Uniqueness of Spontaneously Broken Gauge Theories*

John M. Cornwall,† David N. Levin, and George Tiktopoulos
Department of Physics, University of California, Los Angeles, California 90024
(Received 25 April 1973)

We have made a systematic search for theories of interacting heavy vector mesons which have unitarily bound trees. In simple cases (four vector mesons and one scalar particle) the *only* unitarily bound models are spontaneously broken gauge theories. Evidently, a unitarity bound, which controls high-energy behavior, imposes internal symmetry on heavy-vector-boson interactions.

Formal manipulation and detailed calculations indicate that spontaneously broken gauge theories (SBGT's)[1] are renormalizable theories of heavy vector mesons. In the unitary gauge,[2] characterized by the absence of nonphysical scalar excitations in the Lagrangian, an essential aspect of renormalizability is that tree graphs have a high-energy behavior consistent with the unitary bound on imaginary parts. Loosely speaking, the invariant amplitude T_N for the N-point tree graphs should be of order E^{4-N} or less when all scalar invariants are large and proportional to a characteristic squared energy (E^2). In this way, the imaginary part of one-loop N-point amplitudes, when calculated from unitarity, will have the same high-energy behavior (up to logarithms). A large number of authors[3] have shown that SBGT's satisfy this high-energy condition, which we term the tree-unitarity condition. The satisfaction of the unitarity requirement is made possible by coupling-constant and mass relations, which are a remnant of the original gauge symmetry of the SBGT.

In this paper, we investigate the inverse question: Are all heavy-vector-meson theories which satisfy the tree-unitarity condition of the SBGT type? The interest in this question is threefold: (1) At present, no known SBGT embraces all the experimental weak-interaction phenomenology in a clean-cut way; is there another class of renormalizable theories that does? (2) It is of considerable phenomenological interest to study possibly nonrenormalizable theories where it is only known that the tree-unitarity condition is satisfied for N-point trees with $N \leq M$, where M is an interger like 4 or 5. In such cases, effects associated with $G_F\Lambda^2 \sim 1$ (where Λ is a cutoff and G_F is the Fermi coupling constant) occur only in amplitudes of $O(G_F^2)$ or higher. (3) It is remarkable to see the degree of internal symmetry which emerges simply by imposing unitarity, with no *a priori* symmetry constraints.

We have derived the requisite relations between masses and coupling constants to satisfy the tree-unitarity condition in two ways. The first, reported here, is the brute-force technique of calculating the four- and five-point trees explicitly. The second way is to exploit the geometry induced on the internal (longitudinal vector and scalar) degrees of freedom by the structure of the Lagrangian, and to demand that the curvature of this internal space be zero.[4] Details of the second approach will be reported in a lengthier publication.[5]

The most general interaction Lagrangian for vector fields $W_{a\mu}$ (mass M_a) and scalar fields φ_k (mass m_k), which would be (naively) renormalizable if all the M_a were zero (i.e., which has dimension less than or equal to four), is[6]

$$-L^1 = A_{abc}\epsilon^{\mu\nu\lambda\rho}\partial_\rho W_{a\mu}W_{b\nu}W_{c\lambda} + B_{abcd}\epsilon^{\mu\nu\lambda\rho}W_{a\mu}W_{b\nu}W_{c\lambda}W_{d\rho} + C_{abc}\partial_\nu W_{a\mu}W_b{}^\mu W_c{}^\nu + D_{abcd}W_{a\mu}W_b{}^\mu W_{c\nu}W_d{}^\nu$$
$$+F_{ab}{}^{(k)}\varphi_k W_{a\mu}W_b{}^\mu + G_a{}^{kl}W_{a\mu}\varphi_k\partial^\mu\varphi_l + H_{ab}{}^{kl}\varphi_k\varphi_l W_{a\mu}W_b{}^\mu + P^{klm}\varphi_k\varphi_l\varphi_m + R^{klmn}\varphi_k\varphi_l\varphi_m\varphi_n, \qquad (1)$$

