case the continuity equation (3) and the superfluid-acceleration equation (4) provide a complete description and in fact, are isomorphic to the classical Euler hydrodynamics at 0°K. Thus a small disturbance travels at velocity $(\rho\partial\mu/\partial\rho)^{1/2}$ which is the velocity of ordinary sound.

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Attenuation of Zero Sound in Liquid ^{3}He: A Probe of Superfluid Pairing*

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It is shown that the anomalous zero-sound attenuation at the transition "*A*" observed recently in liquid ^{3}He at about 2.7 mK can be understood by postulating pairing in a state of odd relative angular momentum. The attenuation is found to depend strongly on the angular variation and orientation of the gap parameter, providing information on the precise nature of the pairing. It is concluded that (1) Balian-Werthamer p-wave pairing can be ruled out, and (2) Anderson-Morel p-wave pairing is possible.

We have calculated a contribution to the zero-sound attenuation in a postulated superfluid state of ^{3}He, which is caused by direct absorption of phonons by the Cooper pairs, with a view to explaining the recently found anomalies in the zero-sound attenuation of ^{3}He below the transition "*A*."[1] Although both absolute value and temperature dependence of the calculated attenuation are of the correct order of magnitude, it seems that none of the equilibrium states discussed below leads to satisfactory agreement with experiment. The reason for this preliminary report is that our results suggest zero-sound–attenuation measurements as a tool for investigating the angle dependence of the gap parameter. On the basis of the existing data some general conclusions can be drawn, which aid further theoretical searches for the nature of the equilibrium state of superfluid ^{3}He.

It is known from the theory of normal Fermi liquids[2] that zero sound can be damped in two ways. The usual mechanism, which accounts for the attenuation in normal ^{3}He, is collision damping. In the superfluid state, close to the phase transition, the corrections to the normal-state results are small, of order $\langle|\Delta|^2\rangle/(\pi T)^2$, where $\langle|\Delta|^2\rangle$ is an appropriate angular average of the gap parameter at temperature T. Furthermore, since the expression for the (collision-induced) zero-sound attenuation involves multiple integrations it can be expected to vary smoothly with temperature on a scale given by T_c, whereas the observed anomalous attenuation is confined to a small region within some $10^{-2}T_c$ of the transition temperature. It therefore seems reasonable to identify the experimentally observed background attenuation with collision damping. Another mechanism for zero-sound attenuation is "Landau damping": the direct absorption of phonons by pairs of single-particle excitations. There is no Landau damping in the normal state for reasons of energy and momentum conservation, because the zero-sound velocity in ^{3}He is much greater than the Fermi velocity. In the superfluid state, the absorption of a phonon by a Cooper pair, which subsequently breaks up into two quasiparticles of nearly opposite momenta, takes place whenever the energy conservation requirement

$$\omega = 2E_k \qquad (1)$$

is met (ω is the phonon frequency, E_k the BCS single-particle energy; $\hbar = k_B = 1$). We have neglected the phonon momentum q in Eq. (1), which would lead to small corrections. For sound frequencies such that $\omega \ll T_c$ (in Ref. 1, $\omega = 2\pi\times10^7$ sec^{-1} = 0.48 mK and T_c = 2.7 mK), Eq. (1) can be satisfied only very close to T_c.

At present there exists no complete equilibrium theory for the newly discovered phases of ^{3}He. There is accumulating evidence[3-7] that liquid ^{3}He undergoes a phase transition into a BCS-type state, where the pairing occurs in a state of odd relative angular momentum L (spin triplet). Up to now the more detailed discussions of spin-triplet pairing have been concentrated on the p-wave case. Balian and Werthamer[8] (BW) have shown that for spin-independent p-wave interaction the isotropic solution of the gap equation belongs to the lowest free energy. The single-particle energy is $E_{k\alpha}^2 = \epsilon_k^2 + \Delta^2(T)$, with $\Delta(T) \approx 3.06 T_c t^{1/2}$ for $t \ll 1$ [t is the reduced temperature $(T_c - T)/T_c$; $\epsilon_k = k^2/2m^* - \epsilon_F$; m^* is the effective mass]. Ambegaokar and Mermin[9] extended this analysis to include the effects of a static magnetic field H. They found that the transition "A" is split.

