

from the higher of the two pulses are shown in Fig. 2. In Fig. 2(b) the diode voltage has been corrected for induced voltage.¹⁰ The peak voltage was 0.69 MV, which coincided with the peak indicated Faraday-cup current. The diode space was shorted by debris or plasma at $t = t_0 + 30$ nsec.

Figure 2(c) shows an overlay of the diode and Faraday-cup currents. The Faraday-cup trace indicates that only a very small fraction of the diode current entered the aperture before $t = t_0 + 7$ nsec, when the beam focused sharply and more than 90% of the entire beam passed through the aperture. At $t = t_0 + 14$ nsec the Faraday-cup surface apparently became conductive and probably was reading a current less than the total incident beam current.¹¹ The peak current positively diagnosed was 2.71 MA/cm², and the peak power flux was 1.87 TW/cm². The rate of rise of power on the target was 9×10^{20} W/cm² sec. Figure 3 shows the aperture after the shot and indicates that the area where the tantalum was vaporized away had a radius of approximately 1 mm. The tantalum was lightly damaged to a radius of 3.5 mm. This probably occurred in the first 7 nsec of the beam pulse, before the beam self-pinched into the aperture. The current after 7 nsec was always considerably in excess of the minimum current¹² for a diode self-pinch, and

one would not expect subsequent defocusing.

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¹¹An alternative explanation is that the 1-mm-diam aperture was being partially closed by the edges of the aperture blowing inward. Image-converter camera photographs have shown luminescent material expanding from targets struck by this beam at velocities of 1 to 4×10^6 cm/sec. This is large enough to provide significant area reduction in 5 to 10 nsec.

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Linear Contribution to Spatial Dispersion in the Spin-Wave Spectrum of Ferromagnets

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The implications of magnetic symmetry on the spin-wave spectrum of ferromagnets are examined. Contrary to the usual result, the spin-wave dispersion relation is found to contain a term linear in the wave vector for a definite set of magnetic symmetries. This linear dispersion is shown to be a consequence of antisymmetric exchange.

One of the most widely accepted results of spin-wave theory in ferromagnets¹⁻³ is the quadratic dispersion law

$$\omega = a + ck^2. \quad (1)$$

Here, ω is the angular frequency of a spin wave and k is its wave number. Although the constants a and c depend upon the exact form of the magnetic anisotropy, the exchange energy, external magnetic fields, sample shape, and the direction of propagation of the spin wave, the quadratic dependence on k remains unaltered. The purpose of this paper is to show that the above quadratic

dispersion law is a consequence of assumptions regarding the magnetic symmetry of the medium and/or the symmetry of the exchange coupling. The apparent generality of Eq. (1) can lead to the incorrect conclusion that this quadratic dispersion law is fundamental to the nature of magnetic excitations in ferromagnetic media. The approach used here is largely phenomenological and is based upon the classical continuum model of a ferromagnet as discussed in Ref. 3. However, a microscopic example of a relevant system is also discussed.

The energy density of a ferromagnet can be

written in the following form:

$$w = \Sigma (M_i, \partial M_i / \partial x_j) + (8\pi)^{-1} (H^2 + \vec{E} \cdot \vec{D}) - \vec{H}^e \cdot \vec{M}. \quad (2)$$

Here, Σ is the stored or potential energy density which in the long-wavelength limit is taken to be a function of the components of the magnetic moment per unit volume M_i and their first spatial derivatives $\partial M_i / \partial x_j$. The Maxwellian magnetic field, electric field, and electric displacement are denoted respectively by \vec{H} , \vec{E} , and \vec{D} and the external field is denoted by \vec{H}^e . The total energy W is just the volume integral $\int_V w d^3r$.