where all coefficients are real and $A_{abc} = -A_{acb}$, B_{abcd} is totally antisymmetric, $D_{abcd} = D_{bacd} = D_{cdab}$, $F_{ab}{}^{(k)} = F_{ba}{}^{(k)}$, $H_{ab}{}^{kl} = H_{ba}{}^{kl} = H_{ab}{}^{lk}$, P^{klm} is totally symmetric, and R^{klmn} is totally symmetric. Note the CP-nonconserving terms involving A_{abc} and B_{abcd}. Fermions may be added without difficulty; for brevity we omit them. They do not affect our conditions (2)–(9) below. Some of the vectors may be massless, in which case they must couple to conserved currents in order to preserve Lorentz invariance of the S matrix.

Consider the four-point amplitude for $WW \to WW$. When all vectors are longitudinal, this amplitude is of $O(E^6)$ at large energy, while tree unitarity requires it to be of $O(E^0)$. Straightforward calculation yields the following conditions for the tree unitarity of all $WW \to WW$ amplitudes:

$$A_{abc} = 0, \tag{2}$$

$$B_{abcd} = 0, \tag{3}$$

$$C_{abc} \text{ is totally antisymmetric,} \tag{4}$$

$$C_{abe}C_{cde} - C_{ace}C_{bde} - C_{ade}C_{cbe} = 0, \tag{5}$$

$$8D_{abcd} = C_{ace}C_{bde} + C_{ade}C_{bce}, \tag{6}$$

$$r_{abcd} = r_{acbd}, \tag{7}$$

where

$$r_{abcd} = 4F_{ab}{}^{(k)}F_{cd}{}^{(k)} + (C_{abe}C_{cde}/M_e{}^2)(M_a{}^2 - M_b{}^2)(M_c{}^2 - M_d{}^2) + (M_a{}^2 + M_b{}^2 + M_c{}^2 + M_d{}^2 - 3M_e{}^2)C_{ace}C_{dbe}.$$

In the last relation those terms divided by a vector-meson mass are summed only over massive vector particles. Equations (4) and (5) mean that the C_{abc} are the structure constants of a Lie algebra. Conditions (2)–(6) specify that the purely vector-meson Lagrangian (other than mass terms) is invariant under the local gauge transformations of that Lie group, i.e., the purely vector-meson terms are of the (possibly massive) Yang-Mills type, and no CP-nonconserving terms are allowed. On the other hand, Eq. (7) shows that, if the vector-meson scattering amplitude obeys tree unitarity, then either there must exist scalar particles appropriately coupled to the vectors or all vector particles are massless. Next, the same procedure is applied to the scattering amplitude for $W_aW_b \to \varphi_k\varphi_l$; the resulting tree-unitarity conditions are as follows:

$$\hat{G}_a{}^{kn}\hat{G}_b{}^{nl} - \hat{G}_b{}^{kn}\hat{G}_a{}^{nl} + C_{abn}\hat{G}_n{}^{kl} = M_n{}^{-2}(F_{an}{}^{(k)}F_{nb}{}^{(l)} - F_{an}{}^{(l)}F_{nb}{}^{(k)}), \tag{8}$$

$$4H_{ab}{}^{kl} = \hat{G}_a{}^{kn}\hat{G}_b{}^{nl} + \hat{G}_b{}^{kn}\hat{G}_a{}^{nl} - M_n{}^{-2}(F_{an}{}^{(k)}F_{nb}{}^{(l)} + F_{an}{}^{(l)}F_{nb}{}^{(k)}) - C_{abn}\bar{G}_n{}^{kl}(M_a{}^2 - M_b{}^2)/M_n{}^2, \tag{9}$$

where

$$\hat{G}_a{}^{kl} = \tfrac{1}{2}(G_a{}^{kl} - G_a{}^{lk}), \quad \bar{G}_a{}^{kl} = \tfrac{1}{2}(G_a{}^{kl} + G_a{}^{lk}).$$

