[7]In other words, we assume that the current drawn from the source depends only on the electric field at the source, and not on the amount of vorticity present. While this assumption is often made, there is indirect evidence that it may not be entirely correct [J. A. Northby and R. J. Donnelly, Phys. Rev. Lett. 25, 214 1970)].

[8]D. J. Tanner, Phys. Rev. 152, 121 (1966); see also R. L. Douglass, Phys. Rev. 141, 192 (1966).

[9]SM assume $\sigma = \sigma_T$ in their analysis.

[10]R. E. Packard, Ph.D. thesis, University of Michigan, 1969 (unpublished).

[11]F. Moss (private communication) suggests that the discrepancy may result primarily from SM's use of the ion's thermal rather than drift velocity in their Eq. (6).

[12]For example, if β is increased by a factor of 30, then at 1.4 K l_{SM} is too large by a factor of 3, and σ_{SM} is too large by a factor of 10.

Onset for Superfluid Flow in He⁴ Films on a Variety of Substrates

E. S. Sabisky and C. H. Anderson

RCA Laboratories, Princeton, New Jersey 08540

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The onset of superfluid flow in He⁴ films was measured between 1.37 and 2.13 K on substrates of NaF, CaF₂, SiO₂, Si, and Grafoil. Values of the Van der Waals constant of 19, 26, and 65 K were obtained for NaF, SiO₂, and Grafoil, respectively. In addition, a precise measurement of the critical thickness of the liquid-helium film was obtained, taking into account a small correction for the influence of the substrate.

The conditions under which a film of liquid helium exhibits superfluid flow was studied with substrates of NaF, CaF₂, SiO₂, Si, and Grafoil[1] at temperatures between 1.37 and 2.1 K. The experiments clearly show for the first time the role of the Van der Waals force in this problem. A relative measure of this force was extracted from the data and the results are in good agreement with the Lifshitz theory.[2] In addition, a precise measurement of the critical thickness of the liquid-helium film was obtained, taking into account a small derived correction for the influence of the substrate on the helium.

The basic premise of this paper, essentially confirmed by the data, is that the primary condition for superfluid flow is for the total film thickness d to exceed an onset thickness equal to the sum of two lengths $\zeta(T)$ and δ.[3] The temperature-dependent length $\zeta(T)$, in the Ginzburg-Pitaevskii theory,[4] is directly proportional to the coherence length of liquid helium. (We shall not enter into the current discussion on the constant of proportionality.) The temperature-independent length δ depends solely on the substrate and we assume it is a continuous variable and not, as is normally assumed, limited to integral numbers of an atomic layer of helium (i.e., 3.6 Å).[5]

The onset thickness $\zeta(T) + \delta$ then determines the chemical potential observed at onset through the Van der Waals force as expressed by

$$V = kT \ln[P_0(T)/P] = \alpha/[\zeta(T) + \delta]^3. \qquad (1)$$

Here V is the negative of the chemical potential of the film at onset relative to that of the bulk liquid as measured by the temperature T and the ratio of the vapor pressure of bulk liquid helium, $P_0(T)$, to the helium-vapor pressure P of the film observed at onset. The Van der Waals potential for film thicknesses less than 100 Å (the maximum thickness in the experiments) is described to a good approximation by an inverse-cube law with a constant of proportionality α which depends on the polarizability of the substrate material.

Measurements have recently been published on the dependence of the Van der Waals potential on film thickness with substrates of CaF₂, SrF₂, and BaF₂ using an acoustic interferometric technique.[6] The results have been shown to be in excellent agreement with the Lifshitz theory of the Van der Waals force.[7] Because of this detailed knowledge of the properties of He⁴ films on CaF₂, it has been used as a standard to which the measured potentials at onset on the other substrates have been compared through the parameter R defined by

$$R^{1/3} = (V_s/V_x)^{1/3}$$
$$= (\alpha_s/\alpha_x)^{1/3}[1 + (\delta_x - \delta_s)/d_s]. \qquad (2)$$

The subscript s refers to CaF₂ and x to the other substrates. By plotting $R^{1/3}$ as a function of d_s^{-1}, the inverse thickness of the film on CaF₂ at onset, it is possible to extract the ratio of the Van

der Waals constants[8] as well as the relative change in the boundary term, $\delta_x - \delta_s$.

The basic apparatus used in the experiments was a small copper can which was divided into two equal-size compartments by a vertical copper sheet. Each half contained a carbon resistance thermometer suspended on superconducting leads, and each side was filled with granules which completely covered the thermometers: CaF_2 granules on one side and one of the other materials on the other side. The granules of the crystalline materials were obtained by crushing single crystals in air and screening the particles to a size $\sim \frac{1}{4}$ mm on a side. Helium gas could freely exchange between the two sides and a line to the can allowed gas to be introduced and removed. A second line went to a high-precision differential-capacitance pressure gauge, which measured the difference between the pressure in the can and the pressure over the helium bath surrounding the can.