The stored energy density Σ is generally assumed to be expressible as a polynomial in M_i and $\partial M_i / \partial x_j$, with the constraint that $M_i M_i = M_0^2$ is a constant of the motion. (Summation over repeated indices is assumed throughout this paper.) Defining $\alpha_i \equiv M_i / M_0$ we have to second order in α_i and $\partial \alpha_i / \partial x_j$ the following expression for Σ :

$$\Sigma = \frac{1}{2} K_{ij} \alpha_i \alpha_j + A_{ijk} \alpha_i \partial \alpha_j / \partial x_k + \frac{1}{2} J_{ijkl} (\partial \alpha_i / \partial x_j) (\partial \alpha_k / \partial x_l). \quad (3)$$

K_{ij} is the second-order magnetic anisotropy tensor, J_{ijkl} is the usual symmetrical exchange-constant tensor,³ and A_{ijk} (as is shown later) is the antisymmetrical exchange tensor. The three material tensors appearing on the right-hand side of Eq. (3) must be invariant to the symmetry operations of the material. The torque equation

$$\partial \vec{M} / \partial t = \gamma (\vec{M} \times \vec{H}^{\text{eff}}) \quad (4)$$

describes the time variation of the magnetization. The effective field is³

$$\vec{H}^{\text{eff}} \equiv -\delta W / \delta \vec{M} = -(1/M_0) \delta W / \delta \vec{\alpha}, \quad (5)$$

where $\delta / \delta \vec{M}$ denotes the functional derivative with respect to \vec{M} . It is easily shown that as a consequence of Eqs. (4) and (5), one can, without loss of generality, restrict the tensor A_{ijk} to be antisymmetric in the first two indices:

$$A_{ijk} = -A_{jik}. \quad (6)$$

Since the existence of linear spatial dispersion in the spin-wave spectrum is shown below to be a consequence of antisymmetric exchange, it is useful to examine the restrictions imposed by material symmetry on the existence of polar tensors of the form $A_{ijk} = -A_{jik}$. Eighteen of the 21 noncentrosymmetric crystallographic point groups (excluding $\bar{6}$, $\bar{6}m2$, and $\bar{4}3m$ for which $A_{ijk} = A_{jik}$) allow the existence of nonzero components of $A_{ijk} = -A_{jik}$. These eighteen are just those crys-

tallographic symmetries for which optical activity is allowed. Combining the geometric operations of these eighteen crystallographic groups with that of time reversal leads to nineteen magnetic point groups which are consistent with a net ferromagnetic moment and with a nonzero antisymmetric exchange tensor $A_{ijk} = -A_{jik}$.

The remainder of this paper is confined to a discussion of uniaxial ferromagnetic materials for which antisymmetric exchange can affect the propagation of spin waves along the unique axis. This restriction further limits the discussion to materials with one of the following six magnetic symmetry groups:

$$3, \quad 32, \quad 4, \quad 422, \quad 6, \quad 622. \quad (7)$$

The most general stored energy density (to second order in the direction cosines α_i and their gradients) for this special case is given by

$$\Sigma = -\frac{1}{2} K \alpha_z^2 + A_{123} \left(\alpha_x \frac{\partial \alpha_y}{\partial z} - \alpha_y \frac{\partial \alpha_x}{\partial z} \right) + \frac{1}{2} J_{33} \left(\frac{\partial \alpha_x}{\partial z} \right)^2 + \frac{1}{2} J_{44} \left[\left(\frac{\partial \alpha_x}{\partial z} \right)^2 + \left(\frac{\partial \alpha_y}{\partial z} \right)^2 \right]. \quad (8)$$

