Phonon and Roton Propagation in He II under Pressure

R. C. Dynes, V. Narayanamurti, and K. Andres
Bell Laboratories, Murray Hill, New Jersey 07974

(Received 19 March 1973)

We have studied the transition from second sound to ballistic phonon flow in He II up to pressures of 24 bar. At saturated vapor pressure the phonon-phonon scattering rate $\tau_{pp}^{-1} \cong 1.7\times10^6 T^3$ sec^{-1} for $T<0.6$ K. At 24 bar, $\tau_{pp}^{-1}<8\times10^4$ sec^{-1} for $T<0.7$ K. The phonon-roton scattering rate τ_{pr}^{-1} is $\sim10^6$ sec^{-1} at 0.75 K and almost pressure independent. At 24 bar between 0.4 and 0.7 K, in addition to ballistic phonons, we observe a second pulse traveling at 16 m/sec, which we believe is the propagation of interacting rotons.

Heat pulses in liquid helium II, depending on the temperature, will propagate in a variety of different ways. When the mean free path of the elementary excitations is long compared with the propagation length, they are expected to propagate ballistically (as phonons and/or rotons); while as the temperature is raised and interactions set in, such pulses can propagate as second sound in the phonon or roton gas, or as a collective mode of both phonons and rotons. This transition from ballistic to second-sound flow is determined by the various collision rates τ_{pp}^{-1} (phonon-phonon), τ_{rr}^{-1} (roton-roton), and τ_{pr}^{-1} (phonon-roton) which determine the various equilibrium times.

In this Letter we report some experimental results on the transition from ballistic to second-sound propagation in liquid He II under pressure. We present a picture of second-sound propagation in terms of the elementary excitation spectrum (as distinct from the usual two-fluid model) and obtain information on the different scattering rates by the application of pressure, which allows us to vary the relative phonon and roton contributions.

Previous experimental work in the ballistic regime was limited because of broad pulse widths employed and the slow response time of the carbon bolometers used[1] and/or the general use of long, narrow tubes causing most of the signal to arrive by diffuse or specular reflection at the walls of the sample chamber. In our experiment, we minimize wall reflections and use fast generators and detectors developed previously for heat-pulse propagation studies in solids.[2] The generator consisted of a 500-Å-thick Constantan film of nominal resistance 50 Ω and size 3.7×3.7 or 1.5 ×1.5 mm^2. The detector was a 1000-Å-thick indium bolometer of size 3.5×3.5 mm^2 or a Sn tunnel junction of size 1×1 mm^2, both biased in a large parallel magnetic field. The typical propagation length L was 0.24 cm. The helium sample chamber was cooled by means of a dilution refrigerator. Typical pulses were of duration 0.05 to 0.5 μsec and of amplitude 0.5–25 V.

A Biomation 8100 transient recorder with 10-nsec resolution and a Fabritek multichannel analyzer were used to time analyze and signal average the received pulses.

At the top of Fig. 1(a) we show typical ballistic heat pulses in He II under the saturated vapor pressure (SVP) as a function of ambient temperature of the bath. The data were found to be independent of excitation energy for pulse energies less than about 10 erg/cm^2. At the lowest temperatures (below 0.3 K) one observes a well-defined single pulse propagating with a velocity of 236 ± 4 m/sec close to that expected for ballistic sound. Above about 0.35 K, the pulse broadens