The experimental procedure involved first cooling the system to 1.4 K. No helium gas was permitted in the interior for about an hour after cooling. Under these conditions the thermometers were observed to stay at about 200 K so that gas molecules which were weakly absorbed on the granules would be deabsorbed and frozen onto the walls of the cold copper can. Then at a fixed bath temperature, helium gas was added until saturation was reached. Following this, the temperature of one of the resistors was recorded as a function of the change in pressure from saturation as the helium gas was slowly pumped out of the can. A typical run near 2 K would take around an hour. The temperature of the resistor would remain constant until a critical value of the pressure was reached, and then a relatively sharp rise in temperature occurred. Traces of the data for the CaF_2 substrate at several temperatures are shown in Fig. 1(a), and the difference observed between two substrates at a given temperature is illustrated in Fig. 1(b). Using a dc bridge it was possible to detect changes in the temperature of 20 μK with a typical power input of 50 nW to the carbon resistors.

This well-defined break point is taken as the condition for the onset of superfluid flow of the film on the granules surrounding the resistor because of the following: (1) It was established through measurements with and without the granules that the film on the granules rather than that on the leads carries most of the heat. (2) The position of the break is the same when the car-
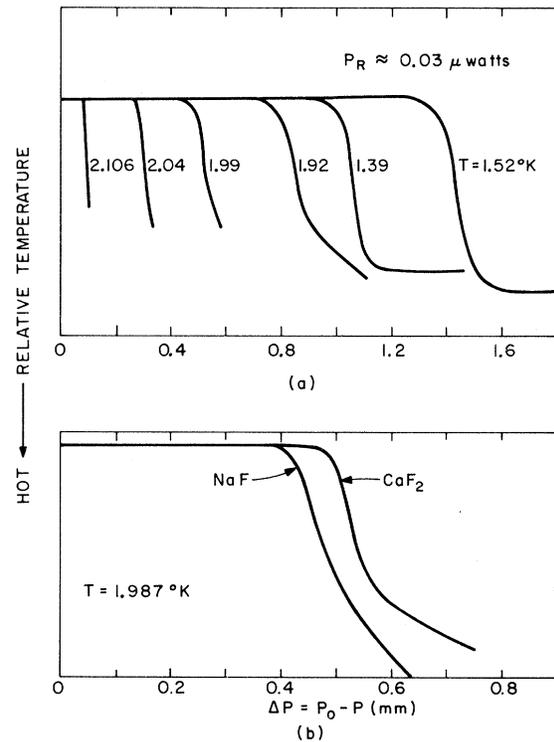

FIG. 1. (a) Traces exhibiting the temperature dependence of the onset of superfluidity as a function of the helium-gas pressure over the He[4] film for CaF_2 substrate. (b) Traces showing the dependence of the onset on the substrate material at a fixed temperature.

bon resistors are replaced by crystals of SrF_2 : Tm^{2+}, which can be used as sensitive thermometers using an optical probe and no electrical leads (see Ref. 6). (3) The position of the break depends on the particular composition of the granules [Fig. 1(b)] and, as will be shown, the changes are consistent with Eq. (2) and our knowledge of the Van der Waals force. Care was taken to ensure that the break point was not dependent on the power input nor on the rate at which the helium gas was removed from the can. A more detailed description of the experiments will be published shortly.

