at much higher energies indicate that spallation cross sections from medium-weight nuclei (at least up to $A \approx 60$) do indeed level off above 10 GeV. Thus we infer from this that the spectrum of excitation energies deposited in the nucleus appears to be independent of bombarding energy above 10 GeV for this range of target nuclei. Although pion production increases with energy, most of the pions must escape from the nucleus without deposition of much energy.

It should be recognized that an overall distribution of product yields by itself is not a sensitive indicator of mechanism. More definitive studies, such as recoil-range and angular-distribution measurements of the products from light-, medium-, and heavy-element targets are needed to establish any differences in nuclear reaction mechanisms.

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Origin of j and Q Dependence in Heavy-Ion Transfer Reactions

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A semiclassical treatment of heavy-ion–induced reactions is used to analyze experimental results on the (^{16}O, ^{15}N) and (^{14}N, ^{13}C) reactions at 80 MeV and the (^{12}C, ^{11}B) reaction at 95 MeV, on a ^{54}Fe target. It is shown that kinematical conditions and angular momentum coupling are sufficient to explain the strong j effect and Q dependence observed in the selective population of the various states.

Studies of heavy-ion–induced single-nucleon transfer reactions reveal a strong j dependence in the relative cross sections. This j effect depends on the nature of the projectile and more specifically on the orbital from which the nucleon is transferred. So far, this effect has been accounted for with the aid of the no-recoil distorted-wave Born-approximation (DWBA) selection rules.[1-4] Recently, it has been shown[5,6] that the complete treatment including the recoil terms is necessary in some cases to reproduce the experimental data, and therefore that the no-recoil selection rules cannot be used *a priori*. In this paper, we show that the observed j and Q dependence can be deduced from a simple discussion of the physics of the problem.

We recall that a semiclassical treatment[7] can be used to describe heavy-ion reactions provided the condition $\eta = Z_1 Z_2 e^2/\hbar v \gg 1$ is satisfied. It leads[8,9] to two momentum-matching conditions which must be simultaneously satisfied if the transfer probability is to be large. The first condition comes from conservation of the transferred-nucleon velocity. The second condition comes from total angular momentum conservation. For one-nucleon transfer reactions, taking the transferred-nucleon spin into account, these conditions are[9]

$$l_c - \lambda_i R_2/R_1 = \lambda_f \quad \text{(Condition II)}, \qquad (1)$$

$$L_i + \lambda_i + \sigma_i = L_f + \lambda_f + \sigma_f \quad \text{(Condition I)}, \qquad (2)$$

where $l_c = k_i R_2/\mu_i$ is the average angular momentum due to center-of-mass motion (k_i is the rela-

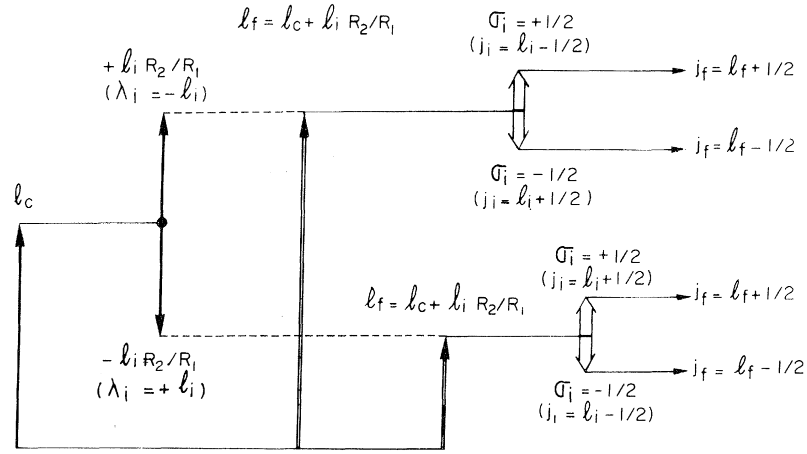

FIG. 1. Angular momentum coupling scheme restricted to the two extreme values $\pm l_i$ of λ_i, which give rise to a maximum j effect. Parameters l_i and l_f (projections λ_i, λ_f) are the orbital angular momenta of the transferred nucleon in the projectile and final nucleus, respectively; σ_i is the spin projection of the transferred nucleon. See text for other notations.

tive wave number at the grazing distance and μ_i is the reduced mass, in the entrance channel); λ_i, σ_i and λ_f, σ_f are the orbital momentum and the spin projection of the transferred nucleon in the initial and final states, respectively; L_i and L_f are the relative orbital angular momenta in the entrance and exit channels, respectively; and R_1 (R_2) is the radius of the heavy ion from which (to which) the transfer occurs.