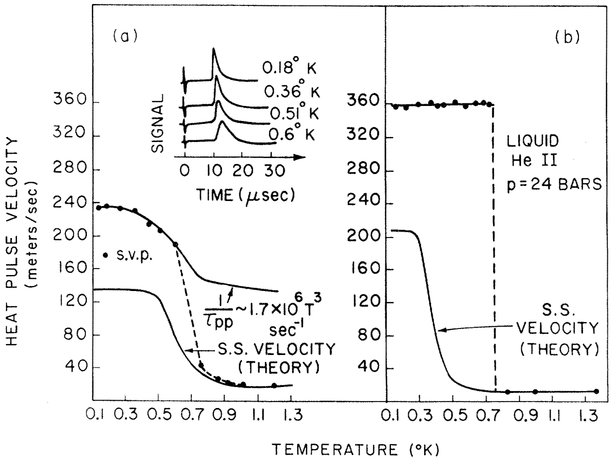

FIG. 1. Peak velocity of heat pulses in He II (a) at saturated vapor pressure and (b) at 24 bar. For a description of the theoretical curves, see the text. Top of (a), traces of experimental heat-pulse data at SVP that have propagated a distance of 2.34 mm.

and retards in a manner reminiscent of the transition region from ballistic to second-sound propagation in solids.[3] However, above about 0.62 K the pulse is sufficiently broad to be unobservable until a temperature of 0.72 K when it again begins to narrow with a velocity of about 40 m/sec as shown in the bottom of Fig. 1(a). Above this temperature we see continued narrowing and well-defined pulses with many echoes and a velocity in excellent agreement with previous second-sound[4] data.

Theoretically, one can calculate the second-sound velocity for an arbitrary excitation spectrum, assuming strong scattering among all the excitations, and small perturbations from the ambient. The velocity of second sound is a function of temperature T as different parts of the excitation spectrum are populated as T is varied. The second-sound velocity v_{II} in a dispersive medium was first given by Kwok[5] for the case of solids:

$$v_{\mathrm{II}}^2 = \tfrac{1}{3}\left(\sum \omega_k S_k^0 \vec{\mathrm{k}}\cdot\vec{\mathrm{u}}_k\right)^2 \left(\sum_k S_k^0 \omega_k^2\right)^{-1} \times \left(\sum_k S_k^0 k^2\right)^{-1}. \qquad (1)$$

In this expression $S_k^0 = N_k^0(N_k^0 + 1)$, where N_k^0 is the equilibrium Bose-Einstein distribution function, and $\vec{\mathrm{u}}_k$ is the group velocity $\nabla_k \omega_k$. We have numerically evaluated[6] (1) for He II using the experimental dispersion curve determined by inelastic neutron scattering[7] at 1.1 K; the result is the solid curve labeled "s.s. velocity (theory)" in Fig. 1(a). Excellent agreement in the second-sound regime (>0.8 K) is achieved. We have not taken into account the T dependence of the ω-k curve so that the calculation is most reliable at 1.1 K. At temperatures higher than 1.6 K, we do not expect good agreement. Physically, in the elementary excitation picture, the low second-sound velocity above about 0.7 K arises as a result of the *combined* effect of the high density of roton states of low group velocity and the near cancelation of the contributions from either side of the dispersion curve about the roton minimum due to the $\vec{\mathrm{k}}\cdot\vec{\mathrm{u}}_k$ term in (1). On the low side of the roton minimum the heat current, which is determined by the group velocity, and the momentum lie in opposite directions, and $\vec{\mathrm{k}}\cdot\vec{\mathrm{u}}_k$ is negative.

The heat-pulse velocity also varies with temperature because the scattering rates are finite and temperature dependent. Khalatnikov and Chernikova[8] have shown for SVP that the dominant scattering mechanism above 0.9 K is phonon-roton scattering. Since this depends on the roton number density, it decreases rapidly with T and by 0.6 K it is not expected to be effective. Below 0.6 K phonon-phonon scattering is expected to be dominant. Assuming a linewidth to the dispersion curve, Pethick and Ter Haar[9] predict an ωT^4 dependence for the three-phonon process, while Khalatnikov and Chernikova have calculated a T^9 dependence for the four-phonon process independent of the dispersion. The relative magnitude of these contributions depends sensitively on the nature of the He^4 dispersion curve in the phonon region.[10]

