ers"). A particular breather solution with $\xi\xi^* = \frac{1}{4}$ is given by

$$u = 4\tan^{-1}\{[(1-\omega^2)/\omega^2]^{1/2}\cos[\omega(T-T_0)]\,\mathrm{sech}[(1-\omega^2)^{1/2}(X-X_0)]\}, \tag{28}$$

where $\omega = -2\,\mathrm{Re}\,\xi$. We note that the eigenvalues corresponding to the modes of a given breather in its own rest frame lie on the circle $\xi\xi^* = \frac{1}{4}$.

When $b(\xi)=0$ the solution is generated by the discrete spectrum only. Following Ref. 8, the solution can be shown to be given by

$$\frac{1}{4}\left(\frac{\partial u}{\partial x}\right)^2 = \frac{d^2}{dx^2}\ln[\det(I+AA^*)], \tag{29}$$

where

$$A_{lm} = \frac{(c_l c_m^*)^{1/2}}{\xi_l - \xi_m^*}\exp[i(\xi_l - \xi_m^*)X]. \tag{30}$$

The phase shifts of kink-kink (kink-antikink) interactions have been discussed already in the literature.[6] Lamb[9] has recently investigated a special class of solutions which correspond to paired complex eigenvalues.

A method for generating an infinite set of conservation laws has been given by Lamb[10] and applied in the context of nonlinear optics.[11]

In general the solution depends on both the discrete and continuous spectrum $[b(\xi)\neq 0]$. The asymptotic behavior of $u$ in these cases can be found by using methods similar to those used by Ablowitz and Newell.[12] That part of the solution corresponding to the continuous spectrum decays algebraically in time. In this regard it should be noted that, corresponding to initial conditions of compact support, the continuous spectrum completely cancels any contribution arising from the discrete spectrum outside the light cone at any $T\neq 0$.

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# Cosmic $\bar{p}$ Production in Interstellar $pp$ Collisions

T. K. Gaisser and R. H. Maurer
*Bartol Research Foundation of The Franklin Institute, Swarthmore, Pennsylvania 19081*
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We place a conservative upper limit of $\sim 4.6\times 10^{-4}$ on the fraction of antiprotons expected in the cosmic-ray flux at the top of Earth's atmosphere due to collisions of primary cosmic rays with interstellar hydrogen. The implication of this result for experiments designed to detect the existence of antimatter in the universe is discussed. We have parametrized existing data on the $\bar{p}$ inclusive cross section from threshold to intersecting-storage-ring energies to obtain the result.

Recent experiments designed to look for antimatter in the cosmic radiation have concentrated on antihelium[1,2] and heavier antinuclei[1] in order to be able to neglect significant contamination of the flux of antinuclei by antiprotons produced in the collision of ordinary cosmic rays with the interstellar gas. Previous calculations of the $\bar{p}$ flux to be expected from this source have been limited largely by the lack of information on the $\bar{p}$ production cross section, and values of $\bar{p}/p$ up to $\sim 10^{-3}$ have been obtained.[3] Recent results on $\bar{p}$ production at the CERN intersecting storage rings (ISR)[4-7] have enabled us to place a conservative upper limit of $\sim 4.6\times10^{-4}$ on this ratio at all energies and to estimate that in the 2–10-GeV range (crucial for cosmic-ray experiments) it is $\sim(0.4\text{–}10)\times10^{-5}$. This has the consequence that experimental observation of a $\bar{p}/p$ ratio of $\sim 10^{-3}$ would be very strong evidence for injection of antiprotons from antimatter sources or for their production in some other astrophysically interesting way. We note that the current upper limit on the $\bar{p}/p$ ratio is[8] $\sim 6\times10^{-3}$ in the several-GeV range.

The flux of antiprotons produced by the passage of primary cosmic rays through several g/cm$^2$ of interstellar hydrogen depends primarily on the cross section for $p+p\to\bar{p}+$anything. We have developed a parametrization of this process based on data at 12.5 GeV/$c$[9] and at 19.2 and 24 GeV/$c$,[10] as well as on the ISR data. We have used the Feynman scaling hypothesis[11] and a Mueller-Regge[12] analysis of approach to the scaling limit to interpolate between data at conventional accelerator energies and at ISR energies (equivalent $p_{\text{lab}}\sim 500\text{–}1500$ GeV/$c$) and to extrapolate beyond ISR energies. Because of the steepness of the primary–cosmic-ray spectrum, the $\bar{p}/p$ ratio below $\sim 150$ GeV does not depend significantly on the extrapolation beyond ISR experiments.