So far, there has not been any evidence for spin-flip processes in heavy-ion transfer reactions and they are usually neglected in DWBA calculations. We use here the same assumption. Hence the spin projection is unaffected by the transfer, so that $\sigma_i = \sigma_f$.

To clarify the physical origin of the observed effects, two further assumptions are made: (i) The component of angular momentum out of the quantization axis, which arises when $|\lambda_i|$ is not equal to its maximum value, is neglected so that $\lambda_f = l_f$ in all cases {instead of $l_f = [l_c^2 + l_i^2(R_2/R_1)^2]^{1/2}$ for $\lambda_i = 0$; see Ref. 9}; (ii) the use of the Clebsh-Gordan coefficients for the coupling of the orbital and spin angular momenta is replaced by a simple classical coupling scheme. These assumptions are not essential but greatly simplify the calculations.

Three consequences can be deduced for each value of λ_i. First, the l value l_f of the shell the nucleon enters is selected through Eq. (1):

$$l_f = l_c - \lambda_i R_2/R_1.$$

Second, the j value results from the coupling scheme shown in Fig. 1. This leads to the following rules[10]: For $j_i = l_i - \frac{1}{2}$

$$\lambda_i = +l_i \to j_f = l_f - \tfrac{1}{2}, \qquad (3)$$

$$\lambda_i = 0 \to j_f = l_f \pm \tfrac{1}{2}, \qquad (4)$$

$$\lambda_i = -l_i \to j_f = l_f + \tfrac{1}{2}; \qquad (5)$$

for $j_i = l_i + \frac{1}{2}$

$$\lambda_i = +l_i \to j_f = l_f + \tfrac{1}{2}, \qquad (6)$$

$$\lambda_i = 0 \to j_f = l_f \pm \tfrac{1}{2}, \qquad (7)$$

$$\lambda_i = -l_i \to j_f = l_f - \tfrac{1}{2}. \qquad (8)$$

Third, the favored Q value (hence, the favored excitation energy E_x) is deduced from Eqs. (1) and (2) by solving the following equation:

$$l_c - \lambda_i R_2/R_1 = \Delta L(Q) + \lambda_i, \qquad (9)$$

where

$$\Delta L(Q) = L_i - L_f(Q).$$

For a given reaction, these rules introduce severe restrictions on the population of the different states.

We now test the above scheme by applying it to experiments performed with 80-MeV ^{16}O and ^{14}N and 95-MeV ^{12}C beams at Orsay.[9,11] The energy spectra recorded in the reactions $^{54}Fe(^{16}O, ^{15}N)^{55}Co$, $^{54}Fe(^{14}N, ^{13}C)^{55}Co$, and $^{54}Fe(^{12}C, ^{11}B)^{55}Co$ are shown in Fig. 2. As expected, only a few levels are strongly populated. The selectivity is found to be different from one reaction to the other. For these reactions, the fulfillment of the kinematical conditions I and II is investigated in Fig. 3. The value $r_0 = 1.5$ fm is used for the cal-