The experimental velocity-profile curve of Fig. 1(a) reflects some of these concepts. The large variation in the velocity between 0.72 and 0.62 K is believed to be due to a change in τ_{pr}^{-1} by an order of magnitude (from 10^{-6} to $\sim 10^{-5}$ sec). That the theoretical second-sound velocity curve shows a large drop in the same temperature region is believed to be accidental as we shall show from the high-pressure data. The increase in the velocity of the heat pulse below 0.6 K must be due to the decrease of the phonon-phonon scattering rate τ_{pp}^{-1}. In the phonon region, the transition from second sound to ballistic flow can be written, in analogy with hydrodynamics,[3] as

$$k^2 v_{\mathrm{II}}^2 = \omega^2 - i\omega k^2 \tau_{pp}(v_{\mathrm{I}}^2 - v_{\mathrm{II}}^2)/(1 - i\omega\tau_{pp}). \qquad (2)$$

In the limit $\omega\tau_{pp} \ll 1$, one would observe a fully developed second-sound wave in the phonon gas traveling with a velocity $v_{\mathrm{II}} = v_1/\sqrt{3}$, where v_1 is the ballistic phonon velocity. A best fit to the data of Fig. 1(a) below 0.7 K is obtained with a scattering rate $\tau_{pp}^{-1} \sim 1.7\times10^6 T^{3\pm0.3}\ \mathrm{sec}^{-1}$. The temperature dependence is to be viewed with caution because of the limited region of the dispersion curve studied. It is, however, similar to the T^3 dependence observed in solid He but slower than the T^4 dependence observed in other solids.[3] Numerically, the relaxation time τ_{pp} agrees with the Poiseuille flow thermal-conductivity data between 0.4 and 0.6 K[11] within a factor of 2.

Considerable additional information on the propagation and lifetime of excitations can be obtained in He II by the study of heat pulses under pressure. With increasing pressure the roton minimum decreases in energy while the ballistic phonon velocity increases. Figure 1(b) shows the results of some of our experiments on He II at the maximum pressure of 24 bar. Below about 0.73 K we observe a ballistic pulse traveling with a speed of 360 ± 3 m/sec, in excellent agreement with ultrasonic data. Above this temperature the ballistic pulse abruptly disappears, and at about 0.8 K we observe the usual second-sound pulses.[12] The main difference between the SVP and high-pressure data is the lack of variation in the phonon velocity at 24 bar below 0.7 K. The temperature of occurrence of the transition from phonon flow to complete second sound is, however, almost the same, but occurs over a much narrower range.

We can detect no temperature dependence of the velocity in the ballistic regime (below 0.7 K) at pressures between 24 and 10 bar. In this pressure and temperature range then the mean free path for phonon-phonon scattering must be larger than 0.5 cm. This result is probably due to the large increase in normal dispersion in the phonon region of the He-II excitation spectrum[7] at high pressure, which effectively decreases the usual three-phonon scattering rate. Then only the much weaker four-phonon process would be allowed.

From our data, it appears that phonon-roton scattering is independent of pressure. Apparently, the effect of the increase in the roton number density with increasing pressure is almost exactly canceled[8] by the approximately v^{-7} dependence in τ_{pr}^{-1}.

This decoupling of the phonon and roton gases below about 0.7 K is dramatically illustrated if one plots the theoretically expected second-sound velocity at 24 bar calculated from Eq. (1). This is shown as the solid curve in the lower part of Fig. 1(b). It is clear that in a region of ballistic phonon flow (~0.45 to ~0.7 K) a large proportion of the heat pulse should consist of low-velocity, *thermally excited* roton components.