For the purposes of this calculation we have found it convenient to parametrize the $\bar{p}$ inclusive cross section integrated over transverse momentum, $F(x,E_0)=\int E(d\sigma_{\bar{p}}/d^3p)\,dp_\perp^{\,2}$, where $E_0$ is the incident proton total lab energy and $x$ is the Feynman variable, here defined by $x\equiv 2p_\parallel^{\text{c.m.}}/\sqrt{s}$.[13] We find that

$$F(x,E_0)=(0.21\text{ mb})[\exp(-18x^2)(1-3.4/E_0^{1/2})+9.3\exp(-36x^2)(1-2.8/E_0^{1/4})\theta(E_0-62)] \qquad (1)$$

gives an adequate representation of the data for $E_0\geq 12.5$ GeV, as shown in Fig. 1. The experimental points in Fig. 1 are obtained from experimental $\bar{p}$ inclusive cross sections by fitting the data at each $x$ value with a Gaussian in transverse momentum, then integrating over $p_\perp^{\,2}$. When the data are presented as $\bar{p}/\pi^-$ ratios[5a,7] we use the ISR data on $p+p\to\pi^-+$anything[14] to extract the $\bar{p}$ cross sections.

In Eq. (1) the form of energy dependence is motivated by the Mueller-Regge approach to asymptotic behavior[12] as discussed, e.g., by Ferbel[15] and Chan.[16] The first term represents the fragmentation region, the second the central region.[17] We show in Fig. 2(a) the $x=0$ data of Banner *et al.*[4] and of Allaby *et al.*[10] plotted versus $1/E_0^{1/4}$.

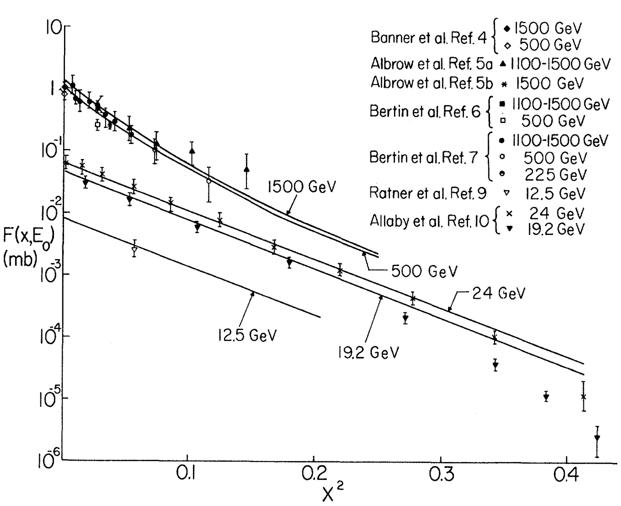


FIG. 1. $F(x,E_0)$ versus $x^2=(2p_\parallel^{\text{c.m.}}/\sqrt{s})^2$. Data points, from Refs. 4–7, 9, 10. Solid lines, parametrization of Eq. (1) for $E_0=12.5$, 19.2, 24, 500, and 1500 GeV/$c$.

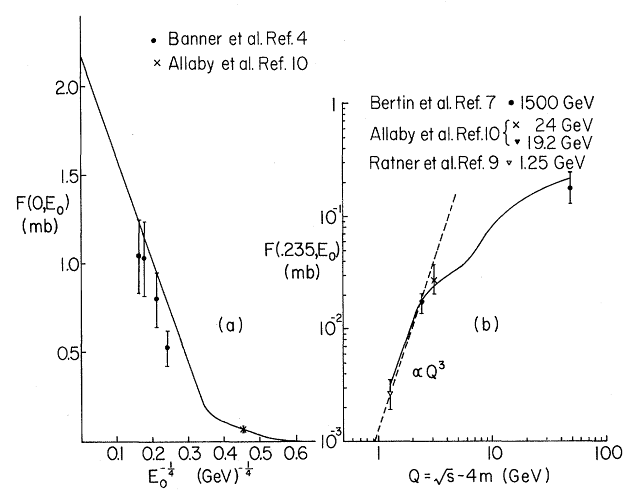


FIG. 2. (a) $F(0,E_0)$ versus $E_0^{-1/4}$ illustrating the Mueller-Regge approach to asymptotic behavior. Data points, from Refs. 4 and 10. Solid line, parametrization of Eq. (1). (b) $F(x=0.235,E_0)$ versus $Q=\sqrt{s}-4m$, primarily illustrating the threshold behavior for the reaction $pp\to ppp\bar{p}$. Data points are from Refs. 7, 9, 10. Solid line, result of Eq. (1); dashed line, proportional to $Q^3$ and normalized to the data at 12.5 GeV/$c$.