The experimental results as reduced according to Eq. (2) are displayed in Fig. 2. The value of the total onset thickness on CaF_2 used in Fig. 2 was obtained from the measured potential at onset and the results of Ref. 7. From the extrapolated intercept on the vertical axis and the known value for the Van der Waals constant of CaF_2 [$\alpha_s = 23$ (layers)^3 K][9] we find the Van der Waals constants of 19 ± 2 for NaF, 26 ± 2 for SiO_2, and 65 ± 5 for Grafoil. The values are in good
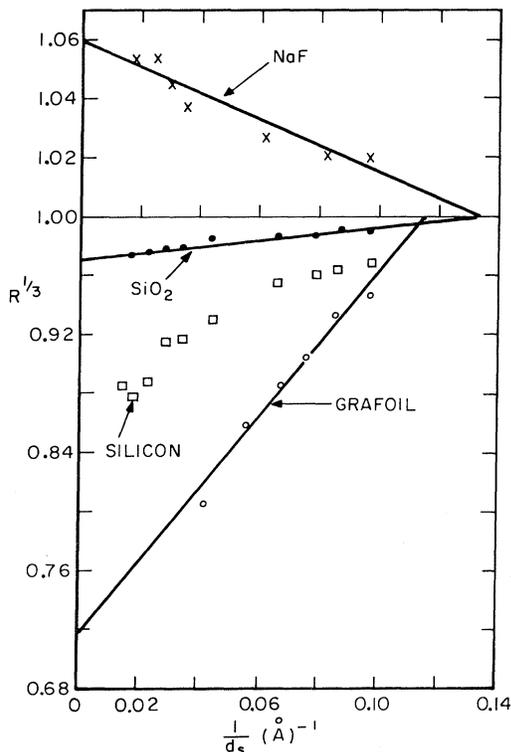
FIG. 2. The cube root of the ratio of the chemical potential of the film at onset for the CaF_2 substrate divided by that for other substrates as a function of the reciprocal onset thickness of He^4 films on CaF_2. (The vertical scale changes at $R^{1/3} = 1$.)
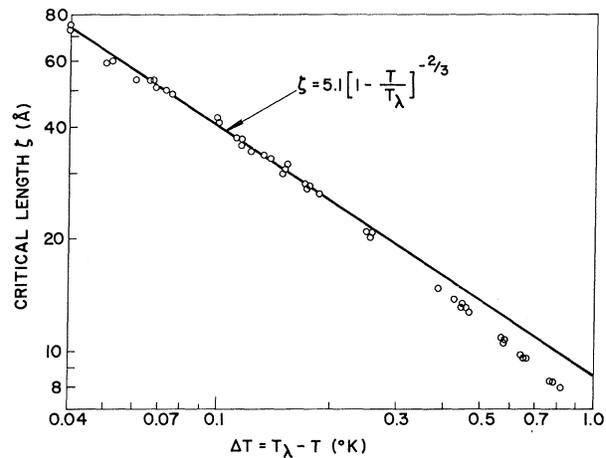

FIG. 3. The temperature dependence of the critical length derived from the total onset thickness for CaF_2 substrates using $\zeta(T) = d_s - 2$.

agreement with those we obtain using the Lifshitz theory, namely, 16, 25, and 55 for NaF, SiO_2, and graphite, respectively. The experimental value for Grafoil is in excellent agreement with the value of 67 reported by Herb and Dash.[10]

The data for the silicon substrate shown in Fig. 2 did not exhibit the same linear behavior as was found for the other substrates. It could, however, be fitted by assuming that there was a 10-Å layer of SiO_2 on the silicon and that the Van der Waals potential could be approximated by simply adding the contribution of this layer to that of the silicon. (The value of α for Si is calculated to be 36 K.) The silicon granules used in these experiments were obtained from crushed-silicon wafers which were treated to reduce the oxide layer to about 7–10 Å.[11]

From the extrapolated intercept with the horizontal line $R^{1/3} = 1$, we find for $\delta_x - \delta_s$ the values -0.5 Å for NaF, 0.25 Å for SiO_2, and 3.5 Å for Grafoil. Within the experimental accuracy, the data suggest that δ increases linearly with α.[12]

We have no explanation for this behavior. Assuming this linear behavior can be extrapolated to the point $\delta = 0$, when $\alpha = 0$, an absolute value for CaF_2 of $\delta_s = 2$ Å is found. It would be difficult to change this by more than ± 0.05 Å.

The length $\zeta(T)$ as obtained by subtracting 2 Å from the onset thickness observed on CaF_2, d_s, is plotted in Fig. 3. The precision of d_s is limited by the width of the break shown in Fig. 1 to a maximum of $\pm 3\%$. The absolute temperature above 2 K was limited to an accuracy of ± 2 mK. Our values for $\zeta(T)$ are more precise than those previously reported[13] because of the better understanding of the film thickness on CaF_2 and the more direct manner in which δ is obtained. At temperatures near T_λ (the bulk-liquid critical temperature), the data are very well described by $\zeta(T) = 5.1(1 - T/T_\lambda)^{-2/3}$ Å.[14] Under the assumption that $\zeta(T)$ is proportional to the bulk-liquid coherence length, it is meaningful to examine the product of $\zeta(T)$ with the fraction of the superfluid density of the bulk liquid.[15] The product $(\rho_s/\rho)\zeta(T) = 5.8T$ over the entire temperature range.[16]

In conclusion, this work significantly clarifies the conditions for the onset of superfluid flow in films of He^4.

We wish to thank Dr A. Goodman for treating the silicon wafers, and also to acknowledge the technical assistance of J. Beherrell in carrying out the experiments.