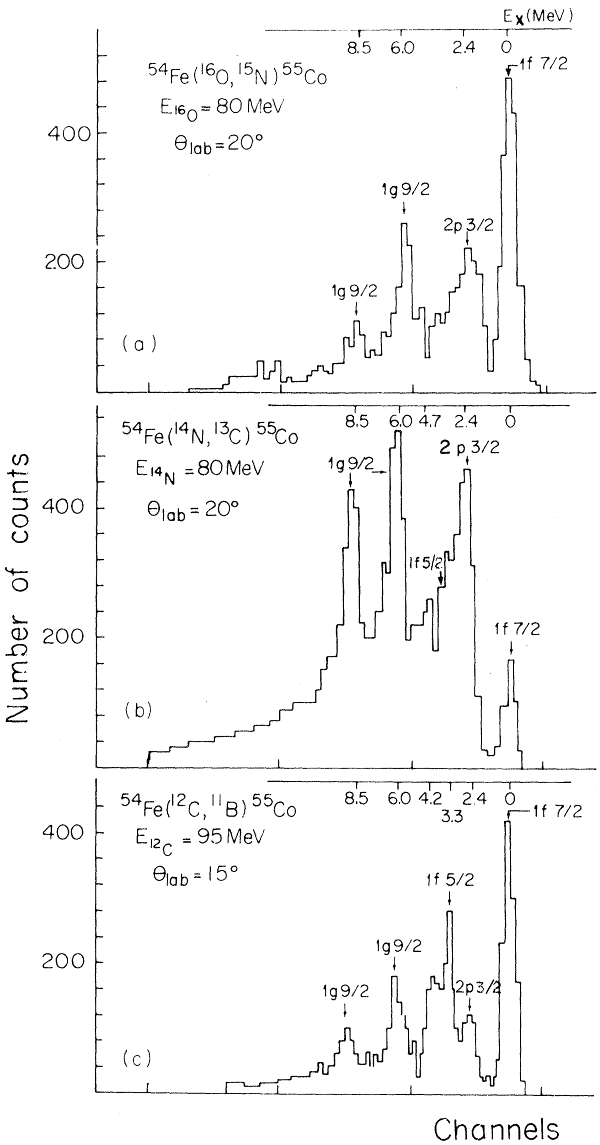

FIG. 2. Spectra of (a) (^{16}O, ^{15}N), (b) (^{14}N, ^{13}C), and (c) (^{12}C, ^{11}B) reactions on ^{54}Fe. The cross sections for the excitation of the ^{55}Co($\frac{7}{2}^-$) ground state are 3.0, 0.6, and 0.9 mb/sr, respectively. The proton single-particle states are labeled according to their experimental excitation energy and shell-model orbitals.

culation of R_1 and R_2.[12] For each reaction and for the three available values of λ_i, the l_f value obtained from condition I [Eq. (1)] and from condition II [Eq. (2)] is plotted against the excitation energy in the final nucleus, so that the graphical solution of Eq. (9) is found. Since the proton transferred in the (^{16}O, ^{15}N) and (^{14}N, ^{13}C) reactions originates from the $1p_{1/2}$ shell ($j_i = l_i - \frac{1}{2}$), relations (3), (4), and (5) must be used to select the j_f value. In the (^{12}C, ^{11}B) reaction, the proton originates from the $1p_{3/2}$ shell ($j_f = l_i + \frac{1}{2}$) and relations (5), (6), and (7) must be used.

For the (^{16}O, ^{15}N) reaction at 80 MeV, the l_f value obtained from condition I with $\lambda_i = -1$ is 3.5 [Fig. 3(a)]. It favors the stripping of the nucleon into an f or g orbital. Since the favored excitation energy is about halfway between the $1f_{7/2}$ (ground state) and the $1f_{5/2}$ (3.3 MeV) levels, both levels should be strongly populated. However, the j selection rule eliminates the $j_f = l_f - \frac{1}{2}$ levels [Eq. (5)]. The experimental spectrum shows that the $1f_{5/2}$ state is indeed very weakly populated, whereas the $1f_{7/2}$ ground state dominates the spectrum. The $2p$ and $1g_{9/2}$ states, which are not favored by the above kinematical conditions (whatever the value of λ_i), are only slightly populated.

For the (^{14}N, ^{13}C) reaction at 80 MeV, the l_c and R_2/R_1 values are slightly larger. Besides, the change in Q_0 values (− 2.5 MeV instead of − 7.1 MeV) shifts the $\Delta L(Q)$ curves towards higher excitation energies [Figs. 3(d), 3(e), 3(f)]. The l_f value calculated with $\lambda_i = -1$ is 3.8 and favors stripping into the g shell. The kinematically-favored excitation energy is 8.0 MeV, as can be seen on the graph of Fig. 3(f). This value lies between the $1g_{9/2}$ $T_< = \frac{1}{2}$ (6.01 MeV) and the $1g_{9/2}$ $T_> = \frac{3}{2}$ (8.5 MeV) states.[13] According to prescription (5), the j selection rule also leads to a $j_f = l_f + \frac{1}{2}$ value. Experimentally, these states are indeed strongly populated. Stripping into the $1f_{7/2}$ state does not fulfill both kinematical conditions; the cross section of this state is very small.