After the above analysis was completed, we repeated our experiments to look simultaneously for slow components. A few results at very low excitation levels ($\lesssim 1$ erg/mm^2) are shown in Fig. 2(a). At 0.1 K we observe only a single ballistic phonon pulse. At 0.4 K, a new broad pulse appears at a time which corresponds to a velocity more than 20 times slower than the ballistic phonon velocity. As the ambient temperature is raised further, this new pulse sharpens and finally merges at about 0.75 K with the usual second-sound pulse as shown in Fig. 2(b). We identify this new pulse with slow-moving roton components. From its velocity and shape[13] we believe it corresponds to interacting particles in the vicinity of the roton minimum and excitations *beyond* the minimum of the roton excitation spectrum. These excitations, according to neutron-scattering measurements,[7] have a short lifetime. Pure roton second sound (consisting of particles from the entire negative- and positive-roton branches) we calculate from Eq. (1) to propagate with a velocity of 6 m/sec at 0.5 K, while second sound comprised of excitations from the positive roton branch alone would propagate with a velocity of 40 m/sec. The experimental velocity of 16 m/sec lies in between.

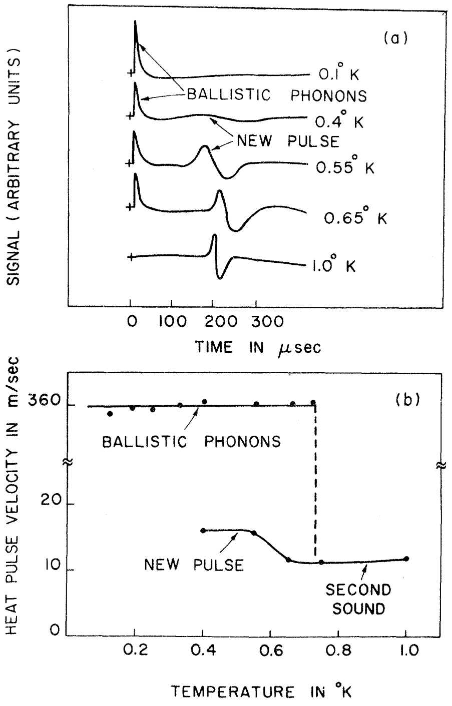

FIG. 2. (a) Typical heat pulses in He II at 24 bar as a function of temperature. The new pulse, which has an anomalously low velocity, is comprised mostly of propagating rotons. (b) Peak velocity of heat pulses at 24 bar showing how ballistic phonons and the collective roton pulse merge into ordinary second sound at higher temperatures.

With increasing heat pulse energies (up to 120 erg/cm^2), the velocity of the "new" pulse increases (as it should with an increasing number of higher velocity excitations) and additional components develop. These additional pulses at higher excitation energies probably correspond to nonthermal propagating rotons from the negative side of the roton dispersion curve. Details will be published in a forthcoming publication.

In summary, we have studied the transition from second sound to ballistic phonon flow in liquid He II as a function of temperature and pressure. The results show that the phonon-phonon scattering rate decreases markedly with increasing pressure while the phonon-roton scattering time is nearly pressure independent. This allowed us to observe the propagation of nearly thermal rotons at high pressure.

We would like to thank M. A. Chin, S. Darack, and J. P. Garno for superb technical assistance. We also thank S. Geschwind, C. C. Grimes, and J. M. Rowell for helpful discussions.

---

[1] K. R. Atkins and D. V. Osborne, Phil. Mag. 41, 1078 (1950); H. C. Kramers, Proc. Kon. Ned. Akad. Wetensch., Ser. B 59, 35, 48 (1956); R. W. Guernsey and K. Lusczynski, Phys. Rev. A 3, 1052 (1971).

[2] See, for example, J. Phys. (Paris), Colloq. 33, C-4 (1972).

[3] V. Narayanamurti and R. C. Dynes, Phys. Rev. Lett. 28, 1461 (1972), and references cited therein.

[4] V. P. Peshkov, Zh. Eksp. Teor. Fiz. 38, 799 (1960) [Sov. Phys. JETP 11, 580 (1960)].

[5] P. C. Kwok, Physics (Long Is. City, N. Y.) 3, 221 (1967).