[1]Grafoil is an exfoliated form of graphite manufac-

tured by Union Carbide Corporation, Carbon Products Division, 270 Park Avenue, New York, N.Y.

[2]E. M. Lifshitz, Zh. Eksp. Teor. Fiz. 29, 94 (1955) [Sov. Phys. JETP 2, 73 (1956)].

[3]This assumption is clearly a simplification of a complicated problem and its validity is questionable when $\zeta(T)$ becomes comparable to δ.

[4]V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. 34, 1240 (1958) [Sov. Phys. JETP 7, 858 (1958)].

[5]We wish to leave open the interpretation of δ, since the way it is obtained from the data it could include a number of effects such as surface impurities or modifications of the macroscopic wave function.

[6]C. H. Anderson and E. S. Sabisky, Phys. Rev. Lett. 24, 1049 (1970).

[7]P. Richmond and B. W. Ninham, J. Low Temp. Phys. 5, 177 (1971); E. S. Sabisky and C. H. Anderson, Phys. Rev. A 7, 790 (1973).

[8]The parameter α is shown in Refs. 6 and 7 to decrease slowly with increasing d, but by using the ratio in Eq. (2) this effect is, to a large extent, compensated.

[9]This value for α is higher than our previously reported values because of a variety of small corrections.

[10]J. A. Herb and J. G. Dash, Phys. Rev. Lett. 29, 846 (1972). Our values for the onset on Grafoil disagree with those reported in this reference.

[11]We are indebted to Dr. A. Goodman of RCA Laboratories for supplying the treated samples.

[12]H. W. Chan, A. W. Yanof, F. D. M. Pobell, and J. D. Reppy, in Proceedings of the Thirteenth International Conference on Low Temperature Physics, Boulder, Colorado, 1972 (to be published). Chan et al. reported an onset thickness of 10 Å for bare Vycor and 7 Å for neon preplated Vycor at 0.57 K. Using the results of the present paper and the calculated value of α for neon, we calculate a difference of 2 Å in the onset thickness in this case.

[13]For a listing of references see Ref. 12 or 10.

[14]This value for the length $\zeta(T)$ agrees exactly with that estimated by Mamaladze for the characteristic length of bulk liquid helium when $\zeta(T)$ is divided by the geometric factor π, which is the value given by Ginzburg and Pitaevskii (Ref. 4) assuming the superfluid density vanishes at both surfaces {Yu. G. Mamaladze, Zh. Eksp. Teor. Fiz. 52, 729 (1967) [Sov. Phys. JETP 25, 479 (1967)]}.

[15]J. R. Clow and J. D. Reppy, Phys. Rev. Lett. 16, 887 (1966).

[16]See, for example, R. A. Ferrell, N. Menyhard, H. Schmidt, F. Schwabl, and P. Szepfalusy, Ann. Phys. (New York) 47, 565 (1968), and references therein.

Low-Temperature Specific-Heat Anomaly of bcc ³He†

S. H. Castles and E. D. Adams

Physics Department, University of Florida, Gainesville, Florida 32601

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The specific heat of bcc ³He has been measured from 25 mK to near the melting curve, with particular attention given to the region of the low-temperature anomaly, $T \lesssim 500$ mK. Here we find that the specific heat has a term linear in T, in addition to the Debye T^3 term. From the specific heat at the lowest temperatures, for the larger—molar-volume samples, values of the exchange energy J in agreement with previous results are found.

Previous measurements on the bcc phase of ³He have indicated that the specific heat C_v does not have the T^3 temperature dependence expected from the Debye theory at temperatures $T < 0.5$ K.[1-3] This "low-temperature anomaly" can be characterized by an excess specific heat. There is also an anomaly in bcc ³He at temperatures $T > 0.5$ K which is characterized by excess specific heat, possibly due to vacancies in the lattice. The cause of the low-temperature anomaly is still open to speculation. Since an apparent low-temperature anomaly was seen in early ⁴He data[4] but not in more recent work,[5] some doubt has persisted as to the validity of the effect in ³He. However, subsequent measurements of $(\partial P / \partial T)_V$[6]

and of thermal conductivity[7] substantiate the existence of the anomaly in bcc ³He.

The object of this work[8] was to make measurements of sufficient precision to show the temperature and volume dependence of the anomalous contribution. For comparison and as a check of our apparatus and procedure, two runs were made in hcp ⁴He.

The cylindrical calorimeter, made of beryllium-copper and copper, has a sample volume of 0.903 cm³. It contains a copper wire brush of 0.06-mm-diam wires with an average spacing of 0.16 mm, which provides 250 cm² surface area for thermal contact to the sample. The stainless-steel filling capillary between the thermal