For the (^{12}C, ^{11}B) reaction at 95 MeV, the value of l_c is much larger than in the two previous cases. With $\lambda_i = -1$ this leads to a value $l_f = 4.6$ which is well matched to stripping into the g orbitals. Moreover, the favored excitation energy derived from Fig. 3(i) is close to the location of the $g_{9/2}$ $T_<$ state. The fact that this state is weakly populated [Fig. 2(c)] reflects the effectiveness of the j selection rule; according to prescription (8) a $j_f = l_f - \frac{1}{2}$ value is selected in this case. Although it may be allowed by the j selection rules, stripping into the $1f_{5/2}$ states does not simultaneously fulfill the two other kinematical conditions; such states are definitely less populated than the $1f_{7/2}$ ground state for which good matching conditions can be achieved [see Fig. 3(h) and prescription (7)]. As compared with the reactions discussed above, a striking decrease of the $2p$-state population is observed here (Fig. 2); it may be ascribed to the fact that at least one of the kinematical conditions is

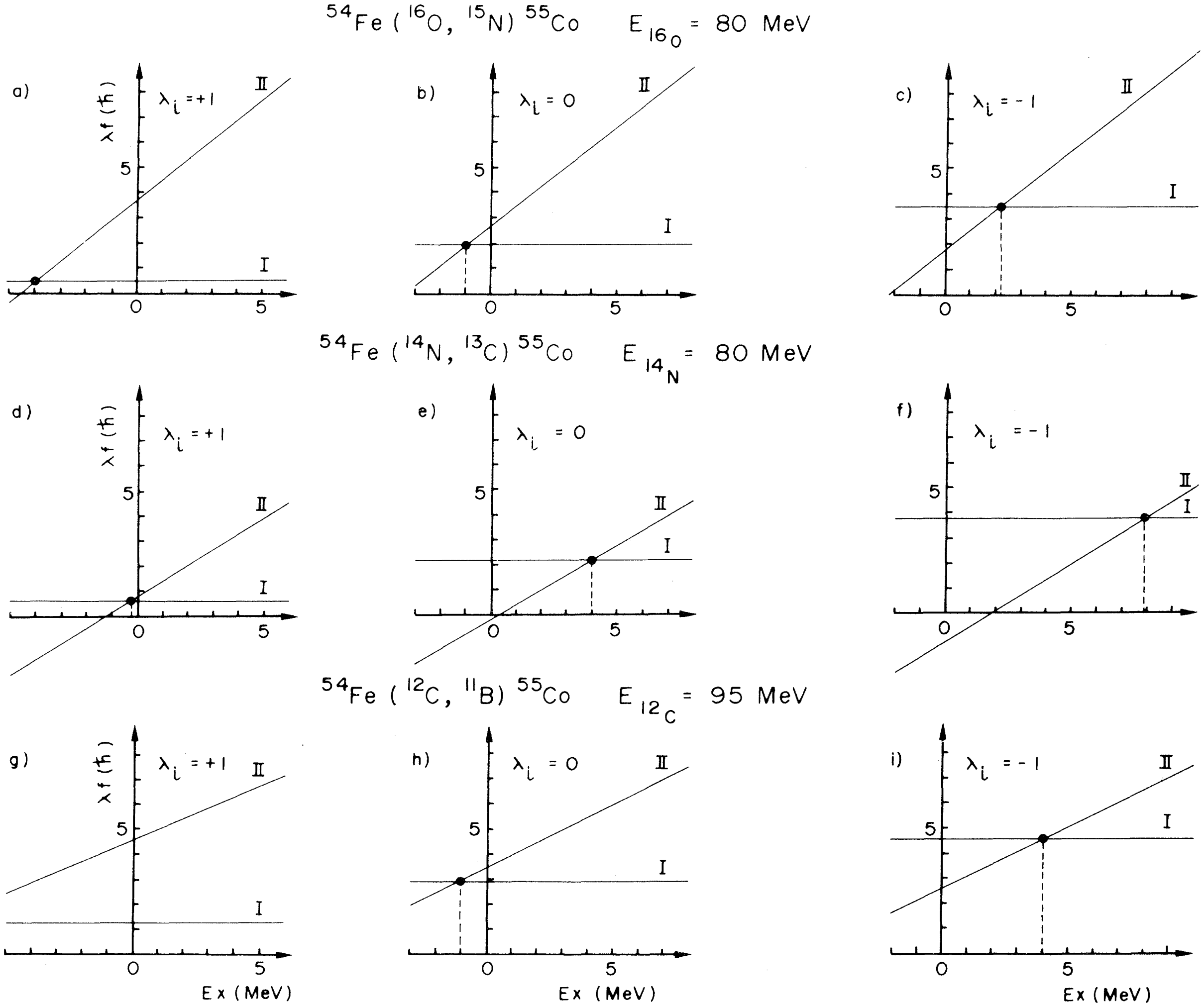

FIG. 3. Graphical investigation of the kinematical conditions I and II discussed in the text. The plots are split according to the different projections λ_i of the angular momentum l_i. The simultaneous fulfillment of conditions I and II (hence a large transfer cross section) is graphically determined by the intersection of the two curves which indicates both the favored shell and the favored excitation energy.

strongly violated for every value of λ_i.

Of course, this qualitative analysis cannot compete with the recoil-included finite-range DWBA treatment. But we believe that it accounts for the j effect and Q dependence found in heavy-ion reactions at energies above the Coulomb barrier. Moreover, it indicates which physical factors play a dominant role in the reaction mechanism. This scheme can be used to select different reactions and their incident energy when only l and j assignments are required.

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Constrained—Hartree-Bogolyubov Treatment of the Pairing-Plus-Quadrupole Model: Discontinuities in Nuclear Moments at High Spins*

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I discuss a self-consistent, microscopic theory of nuclear rotations at high spins and review results obtained previously for the spin dependence of the moment of inertia, deformation, and energy gaps of ^{160}Dy. Additional results given here suggest some striking discontinuities in isomer shifts, gyromagnetic ratios, and $B(E2)$ values at high spins. Theoretical results are consistent with the recent measurements of gyromagnetic ratios and $B(E2)$ values, but a greater precision is needed to test the predicted discontinuities.