The frequency- and wave-vector-dependent susceptibility³ $\chi_{ij}(\omega, k)$ is defined via

$$m_i(\omega, k) = \chi_{ij}(\omega, k) h_j(\omega, k), \quad (9)$$

where $\vec{m}(\omega, k)$ and $\vec{h}(\omega, k)$ are the space-time Fourier transforms of the deviations of \vec{M} and \vec{H} from their equilibrium values \vec{M}_0 and \vec{H}_0 . From Eqs. (4), (5), and (9), one finds all $\chi_{ij} = 0$ except

$$\chi_{xx} = \chi_{yy} = \frac{-\gamma M_0 \omega_s}{[\omega - (2\gamma/M_0) A_{123} k]^2 - \omega_s^2}, \quad (10a)$$

$$\chi_{xy} = -\chi_{yx} = \frac{i\gamma M_0 [\omega - (2\gamma/M_0) A_{123} k]}{[\omega - (2\gamma/M_0) A_{123} k]^2 - \omega_s^2}, \quad (10b)$$

where

$$\omega_s = \gamma [H_0 + H^e + K/M_0 + J_{44} k^2 / M_0]. \quad (11)$$

We have assumed the external field \vec{H}^e to be applied along the z axis and that the sample is uniformly magnetized in that direction. The generalized principle of symmetry of the kinetic coefficients for gyrotropic media⁵ places the following general condition on the susceptibility:

$$\chi_{ij}(\omega, \vec{k}, \vec{H}^e) = \chi_{ji}(\omega, -\vec{k}, -\vec{H}^e). \quad (12)$$

The susceptibility calculated in Eqs. (10) can readily be shown to have this property. Note that assumption of a stable equilibrium state of the material assures that $M_0 \rightarrow -M_0$ when $\vec{H}^e \rightarrow -\vec{H}^e$.

In the magnetostatic limit ($\nabla \times \vec{h} = 0$), $h_x = h_y = 0$,

and solutions to Eq. (7) are possible only for the poles of χ_{ij} . The following dispersion law for ferromagnets is thereby obtained:

$$\omega = \frac{2\gamma}{M_0} A_{123} k \mp \omega_s = \gamma \left[\mp (H_0 + H^e + \frac{k}{M_0}) + \frac{2A_{123}}{M_0} k \mp \frac{J_{44}}{M_0} k^2 \right]. \quad (13)$$

This has the general form:

$$\omega_{\pm} = \mp a + bk \mp ck^2. \quad (14)$$

The eigenvectors associated with ω_{\pm} are $m_{\pm} = m_x \pm im_y$, respectively. These correspond to right- and left-handed precession of the magnetization about the direction of the external magnetic field (i.e., the equilibrium direction of the magnetization). Note that this does not necessarily correspond to right- or left-handed precession about the propagation direction. In Fig. 1, ω_+ and ω_- are plotted versus k for a given set of values for a , b , and c (assuming all three are positive). Only positive frequencies are physically meaningful. By suitably choosing the values of a , b , and c , the dispersion curve (Fig. 1) can be made to intersect the abscissa ($\omega = 0$). The ground state is then no longer the assumed simple ferromagnetic order but has a helical spin arrangement. The present analysis is not adequate to describe the excitations in the helical state. Since the parameter a is dependent upon the external field [Eq. (13)], phase transitions between these two types of magnetic order can be induced by changing the magnitude of H^e . The soft mode for the transition from the ferromagnetic to the helical configuration is $\omega_{\pm}(k = \pm b/2c)$.

As pointed out originally by Stevens,⁶ the origin of antisymmetric exchange lies in the spin-orbit coupling. When the orbital and spin degrees of freedom are coupled, the ground state is not rigorously a product of spin and orbital states but rather a mixed state. Taking this properly into account⁷ leads, in addition to the usual symmetric exchange interaction

$$-\sum_{l>n} J(R_{lm}) \vec{S}(l) \cdot \vec{S}(m),$$

$$\mathcal{H} = -\frac{1}{2} D \sum_l S_z^2(l) + A_3 \sum_l [S_x(l)S_y(l+1) - S_y(l)S_x(l+1)] + J_1 \sum_l [S_x(l)S_x(l+1) + S_y(l)S_y(l+1)]$$

