Anisotropic Superfluidity in ^{3}He: A Possible Interpretation of Its Stability as a Spin-Fluctuation Effect*

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It is proposed that the paramagnon effects which enhance T_c for triplet pairing in He^3 are also important in selecting the particular component of the triplet p-wave state observed. It is found that the component favored is the original Anderson-Morel state.

To an increasing extent most of the experimental data on the recently observed 2.7-mdeg K transition in ^{3}He [1] support the suggestion[2,3] that the stable state just below T_c is an anisotropic superfluid state whose orbital angular momentum L is odd. The observation of a shifted spin-resonance line clearly indicates an odd L while the fact that the susceptibility does not change below the "A" transition indicates an anisotropic state[4] since the pseudoisotropic Balian-Werthamer[5] (BW) state has a strongly reduced susceptibility. Previous weak-coupling calculations[5] have indicated, however, that the BW state should be the most stable state.

As pointed out by a number of authors,[6] the fact that ^{3}He has an exchange-enhanced spin susceptibility and has other characteristic strong spin-fluctuation effects[7] probably has large effects on the nature of any superfluid state; certainly the even-L, singlet states are strongly suppressed, and the odd-L triplet states are relatively favored.

The main point we make in this Letter is that the spin-fluctuation ("paramagnon") behaviors of the various possible p-wave states are different, and that these differences stabilize the original Anderson-Morel state (AM) by an amount we can estimate quantitatively. The new physical effect which we consider is that the superfluid state changes the various susceptibilities and that these changes react back very strongly on the exchange enhancement effects.

We will use the conventional Doniach-Engelsberg[7] exchange-enhancement model in order to quantify these ideas. We assume there is an effective short-range repulsive interaction I which is so strongly affected by exchange, because of its short range, that it does not act between parallel-spin particles. In addition, there is a conventional interaction V which is not so affected.

In a strongly enhanced Fermi liquid such as ^{3}He at high pressure, the usual arguments given originally by Berk and Schrieffer[8] show that it is essential to correct the particle-particle interaction responsible for superconductive pairing for the two kinds of particle-hole diagrams shown in Fig. 1. One can show that the resulting modification of the interaction I for triplet scattering is

$$[I_{\rm eff}(q,\omega)]_{\uparrow\uparrow} = \frac{1}{2}\left(\frac{I}{1+I\chi_{\rho\rho}{}^0(q,\omega)} - \frac{I}{1-I\chi_{zz}{}^0(q,\omega)}\right), \tag{1a}$$

$$[I_{\rm eff}(q,\omega)]_{\uparrow\downarrow} = \frac{1}{2}\left(\frac{I}{1-I\chi_{zz}{}^0} - \frac{2I}{1-I\chi_{xx}{}^0} + \frac{I}{1+I\chi_{\rho\rho}{}^0}\right) \tag{1b}$$

(related equations have been given by Layzer and Fay[6]). We distinguish the different components

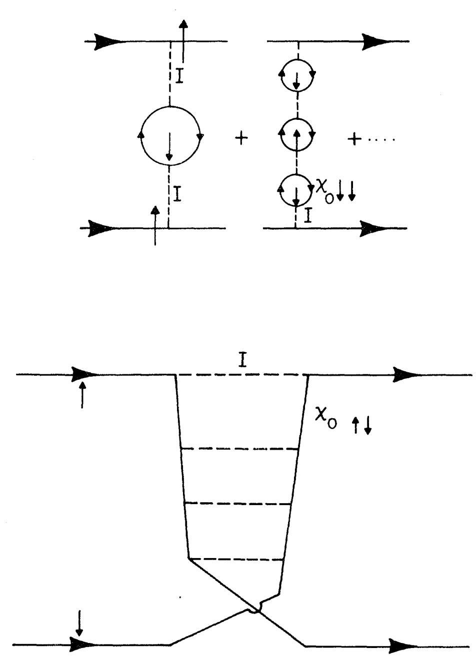

FIG. 1. The two types of diagrams that contribute to the effective interaction involving either a spin or charge-density fluctuation.

of the susceptibility because the anisotropy of the states considered will be essential. The last term is slowly varying and repulsive and we shall simply absorb it into V. $\chi_{\alpha\beta}{}^0(q,\omega)$ is the bare particle-hole susceptibility given by the product of particle and hole propagators:

$$\chi_{\alpha\beta}{}^0(q,\omega)=\tfrac{1}{2}\int d^4p\, G_{ii'}(p+q,p_0+\omega)G_{jj'}(p,p_0)\times\sigma_{ij}{}^{\alpha}\sigma_{i'j'}{}^{\beta}, \qquad (2)$$

and of course if there is an anomalous self-energy (i.e., if the system is superfluid), this must be included in the G's. The ρ indicates the density-density response function.