Anderson and Brinkman[10] have shown how the inclusion of spin fluctuation effects may cause the effective pairing interaction to become spin dependent in such a way as to suppress pairing with magnetic spin quantum number $S_z = 0$.[11] They found that the gap matrix has again the same angular dependence as the BW state, but $\Delta_{\uparrow\downarrow}(T) = 0$ and $\Delta_{\alpha\alpha}(T) = 3.42 T_c t^{1/2}$. The single-particle energy is $E_{k\alpha}^2 = \epsilon_{k\alpha}^2 + \Delta_{\alpha\alpha}^2(T)\sin^2\theta$. All of these gap solutions are degenerate with respect to rotations in momentum space and/or spin space.

From Eq. (1) it follows immediately that absorption can only take place for temperatures such that $\omega > 2\min[\Delta(\vec{k}; T)]$. Consequently, in the isotropic Balian-Werthamer state the anomalous absorption is confined to the temperature range $t < (\omega/6.12 T_c)^2 \approx 8\times 10^{-4}$ (for the experimental parameters of Ref. 1), which is much smaller than experimentally observed (cf. Fig. 1). On the other hand there obviously exists no such rigorous bound for the Anderson-Morel state, in which absorption is possible at any temperature below T_c, if only for a small fraction of states in $\vec{k}$ space in the direction of the nodes of the gap. We can therefore, in the latter case, expect a peak in the absorption somewhere near T_c and a relatively long tail for lower T. This is borne out by the detailed calculation of the absorption described below (cf. Fig. 1). We can also see that the attenuation peak will be split in a sufficiently strong magnetic field, the first peak corresponding to $\omega \approx 2|\Delta_{\uparrow\uparrow}|$. Quite generally, we can infer from the experimentally observed long tail in the attenuation that the gap parameter has to be very anisotropic. In concluding this qualitative discussion we remark that according to the

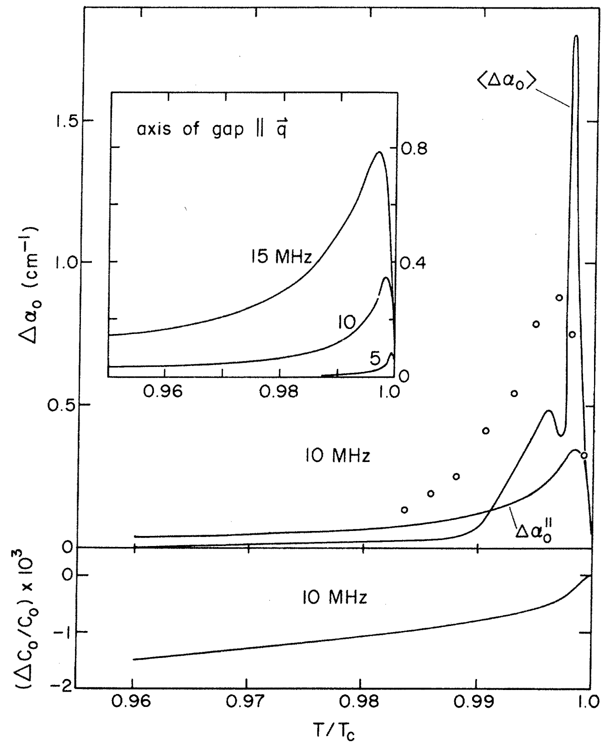

FIG. 1. Anomalous zero-sound attenuation $\Delta\alpha_0$ and change of sound velocity Δc_0 in the Anderson-Morel state below the transition "A" in ^{3}He versus reduced temperature T/T_c ($T_c = 2.7$ mK), for 5-, 10-, and 15-MHz sound. The curve marked $\langle\Delta\alpha_0\rangle$ represents an angular average over all orientations of the gap axis; all other curves are for parallel orientation of the gap axis with respect to the direction of sound propagation $\vec{q}$. The data points are from Ref. 1 (background subtracted).

above considerations there should be no peak in the anomalous attenuation at the transition "A_3" and at the transition "B," if "B" is only a readjustment of the superfluid state.

In the following we briefly describe the analytical results. One way to calculate sound attenuation is to derive the dispersion relation. This is done by writing down and solving the Landau transport equation appropriate for the superfluid state. A convenient way of writing this is[12]

$$\omega\,\delta\underline{n}_{\vec{k}} = \delta\underline{n}_{\vec{k}}\underline{\epsilon}_{\vec{k}_+}^{\,0} - \underline{\epsilon}_{\vec{k}_-}^{\,0}\delta\underline{n}_{\vec{k}} + \underline{n}_{\vec{k}_-}^{\,0}\delta\underline{\epsilon}_{\vec{k}} - \delta\underline{\epsilon}_{\vec{k}}\underline{n}_{\vec{k}_+}^{\,0}, \qquad (2)$$