The solid line is the result of Eq. (1). In Fig. 2(b) we have plotted data for $F(x=0.235, E_0)$ versus $Q \equiv \sqrt{s} - 4m$. The solid line is the result of the parametrization; the dashed line is proportional to $Q^3$ and normalized to the data at 12.5 GeV/$c$. It is apparent from this plot that much of the energy variation in the 12.5–24-GeV/$c$ range is threshold behavior for the reaction $pp \to ppp\bar{p}$ rather than Mueller-Regge variation. Accordingly, for $F(x, E_0)$ between threshold ($E_0 = 6.55$ GeV) and 12.5 GeV, where no data exist, we have taken $F(x, E_0) \propto Q^3$, normalized to its value at 12.5 GeV.

The parametrization (1) for $F(x)$, discussed above, enables us to calculate the $\bar{p}/p$ ratio in a straightforward way. Since the mean path length traversed by primary protons between injection and observation at Earth[18] ($\sim 3-5$ g/cm$^2$) is much less than the interaction length for protons in hydrogen ($\sim 56$ g/cm$^2$), we can assume that each proton interacts at most with one hydrogen nucleus, and we can neglect further interactions of produced $\bar{p}$.[19] With the further assumptions that the primary cosmic-ray flux in interstellar space is primarily protons with the same energy spectrum observed in the vicinity of Earth (but extrapolated beyond the influence of the solar wind), and that $\bar{n}$ production equals $\bar{p}$ production, we have for the differential spectrum of antiprotons

$$\frac{dN_{\bar{p}}}{dE} \approx 2\frac{\langle y\rangle}{m_p}\int_E^\infty \frac{d\sigma_{\bar{p}}}{dE}(E, E')\frac{dN_0}{dE'}\,dE', \tag{2}$$

where $(d\sigma_{\bar{p}}/dE)(E, E')$ is the cross section (in cm$^2$) for producing a $\bar{p}$ of energy $E$ in the collision of a proton of energy $E'$ with an interstellar hydrogen nucleus, $dN_0/dE'$ is the differential primary proton flux, $\langle y\rangle$ is the mean path length of interstellar hydrogen traversed (in g/cm$^2$), and $m_p$ is the proton mass (in grams). We now assume a power law primary spectrum ($dN_0/dE = KE^{-(\gamma+1)}$ with $\gamma \sim 1.6$). Then from Eq. (2) the replacement $E/E' \to R$ leads to

$$\frac{\bar{p}}{p} \approx 2\frac{\langle y\rangle}{m_p}\int_0^1 R^{\gamma-1}E\frac{d\sigma_{\bar{p}}}{dE}\left(E, \frac{E}{R}\right)dR. \tag{3}$$

Because of the experimentally observed sharp transverse momentum falloff of $d\sigma_{\bar{p}}/d^3p$, it is possible to relate the cross section in terms of lab energies that appear in Eqs. (2) and (3) to $F(x)$ of Eq. (1) using $\langle p_\perp^2\rangle$ everywhere that $p_\perp^2$ appears in the relevant transformation equations.[20] In this way we find

$$E\frac{d\sigma_{\bar{p}}}{dE}(E, E_0) \approx \frac{E}{p_\parallel}F(x, E_0)\big|_{x=g(E, E_0, \langle p_\perp^2\rangle)}, \quad g \equiv [p_\parallel - E(1-4m^2/s)^{1/2}]/m, \quad p_\parallel = (E^2 - m^2 - \langle p_\perp^2\rangle)^{1/2}. \tag{4}$$

The results of the calculation of $\bar{p}/p$ using Eqs. (1), (3), and (4) are plotted in Fig. 3, assuming $\langle y\rangle = 5$ g/cm$^2$. It is important to notice that the steepness of the primary spectrum in Eq. (2) tends to emphasize low values of $1/R = E_0/E$. For large $E$ (in practice for $E \gtrsim 20$ GeV) this means that the major contribution to the integral comes from the forward fragmentation region in which $x \approx R$. This implies that

$$\lim_{E\to\infty} \bar{p}/p \to 0.0013\int_0^1 R^{\gamma-1}[\exp(-18R^2) + 9.3\exp(-36R^2)]\,dR \sim 4.6\times 10^{-4}. \tag{5}$$

Note that the existence of this energy-independent limit[21] depends only on the assumption of Feynman scaling *in the forward fragmentation region,* i.e., on the hypothesis of limiting fragmentation.[22] We have also calculated the median primary proton energy $E_M$ for $\bar{p}$ of energy $E$. We find $8 \lesssim E_M/E \lesssim 10$ for all $E > 2$ GeV. Thus our results for $\bar{p}/p$ below $\sim 150$ GeV do not depend significantly on extrapolation beyond ISR energies.

