. These requirements together with the usual SU(3) considerations fix the hadronic charge operator[11]:

$$Q_h = I_3 + \tfrac{1}{2}Y - \tfrac{2}{3}Z - \tfrac{1}{2}F, \tag{12}$$

where F is the generator of a U(1) group which commutes with SU(4) $\otimes$ SU(4), and which must take the value 1 for the hadronic quarks. We shall take F to be the fermion number,

$$F = B + L \tag{13}$$

(where B and L refer to baryon and lepton number), so that formula is exactly correct for the hadrons and yields the correct charge spectrum for the leptons. To assign the correct charge to each individual lepton, however, we must perform an SU(4) rotation on Q_h, in order to obtain the lepton charge operator Q_l. Explicitly, write the electromagnetic currents of the leptons and hadrons as

$$j_{\text{em}}{}^{\mu}(\text{hadron}) = \bar{q}Q_h\gamma^{\mu}q, \tag{14}$$

$$j_{\text{em}}(\text{lepton}) = \bar{l}Q_l\gamma^{\mu}l, \tag{15}$$

where $q = (\mathcal{P}, \mathfrak{N}, \lambda, \mathcal{P}')$, $l = (\nu_1, \nu_2, e, \mu)$, and Q_l and Q_h are the 4×4 matrices representing the charge operators:

$$Q_h = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad Q_l = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{16}$$

Then

$$Q_l = RQ_hR^{-1}, \tag{17}$$

where R is an SU(4) rotation whose 4×4 representation can be chosen as

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{18}$$

This is clearly unique only up to rotations in the (e, μ) and (ν_1, ν_2) subspaces, but these are unmeasurable phases. Next consider the weak currents:

$$j_w{}^{\mu}(\text{hadron}) = \bar{q}W_h\gamma^{\mu}(1-\gamma_5)q, \tag{19}$$

$$j_w{}^{\mu}(\text{lepton}) = \bar{l}W_l\gamma^{\mu}(1-\gamma_5)l, \tag{20}$$

where W_h and W_l are again 4×4 matrices. The weak interaction chooses the physical neutrinos as a mixture of ν_1 and ν_2:

$$\begin{bmatrix} \nu_\mu \\ \nu_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}; \tag{21}$$

therefore W_l can be written as

$$W_l = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ \cos\theta & \sin\theta & 0 & 0 \end{bmatrix}. \tag{22}$$

Now it is clear that the same SU(4) rotation R which relates the hadronic and leptonic electromagnetic currents, *also relates the weak currents:*

$$R^{-1}W_lR = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \cos\theta & 0 & 0 & -\sin\theta \\ \sin\theta & 0 & 0 & \cos\theta \\ 0 & 0 & 0 & 0 \end{bmatrix} = W_h, \tag{23}$$

which does indeed give the correct SU(3) weak hadronic current, when we identify the neutrino mixing angle θ with the Cabibbo angle. Besides this, the full SU(4) weak hadronic current suggested here avoids the undesirable strangeness-changing neutral currents, since

$$[W_h, W_h^+] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{24}$$

To conclude, a lepton-hadron analogy within an SU(4) scheme with a vacuum only SU(3) invariant gives very heavy charmed particles ($\gtrsim 5$ GeV) which may explain the rise in total cross sections at center-of-mass energies higher than 10 GeV. Also a relationship between weak and electromagnetic currents is suggested by this analalogy which predicts first the $V-A$ structure for the weak hadronic current (since the $\mathcal{P}$ and $\mathfrak{N}$ quarks are massless in this scheme), secondly the correct SU(3) Cabibbo structure of this current, and finally avoids neutral strangeness-changing currents.

The authors are grateful to Dr. I. G. Halliday, Dr. J. Huskins, and Dr. P. Schmid for many helpful comments. T.K.K. would like to thank Professor T. W. B. Kibble for his kind hospitality at Imperial College.

---

*Science Research Council Fellow.

†Work supported in part by the U. S. Atomic Energy Commission.

[1]D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Nuovo Cimento 34, 1732 (1964).

[2]B. J. Bjorken and S. L. Glashow, Phys. Lett. 11, 255 (1964); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[3]Z. Maki, T. Maskawa, and I. Umemura, Progr. Theor. Phys. 47, 1682 (1971); S. Meshkov and S. P. Rosen, to be published.

[4]P. Dittner and S. Eliezer, Imperial College Report No. ICTP/72/15 (to be published).

[5]T. K. Kuo, Phys. Rev. D 4, 3620 (1971), and 5, 1033 (1972).

[6]We may summarize the results of Ref. 5 as follows. Given isospin and parity, it is well known that there are two, physically equivalent, parity operators: P and $P' \equiv (-1)^{2I}P$. Let us define an operator W so that $WPW^{-1} = P'$. Then $[W, H] = 0$. A representation of W is the finite chiral rotation given in Eq. (5).

[7]This is just a simple extension of the work by A. McDonald, S. P. Rosen, and T. K. Kuo, Phys. Lett. 40B, 675 (1972). A $(4, 4^*) \oplus (4^*, 4)$ symmetry-breaking Hamiltonian contains only terms transforming like (0, 0) and $(\frac{1}{2}, \frac{1}{2})$ under SU(2) ⊗ SU(2), and the $(\frac{1}{2}, \frac{1}{2})$ is forbidden by W invariance.

[10]J. D. Hansen *et al.*, CERN Report No. CERN-HERA 70-2 (unpublished).

[11]For hadrons this is just one of the formulas given in Ref. 1.

[8]R. Dashen, Phys. Rev. 183, 1245 (1969).

[9]U. Amaldi *et al.*, to be published. Their Eq. (7) gives $\Delta\sigma_T = 4.1 \pm 0.7$ mb as $E_{\mathrm{c.m.}}$ increases from 23 to 53 GeV. S. R. Amendolia *et al.*, to be published.