where $\vec{k}_\pm = \vec{k} \pm \vec{q}/2$ and the underlined quantities are 4×4 matrices in Nambu and spin space. The quasiparticle distribution function and energy matrices are defined as

$$\delta\underline{n}_{\vec{k}} = \begin{pmatrix} \delta f_{\vec{k}}(\vec{q},\omega) & \delta g_{\vec{k}}(\vec{q},\omega) \\ \delta g_{\vec{k}}^*(-\vec{q},-\omega) & -\delta f_{-\vec{k}}(\vec{q},\omega) \end{pmatrix}, \quad \delta\underline{\epsilon}_{\vec{k}} = \begin{pmatrix} \delta\epsilon_{\vec{k}}(\vec{q},\omega) & \delta\Delta_{\vec{k}}(\vec{q},\omega) \\ \delta\Delta_{\vec{k}}^*(-\vec{q},-\omega) & -\delta\epsilon_{-\vec{k}}(\vec{q},\omega) \end{pmatrix},$$

with

$$\delta f_{\vec{k},\alpha\beta}(\vec{q},\omega)=\int dt\, e^{-\omega t}\delta\langle a_{\vec{k}_-\alpha}{}^{\dagger}(t)a_{\vec{k}_+\beta}(t)\rangle, \qquad \delta g_{\vec{k},\alpha\beta}(\vec{q},\omega)=\int dt\, e^{-i\omega t}\delta\langle a_{-\vec{k}_-\alpha}(t)a_{\vec{k}_+\beta}(t)\rangle,$$

$$\delta\epsilon_{\vec{k},\alpha\beta}(\vec{q},\omega)=\sum_{\vec{k}',\alpha'} f_{\vec{k}\alpha,\vec{k}'\alpha'}\delta f_{\vec{k}',\alpha'\beta}(\vec{q},\omega), \quad \delta\Delta_{\vec{k},\alpha\beta}(\vec{q},\omega)=\sum_{\vec{k}'} g_{\vec{k}\alpha,\vec{k}'\beta}\delta g_{\vec{k}',\alpha\beta}(\vec{q},\omega).$$

Here $f_{\vec{k}\alpha,\vec{k}'\beta}$ is the usual Fermi-liquid interaction[2] and $g_{\vec{k}\alpha,\vec{k}'\beta}$ is the pairing interaction. The quantity $\delta\epsilon_{\vec{k}}$ also contains the direct energy gain in an external field. $\underline{\epsilon}_{\vec{k}}{}^0$ is the equilibrium quasiparticle energy matrix

$$\underline{\epsilon}_{\vec{k}}{}^0=\begin{pmatrix}\delta_{\alpha,\beta}\epsilon_{k\alpha} & \Delta_{\vec{k}\alpha\beta}\\ -\Delta_{-\vec{k}\alpha\beta}{}^* & \delta_{\alpha,\beta}\epsilon_{k\alpha}\end{pmatrix},$$

and the equilibrium distribution function $\underline{n}_k{}^0$ for states with diagonal $\Delta_{\alpha\beta}$ and for BW-type states without magnetic field is given by $\underline{n}_k{}^0=\frac{1}{2}-\underline{\epsilon}_k{}^0\theta_k$, where $\theta_k=\tanh(E_k/2T)/2E_k$ and $E_k{}^2=\epsilon_k{}^2+\Delta_{\vec{k}}\Delta_{\vec{k}}{}^{\dagger}$.

In order to solve Eq. (2) we assume as usual that the energy dependence of the interaction parameters $f_{kk'}$ and $g_{kk'}$ is negligible near the Fermi surface. We solve for δf and δg in terms of $\delta\epsilon$ and $\delta\Delta$ and operate with $\sum_{\vec{k}',\beta}\delta f_{\vec{k}\alpha,\vec{k}'\beta}$ and $\sum_{\vec{k}'} g_{\vec{k}\alpha,\vec{k}'\beta}$, respectively, on these. The result, after expanding for $q \ll k_F$ and $\omega \ll \epsilon_F$,[3] is

$$\delta\epsilon_{\vec{k}\alpha}=\tfrac{1}{2}\sum_{\beta}(4\pi)^{-1}\int d\Omega_{\vec{k}'}F_{\vec{k}\alpha,\vec{k}'\beta}$$
$$\times\{[\eta/(\omega-\eta)](1-2\lambda_{\vec{k}'\beta})\delta\epsilon_{\vec{k}'\beta}-\lambda_{\vec{k}'\beta}(\delta\epsilon_{\vec{k}'\beta}+\delta\epsilon_{-\vec{k}'\beta})-\tfrac{1}{2}(\omega+\eta)\bar{\lambda}_{\vec{k}'\beta}(\delta\Delta\Delta^*-\Delta\,\delta\Delta^*)_{\vec{k}'\beta}\}, \quad (3a)$$