Finally, we wish to emphasize that, because $\sigma_{\bar{p}}$ rises to its asymptotic limit, it is unlikely that one can raise the expected $\bar{p}/p$ ratio above the asymptotic limit of Eq. (5) by modifying the astrophysical model of cosmic-ray injection and acceleration. For example, if the primaries pass through the bulk of the 5 g/cm$^2$ before they

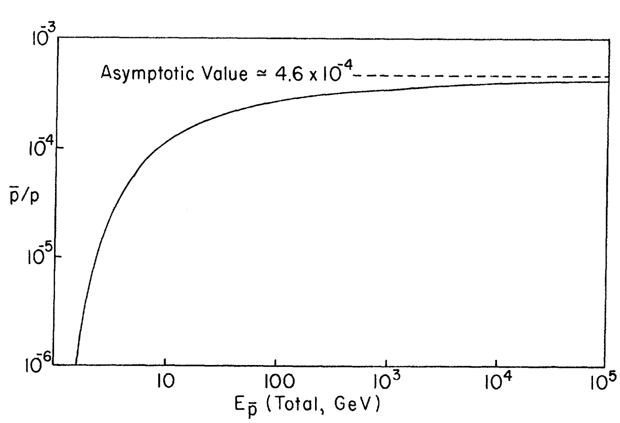


FIG. 3. $\bar{p}/p$ versus total antiproton energy (GeV) representing the results of this calculation from Eqs. (1), (3), and (4).

are fully accelerated, fewer $\bar{p}$'s will be produced.[23] We also note that if $F(x,E_0)$ rises faster between 24 GeV and ISR energies, then our parametrization suggests this will only lead to a more rapid approach of $\bar{p}/p$ to its asymptotic limit. The $\bar{p}/p$ ratio is also insensitive to wide variations in $\gamma$, the primary spectral index.[24] Thus the result of Eq. (5) gives a firm upper limit to $\bar{p}/p$ at all energies.

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[17]The Gaussian $\chi^2$ dependence in the second term is not that indicated by a Mueller-Regge analysis of the central region. Rather, it is a convenient way, consistent with the data, of joining the central and fragmentation regions. The $\theta$ function in energy indicates that below $E_0 = 62$ GeV the whole $x$ range is fragmentation. It would, of course, be possible to fit the $x=0$ data in Fig. 2(a) down to and including the 24-GeV point by including lower lying trajectories in the Mueller-Regge analysis. But this is not warranted in view of the significant threshold behavior present there.

[18]M. M. Shapiro, in *Proceedings of the Twelfth International Conference on Cosmic Rays, Hobart, 1971,* edited by A. G. Fenton and K. B. Fenton (University of Tasmania, Hobart, Australia, 1972).

[19]The $\bar{p}p$ total inelastic cross section is larger than that for $pp$ at low energies, largely because of annihilation, and corresponds to a $\bar{p}$ interaction length of $\sim 24$ g/cm$^2$ at total $E_{\bar{p}} \sim 1.5$ GeV. However, this interaction length increases at higher energies and we therefore neglect subsequent $\bar{p}$ interactions, which would only reduce our stated upper limit slightly for $E_{\bar{p}} < 10$ GeV if included.

[20]This approximation is justified at all energies for $\bar{p}$ production because the transverse momentum always appears in the combination $\mu_\perp^2 = p_\perp^2 + m_p^2$, and $\langle p_\perp^2 \rangle < m_p^2$.

[21]Calculations using Fermi or Cocconi-Koester-Perkins models give varying results for the $\bar{p}/p$ ratio. C. S. Shen and G. B. Berkey [Phys. Rev. 171, 1344 (1968)] find that $\bar{p}/p$ is constant at approximately $10^{-4}$ for $E_{\bar{p}} \geq 5$ GeV, while M. C. Chen [Nuovo Cimento 8B, 343 (1972)] finds $\bar{p}/p \sim E_{\bar{p}}^{-1/8}$ so that this ratio decreases from $\sim 4\times10^{-3}$ at $10^3$ GeV to $7\times10^{-4}$ at $10^9$ GeV. See also Ref. 3.

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[24]Variation of $\gamma$ between 1.5 and 1.7 leads to values of $\bar{p}/p$ varying by $\lesssim 20\%$ from the value at $\gamma = 1.6$. A $\gamma$ of 1.3 (1.9) is required to increase (decrease) the ratio by a factor of 2. See, e.g., W. R. Webber, in *Handbuch der Physik,* edited by S. Flügge (Springer, Berlin, 1967), Vol. 46, Pt. 2, p. 181, for a discussion of the primary spectrum at Earth.