The backbending phenomenon at high spins, observed[1] in the energy-level spectra of deformed, even-even nuclei with $A = 158-168$, has renewed the interest of nuclear physicists in the problem of rotational-type states. The sudden decrease in the spacing of I and $I-2$ levels in a rotational-type spectrum ($I = 0, 2, 4, \ldots$ levels whose level spacing increases with I) at $I \sim 16$ is somewhat[2] surprising from the point of view of the collective model. On the other hand, it is quite surprising that one can talk about collective, rotational-type states at excitation energies of 3–4 MeV. According to the conventional picture of heavy, even-even nuclei, the two-quasiparticle states should start at about 2 MeV and many other noncollective states should become prominent at 3–4 MeV. The lack of observation of the expected high density of states is an unsolved puzzle.

Many nuclear physicists are now engaged in attempting to fit the observed energy spectra (for recent reviews, see papers by Johnson and Szymański, and Sorensen[3]). It is already clear that the backbending type of effect can be reproduced at least qualitatively in a variety of ways. Hence, it is obvious that we need to study the behavior of various nuclear moments [for instance, $B(E2)$ values, gyromagnetic ratios, isomer shifts, and particle-transfer amplitudes] at high spins in order to discriminate between the different models.

The main purpose of the present note is to provide semiquantitative estimates of the behavior of nuclear moments at high spins. Although such measurements are at present extremely difficult, they are already being attempted. Comparison with the available data on $B(E2)$ values[4] up to $I = 18$ and with the gyromagnetic ratios[5] up to $I = 10$ will be given.

Several microscopic treatments of nuclear rotations at high spins of heavy nuclei have been developed: the Nilsson model plus pairing and cranking,[6] the Nilsson-Strutinsky model plus pairing and cranking,[7] the pairing-plus-quadrupole (PPQ) model combined with generalized Hartree-Fock,[8] the PPQ model with Hartree-Fock-Bogolyubov (HFB) and cranking,[9] the PPQ model with HFB and projection techniques,[10] the PPQ model with constrained Hartree-Bogolyubov (CHB),[11] and the particles-plus-rotor models.[12] These methods have been compared in the excellent review article of Sorensen.[3]

The three main steps of the present method[11] (PPQ model with CHB) are the following: (i) A transformation is made from a spherical, single-particle basis to a deformed, single-particle basis. The transformation coefficients are determined by diagonalizing the average part of the two-body quadrupole force, $-DQ = -X\langle Q\rangle Q$, where X is the strength of the quadrupole force (same for p-p, n-n, and n-p) and $Q = r^2 Y_2$ is the quadrupole operator. The usual Hartree-Fock