$$\delta\Delta_{\vec{k}\alpha\beta}+(4\pi)^{-1}\int d\Omega_{\vec{k}'}G_{\vec{k}\alpha,\vec{k}'\beta}\int_{-\omega_0}^{\omega_0}\theta_{\vec{k}'\beta}\,d\epsilon_{\vec{k}'}\delta\Delta_{\vec{k}'\alpha\beta}=(4\pi)^{-1}\int d\Omega_{\vec{k}'}G_{\vec{k}\alpha,\vec{k}'\beta}\lambda_{\vec{k}'\alpha}$$
$$\times\{-\tfrac{1}{2}\Delta_{\vec{k}'\alpha\beta}[(\omega+\eta)\delta\epsilon_{\vec{k}'\beta}+(\omega-\eta)\delta\epsilon_{\vec{k}'\beta}]-\tfrac{1}{2}(\omega^2-\eta^2-2|\Delta_{\vec{k}'}|^2)\delta\Delta_{\vec{k}'\alpha\beta}+(\Delta\,\delta\Delta^*\Delta)_{\vec{k}'\alpha\beta}\}, \quad (3b)$$

where $|\vec{k}|=|\vec{k}'|=k_F$, $\eta=\vec{v}_{k'}\cdot\vec{q}$ ($\vec{v}_k$ is the quasiparticle velocity on the Fermi surface), $F_{\vec{k}\alpha,\vec{k}'\beta}=\nu(0)\times f_{\vec{k}\alpha,\vec{k}'\beta}$, $G_{\vec{k}\alpha,\vec{k}'\beta}=\frac{1}{2}\nu(0)g_{\vec{k}\alpha,\vec{k}'\beta}$ [$\nu(0)$ is the density of states of both spins], ω_0 is the cutoff parameter appearing in the gap equation. We have assumed $\delta\epsilon_{\alpha\beta}$ as well as $(\delta\Delta\Delta^*)_{\alpha\beta}$ to be diagonal. $\lambda_{\vec{k}\alpha}$ is given by

$$\lambda_{\vec{k}\alpha}=-2|\Delta|_{\alpha}{}^2\int_{-\infty}^{\infty}d\epsilon_{\vec{k}}\,[\omega^2\theta_{\vec{k}\alpha}+\eta^2\epsilon_{\vec{k}}\,d\theta_{\vec{k}\alpha}/d\epsilon_{\vec{k}}]D_{\vec{k}\alpha}{}^{-1},$$

with

$$D_{\vec{k}\alpha}=\omega^2(\omega^2-4E_{\vec{k}\alpha}{}^2)-\eta^2(\omega^2-4\epsilon_{\vec{k}\alpha}{}^2), \quad \bar{\lambda}=\lambda/|\Delta|^2.$$

For $\Delta=0$ the equation for $\delta\epsilon$ reduces to the correct result for a normal Fermi liquid. We now expand Eqs. (3) in spherical harmonics and adopt the usual approximation[2] of setting the Fermi-liquid parameters F_l equal to zero for $l\geq 2$. In the case of p-wave pairing, we can also set $G_l=0$ for $l\geq 2$ [for odd L pairing, by inspection of Eq. (3b), even l components of $\delta\Delta$ do not couple to $\delta\epsilon$ and we can set $G_l=0$ for even l]. By virtue of the gap equation

$$\Delta_{\vec{k}\alpha\beta}=\int_{-\omega_0}^{\omega_0}d\epsilon_{\vec{k}'}(4\pi)^{-1}\int d\Omega_{\vec{k}'}G_{\vec{k}\alpha,\vec{k}'\beta}\theta_{\vec{k}'\beta}\Delta_{\vec{k}'\alpha\beta},$$

the dependence on G drops out of Eq. (3b). For any particular choice of Δ we can now solve Eqs. (3). The resulting dispersion relation is ($s=\omega/v_Fq$)

$$s^2=s_0{}^2[1+\xi(1+\tfrac{1}{3}F_1)],$$

with $s_0{}^2\approx(\frac{1}{3}F_0+\frac{3}{5})(\frac{1}{3}F_1+1)$. Since $s_0{}^2\approx 171$ from the experimental zero-sound velocity,[1] we can expand ξ in powers of $s_0{}^{-2}$ and deep only the first term. We have not evaluated ξ for the BW p-wave state because the result for the zero-sound attenuation is in any case incompatible with experiment. Instead we have calculated the attenuation for the Anderson-Morel state, which according to Anderson and