Once again, terms with a vector-meson mass in the denominator are summed only over massive vector particles. Notice that, *if* the right-hand side of Eq. (8) were zero (e.g., if all vectors were massless), then the matrices G_a would represent the Lie algebra specified by C_{abc}. Equation (9) determines the vector-scalar "seagull" in terms of other coupling constants. Needless to say, conditions (2)–(9) are satisfied by SBGT theories, as one may show by direct calculation. Other conditions emerge from the process $WW \to W\varphi$ and from the five-point trees, which uniquely determine the φ^3 and φ^4 interactions. Rather than write these out in full generality, we deal with specific examples next.

We do not yet know how to solve these nonlinear equations in general. Therefore, let us consider a special particle spectrum, which includes most of the popular models in which the group structure is based on SU(2), U(1), or their direct product.

Assume that there are three massive vectors, one massless vector, and one scalar. The massless vector (W_4) must couple to a conserved current. The other fields ($2^{-1/2}W_{1\mp i2}$, w_3, φ) have the conserved charge assignments ±1, 0, 0. Charge conservation imposes conditions on the Lagrangian such as (suppressing scalar indices)

$$C_{134} = 0, \quad C_{234} = 0, \quad F_{11} = F_{22}, \quad F_{44} = 0, \quad F_{ab} = 0, \text{ if } a \neq b;$$
$$G_1 = G_2 = G_4 = 0, \quad H_{11} = H_{22}, \quad H_{44} = 0, \quad H_{ab} = 0, \text{ if } a \neq b. \tag{10}$$

The net effect of these restrictions is to reduce the Lagrangian (1) to a form with 26 independent coupling constants. Now we can demonstrate our main result: Tree unitarity is satisfied if and only if these 26 coupling constants can be described by a few independent parameters, exactly in accordance with the SBGT prescription. First notice that Eq. (7) implies

$$4F_{11}{}^2 = (4M_1{}^2 - 3M_3{}^2)C_{123}{}^2 + 4M_1{}^2C_{124}{}^2,$$
$$4M_1{}^2F_{11}F_{33} = M_3{}^4C_{123}{}^2, \tag{11}$$

and that Eq. (9) can be written as

$$F_{33}{}^2 + 2M_3{}^2H_{33} = 0, \quad F_{11}{}^2 + 2M_1{}^2H_{11} = 0. \tag{12}$$

Next, the tree-unitarity condition is imposed on the scattering amplitudes for $W_1W_1 \to \varphi\varphi\varphi$ and

$W_3W_3 \to \varphi\varphi\varphi$ with the result

$$\begin{aligned} &F_{11}G_3^{\,2}=0,\\ &F_{11}(4M_1^{\,2}R-2m^2H_{11}+3PF_{11})=0,\\ &F_{33}G_3^{\,2}=0,\\ &F_{33}(4M_3^{\,2}R-2m^2H_{33}+3PF_{33})=0. \end{aligned} \tag{13}$$

Similarly, the tree-unitarity behavior of $W_1W_1 \to W_1W_1\varphi$ and $W_3W_3 \to W_3W_3\varphi$ is guaranteed if

$$\begin{aligned} &F_{11}^{\,2}(2M_1^{\,2}P+m^2F_{11})=0,\\ &F_{33}^{\,2}(2M_3^{\,2}P+m^2F_{33})=0. \end{aligned} \tag{14}$$

A set of coupling constants and masses, which satisfies all of the above equations, must fall into one of five categories:

(a) $G_3=F_{11}=F_{33}=0$. It follows that *all* vector particles are free and that the scalar has φ^3 and φ^4 interactions only.

(b) $G_3=F_{11}=0$; $F_{33}\neq 0$. Now W_1, W_2, and W_4 are free; the W_3-φ system is identical to the original Higgs[7] [U(1)] SBGT.