[6] The thermal properties at SVP have previously been calculated from the excitation spectrum by P. J. Bendt, R. D. Cowan, and J. L. Yarnell, Phys. Rev. 113, 1386 (1959). In calculating v_{II} they, however, used the thermodynamic relations in therms of ρ_s and ρ_n. J. C. Ward and J. Wilks [Phil. Mag. 42, 314 (1951), and 43, 48 (1952)] have described second sound in the phonon gas only ($v_{\mathrm{II}}=u/\sqrt{3}$) in terms of strongly interacting phonons.

[7] O. W. Dietrich, E. H. Graf, C. H. Huang, and L. Passell, Phys. Rev. A 5, 1377 (1972); E. C. Svensson, A. D. B. Woods, and P. Martel, Phys. Rev. Lett. 29, 1148 (1972); A. D. B. Woods, private communication.

[8] I. M. Khalatnikov and D. M. Chernikova, Zh. Eksp. Teor. Fiz. 49, 1957 (1965) [Sov. Phys. JETP 22, 1336 (1966)].

[9] C. J. Pethick and D. ter Haar, Physica (Utrecht) 32, 1905 (1966).

[10] H. J. Maris, Phys. Rev. Lett. 28, 277 (1972); J. Jackle and K. W. Kehr, *ibid.* 27, 654 (1971).

[11] R. W. Whitworth, Proc. Roy. Soc., Ser. A 246, 390 (1958).

[12] R. D. Maurer and M. A. Herlin, Phys. Rev. 81, 449 (1951).

[13] This "differentiated" shape is observed for our geometry in the case of collective excitations and not for ballistic flow. For an explanation, see R. W. Guernsey, K. Luszczynski, and W. Mitchell, Cryogenics 7, 110 (1967).

Microscopic Investigation of the Coexistence of Superconductivity and Magnetism

P. Steiner, D. Gumprecht, and S. Hüfner
Fachbereich Physik, Freie Universität Berlin, Berlin-Dahlem, Germany
(Received 15 March 1973)

Mössbauer-effect measurements have been performed on the alloy system $\mathrm{Eu}_x\mathrm{La}_{1-x}$ showing that for a sample with $x=1.2\%$ there is a transition to a magnetically ordered state at 660 ± 25 mK. The same sample shows a superconducting transition temperature of $T_s=2.15(10)$ K.

From an investigation of the magnetic and superconducting properties of the $\mathrm{Gd}_x\mathrm{La}_{1-x}$ system and of the $\mathrm{RE}_x\mathrm{Ce}_{1-x}\mathrm{Ru}_2$ system Matthias and coworkers[1-4] suggested a coexistence of magnetism and superconductivity in these systems. Early theoretical work[5] on this problem seemed to exclude coexistence of these two cooperative phenomena, but later work[6] showed that they could indeed coexist. In the meantime much of the work on that question has been performed on the $\mathrm{Gd}_x\mathrm{Ce}_{1-x}\mathrm{Ru}_2$ system and the systematic measurements of Peter *et al.*[7] do show indeed a coexistence of magnetism and superconductivity in this system. Taylor *et al.*[8] have performed a Mössbauer-effect measurement with sources of Co^{57} implanted into $\mathrm{Gd}_x\mathrm{Ce}_{1-x}\mathrm{Ru}_2$ and find that in the concentration range in which, from the bulk measurements, a coexistence of superconductivity and magnetism is expected, a fraction of the iron atoms indeed shows a magnetic hyperfine splitting, showing the existence of magnetism on a microscopic basis in the superconducting state. Mössbauer-effect measurements in these systems are especially suited for a detection of magnetic order because they are performed in zero external field and also these measurements are not affected by the superconductivity. It was therefore decided to look for a system where one could perform such measurements directly with the magnetic ions. Starting out with Phillips and Matthias's[4] suggestion for the $\mathrm{Gd}_x\mathrm{La}_{1-x}$, it was decided to investigate the $\mathrm{Eu}_x\mathrm{La}_{1-x}$ system be-