Brinkman[10] is a possible equilibrium state. ξ is found to be

$$\xi=\frac{3}{S_0^2}\sum_\alpha\left\{-\langle\lambda_\alpha\cos^4\theta\rangle+\frac{\langle\lambda_\alpha\cos^2\theta\rangle^2}{\langle\lambda_\alpha\rangle}+\frac{\omega^2\langle\lambda_\alpha\Delta_\alpha{}^*\cos^2\theta/\Delta_\alpha\rangle^2}{[\omega^2\langle\lambda_\alpha\rangle-2\langle|\Delta_\alpha|^2\lambda_\alpha\rangle]}+\frac{\omega^2[(\bar{\lambda}_\alpha\cos^2\theta)_{10}]^2}{[\omega^2\bar{\lambda}_{00}-2\lambda_{00}+\theta_{00}-\theta_{11}]}\right\},$$

where $\cos\theta$ is $\vec{v}_{\vec{k}}\cdot\vec{q}/v_F q$, the brackets denote angular averages, and $\lambda_{mm'}=\int d\Omega\, Y_1^{m*}\lambda Y_1^{m'}$, etc. (the Y_l^{m}'s are spherical harmonics whose axis is parallel to the axis of the gap). The zero-sound attenuation $\Delta\alpha_0=-q\,\mathrm{Im}(s/s_0)$ is directly related to the imaginary part of λ. If in the expression for λ we drop the terms involving η as being of higher order in s_0^{-2}, we find

$$\mathrm{Im}\lambda_{\vec{k}\alpha}=\pi|\Delta_{\vec{k}}|_\alpha{}^2(\omega^2-4|\Delta_{\vec{k}}|_\alpha{}^2)^{-1/2}\tanh(\omega/4T)/\omega$$

for $\omega^2>4|\Delta_{\vec{k}}|_\alpha{}^2$ and zero otherwise. For $T\approx T_c$ we have

$$\mathrm{Re}\lambda_{\vec{k}\alpha}=\pi|\Delta_{\vec{k}}|_\alpha{}^2(4|\Delta_{\vec{k}}|_\alpha{}^2-\omega^2)^{-1/2}\tanh(\omega/4T)/\omega$$

for $\omega^2<4|\Delta_{\vec{k}}|_\alpha{}^2$ and zero otherwise. Not too surprisingly, it turns out that $\Delta\alpha_0$ depends strongly on the relative orientation of $\vec{q}$ and the axis of the gap. While for parallel orientation ($\Delta\alpha_0{}^{\parallel}$) the attenuation peak is small and broad, it is extremely narrow and high for perpendicular orientation; this shows up in the angular average $\langle\Delta\alpha_0\rangle$ (Fig. 1). It is not clear whether the axis of the gap orients itself by residual interactions, e.g., with the container walls, the zero sound itself, etc., or whether there exist randomly oriented domains. It is, however, likely that at least in a magnetic field the dipole interaction plays an important role. This would favor orientation of the gap axis perpendicular to the field,[13] which puts no restriction on the gap orientation with respect to $\vec{q}$, as long as the magnetic field $\vec{H}$ is perpendicular to $\vec{q}$, as in the experiments.[1] Further experiments in which the angle between $\vec{q}$ and the magnetic field is varied would help to resolve the problem of orientation. A comparison of theory and experiment (circles in Fig. 1) shows that the agreement is fair, but not completely satisfactory. We can expect spin fluctuation effects to modify the results somewhat. We have not plotted the attenuation in a magnetic field, because it is simply obtained by superposing two shifted zero-field attenuation curves with different weight according to the difference in spin population. The insert in Fig. 1 shows the strong frequency dependence of the attenuation, the peak height varying approximately with ω^2. The sound velocity c_0 decreases monotonically with decreasing temperature (Fig. 1) until a frequency-dependent saturation value is reached for $T\ll T_c$ (where $\lambda=\frac{1}{2}$). For ω, $T\ll T_c$, we recover the Anderson-Bogoliubov mode with $s_0^2=(\frac{1}{3}F_0+\frac{1}{3})(\frac{1}{3}F_1+1)$, as predicted by Leggett.[14]

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