(c) $G_3=F_{33}=0$; $F_{11}\neq 0$. In this case W_3 is free while the (W_1, W_2, W_4, φ) interaction is identical to the Georgi-Glashow[8] [SU(2)] SBGT.

(d) $G_3=0$; $F_{11}\neq 0$; $F_{33}\neq 0$. All five fields interact exactly as in Weinberg's[9] [SU(2) $\otimes$ U(1)] SBGT.

(e) $G_3\neq 0$. Here again all vector particles are decoupled, and the scalar is self-interacting. Thus for the above-mentioned particle spectrum, all tree-unitary theories are of SBGT type, a feature which we conjecture is true in general.

As another example, consider the most general theory of one scalar and three vectors *with any masses a priori*. In this case unitarity requires that the model be either (1) the Georgi-Glashow model[8] (one massless vector, two degenerate massive vectors, one scalar) or (2) the Weinberg model with no coupling to "hypercharge"[10] (three degenerate massive vectors, one scalar). Notice that the unitarity condition is compatible with *only* two vector mass spectra in models of this sort.

Of course, imposing the tree-unitarity condition is not sufficient to guarantee renormalizability. There is the problem of triangle-loop anomalies, which would violate unitarity in the real part of scattering amplitudes. We shall discuss this problem in our lengthier work.[5]

We close with two questions. (1) Is it possible to construct a Veneziano-like model with a sensible mass spectrum by imposing the tree-unitarity condition? Note the similarity of the ghost-free Veneziano model mass spectrum to that which would be *naively* inferred from Higgs models if their spontaneously broken character were not recognized, namely, scalar tachyons and zero-mass vectors. (2) General relativity is not the massless limit of a pure tensor theory[11]; the correct limit is only achieved when scalar excitations are included. Is the correct mixture of scalar and tensor fields determined by a boundedness condition at high energy?

*Work supported in part by the National Science Foundation.

†Alfred P. Sloan Foundation Fellow.

[1]For a review and a list of references, see B. W. Lee, in Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Illinois, September 1972 (to be published).

[2]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972); S. Weinberg, Phys. Rev. D 7, 1069 (1973).

[3]S. Weinberg, Phys. Rev. Lett. 27, 1688 (1971); H. R. Quinn (unpublished except as quoted in the Appendix of Ref. 1); A. I. Vainshtein and I. B. Khriplovich, Yad. Fiz. 13, 198 (1971) [Sov. J. Nucl. Phys. 13, 111 (1971)]; J. C. Taylor, to be published; D. A. Dicus and V. S. Mathur, to be published; J. Schechter and Y. Ueda, to be published; C. H. Llewllyn Smith, to be published, refers to some (apparently) unpublished work by J. D. Bjorken, A. I. Vainshtein and I. B. Khriplovich, and J. S. Bell.

[4]G. Ecker and J. Honerkamp [Phys. Lett. 42B, 253 (1972)] have used similar techniques in a different connection.

[5]J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, to be published. Preliminary results of the present paper were reported by J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Bull. Amer. Phys. Soc. 18, 599 (1973).

[6]Notation: Greek indices refer to space-time and are summed from 0 to 3; the metric is taken to be $g_{00}=1=-g_{11}=-g_{22}=-g_{33}$. Roman indices denote internal degrees of freedom and are summed (when repeated) over the (arbitrary) number of vector or scalar particles. Notice that the coupling constants in the Lagrangian have scalar particle superscripts and vector particle subscripts.

[7]P. W. Higgs, Phys. Rev. 145, 1156 (1966); T. W. B. Kibble, Phys. Rev. 155, 1554 (1967); G. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1964); R. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964).

[8]H. Georgi and S. L. Slashow, Phys. Rev. Lett. 28, 1494 (1972).

[9]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).

[10]H. Schnitzer, to be published).

[11]D. G. Boulware and S. Deser, Phys. Rev. D 6, 3368 (1972).