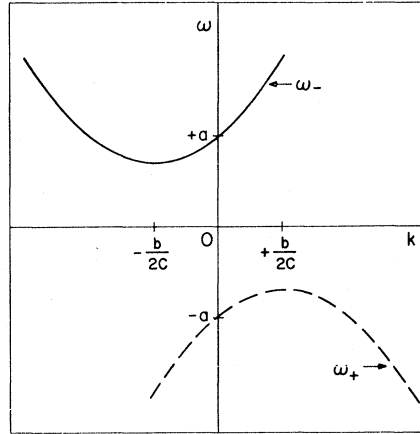


FIG. 1. The dispersion relation $\omega_{\pm} = \mp a + bk \mp ck^2$ is plotted for positive a , b , and c . Positive frequencies are plotted as a solid curve and negative frequencies as a dashed curve. The assumption is made that $a + ck^2 > -bk$ for all k .

to an antisymmetric contribution of the form

$$\mathcal{H}_{AS} = \sum_{l>m} \vec{A}(R_{lm}) \cdot \vec{S}(l) \times \vec{S}(m). \quad (15)$$

$\vec{A}(R_{lm})$ is the antisymmetric exchange constant and is dependent only upon the distance R_{lm} between the spins $\vec{S}(l)$ and $\vec{S}(m)$ on the l th and m th sites, respectively. In order of magnitude $|\vec{A}|$ and the symmetric exchange constant J are related by

$$|\vec{A}| \simeq J(\lambda/\Delta). \quad (16)$$

Here λ is the spin-orbit coupling constant and Δ is the appropriate crystal-field splitting parameter. The third-rank antisymmetric tensor, A_{ijk} , is just an integral measure of the axial vector $\vec{A}(R_{lm})$.

The microscopic origin of linear dispersion in the antisymmetric exchange coupling and the associated symmetry requirements is readily seen in the following example. By analogy to Eq. (8) the microscopic Hamiltonian of a linear chain of magnetic ions whose site symmetry is included in Eq. (7) can be written

$$+ J_3 \sum_l S_z(l)S_z(l+1) - g\beta H^e \sum_l S_z(l). \quad (17)$$

Examples of such a system include materials with one magnetic ion per unit cell with nonmagnetic ions determining the site symmetry and systems for which each unit cell contains n magnetic ions which are equivalent under an n -fold screw ($n = 3, 4, 6$) along the \hat{z} axis. The dispersion relation is readily

found to be

$$\omega = 2A_3 \langle S_z \rangle \sin kc \pm [g\beta H^e + (D - 2J_3 + 2J_1 \cos kc) \langle S_z \rangle], \quad (18)$$

where c is the lattice constant. For $kc \ll 1$ this result is equivalent to Eq. (13). The absence of constant terms in the equations of motion determined from Eq. (17) and the commutation of $\sum_i S_z(i)$ with \mathcal{H} shows that for $\omega > 0$ (all k) the ground state consists of all spins aligned ferromagnetically along the \hat{z} axis.

Antisymmetric interactions between spins along the chain (\hat{z} axis) do not affect the ferromagnetic ground state (except as noted above). However, antisymmetric interactions between ions in the xy plane can cause the ground state to have a more complicated sublattice structure with a net z component of magnetization per unit cell. The excitations of such systems must be described in terms of a multisublattice model and have not been treated here.

Antisymmetric exchange has been shown by Dzialoshinski⁸ to explain the appearance of weak ferromagnetism in otherwise compensated antiferromagnets (see also Ref. 7). The intersublattice antisymmetric exchange in such systems does not in general lead to linear dispersion.⁹ The effect of antisymmetric exchange on the spin-wave spectrum of simple ferromagnets has apparently not been previously discussed. The linear dispersion often derived for spin waves in antiferromagnets is a consequence of the assumption of magnetic isotropy¹ and is in no way related to the linear dispersion discussed in this paper. This holds also for the so-called "nonactivated spin-flop mode" as has been discussed previously.¹⁰ It is no coincidence that the material

symmetry requirements for the existence of optical^{11,12} and acoustical¹³ activity are closely related to those for the effects discussed in this paper.

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