First we note that at T_c (1a) and (1b) are the same because $\chi_{zz}=\chi_{xx}=\chi_N{}^0$, and the result is a very large enhancement of T_c for triplet pairing. For instance, let us calculate the lth component of the effective interaction λ_l at $\omega=0$:

$$\lambda_l=N(0)V_l-\frac{\chi_N{}^0 I}{4}\int_{-1}^{1}d\mu\,\frac{P_l(\mu)}{1-I\chi_N{}^0(q,0)}, \qquad (3)$$

$$\mu=1-q^2/2k_F{}^2;$$

and making the assumption $1/[1-I\chi_N{}^0(0,0)]\gg 1$ which is characteristic of strong enhancement (this is the enhancement factor for the Pauli susceptibility which, in the simple theory, is about 20 at the melting curve in ^{3}He), we can write

$$\begin{aligned}\lambda_{l=1}(0)&=N(0)V_1+\frac{I\chi_N{}^0}{4k_F{}^2}\int\frac{q\,dq}{1-I\chi_N{}^0+\bar{I}q^2/12k_F{}^2}\\&=N(0)V_1+\tfrac{3}{2}\ln[(1-I\chi_N{}^0)^{-1}]\end{aligned} \qquad (4)$$

[defining $\chi_N{}^0=\chi_N{}^0(0,0)$]. The result for the paramagnon term is large. There is every reason to expect that the net effect of the ordinary interaction is also large and repulsive (negative V_1). The enhanced λ will disappear and the interaction become normal and repulsive at $\omega_s=E_F(1-I\chi_N{}^0)$. Using the experimental results for He^3 at 27 atm atm,[9,10] $1-I\chi_N{}^0=\frac{1}{20}$ and $\omega_s=E_F/20=0.3°K$, we find that $\lambda_{sf}=-4.5$, where λ_{sf} is the second term in Eq. (4). Defining an effective coupling constant as $\lambda_{eff}=[\ln(\omega_s/T_c)]^{-1}$ we find $\lambda_{eff}=\frac{1}{5}$. There is clearly a large cancelation of attractive and repulsive terms.

A remark should be made regarding the validity of the spin-fluctuation model being used. In the past few years most of the results of spin-fluctuation theory have been incorporated into the language of Fermi-liquid theory.[11] One of the results of this work is that the characteristic frequency involved in the spin fluctuations is the same experimental number, namely $\omega_s=(\chi_N/\chi_N{}^F)E_F$ (the superscript F denotes the noninteracting values) in both theories. Fermi-liquid theories do not attempt to calculate cutoff-sensitive results such as λ_{sf} above and therein lie the uncertainties (usually overestimates) of many calculations using spin-fluctuation theory. The estimates made below of the free-energy differences will not be so sensitive to this cutoff problem but there is some uncertainty as to how to properly interpret the result.

What we do now is to insert the change in χ due to the superfluidity into formulas (1) in order to compute λ_l to the first order in Δ^2. We examine the BW state and the original (AM) state which is equivalent to the state with $\vec{d}(\vec{k})=(k_x+ik_y)\hat{z}$, where $\vec{d}(\vec{k})$ is defined by BW as $\vec{d}(\vec{k})\cdot\vec{\sigma}\sigma_y=\Delta(\vec{k})$. This state is not the same as the AM-type state used by BW and subsequent work by one of us[3] in discussing this problem. Following standard approximations used by BW, plus techniques for evaluating $\chi(q)$ used elsewhere,[12] we write

$$\chi^0=\chi_N{}^0(q,\omega)-\delta\chi^0, \qquad (5)$$

$$\delta\overleftrightarrow{\chi}^0=[\chi_N{}^0K(q,\omega)/K(0)]\overleftrightarrow{\tau}(\Delta/T). \qquad (6)$$

Here $K(q,\omega)$ is the well-known BCS response function which enters into the nonlocal electrodynamics of superconductors:

$$\frac{K(q,\omega)}{K(0,0)}\simeq\frac{3\pi}{4q\xi_0},\quad \xi_0{}^{-1}\ll q\ll k_F, \qquad (7)$$

which is the relevant range, and ξ_0 is the coherence length $\hbar v_F/\pi\Delta_0$. $\overleftrightarrow{\tau}(\Delta)$ is a tensor function in spin space of the gap and the temperature, given by BW. The principal values are

$$\begin{aligned}&\tau_{BW}(\text{isotropic})=\tfrac{1}{3}[1-Y(\Delta/T)],\\&\tau_{AMxx}=\tau_{AMyy}=0,\\&\tau_{AMzz}=1-\langle Y(\Delta_0\sin(\theta)/T)\rangle,\end{aligned} \qquad (8)$$

where Y is Yoshida's function

$$\begin{aligned}Y(\Delta/T)&=\tfrac{1}{2}\beta\int_0^\infty d\epsilon\,\mathrm{sech}^2(\beta E/2)\\&\simeq 1-\tfrac{7}{8}\zeta(3)\Delta^2/\pi^2T^2\cdots.\end{aligned} \qquad (9)$$

When we insert (5) and (6) into (1), we find that the interaction of BW, since it is isotropic, can be written as

$$(\delta I)_{eff}\simeq\frac{I^2\,\delta\chi}{(1-I\chi_N{}^0)^2}\simeq\frac{\frac{1}{6}I^2(1-Y)K(q,\omega)/K(0,0)}{(1-I\chi_N{}^0)^2}.$$

This can be integrated to give an effective change in coupling constant:

$$\delta\lambda_1(\omega=0)=\frac{\Delta^2}{E_F T_c(1-I\chi_N{}^0)^{3/2}}\frac{7\zeta(3)\pi^2\sqrt{3}}{64\gamma}$$
$$\simeq\frac{0.1\Delta^2}{\omega_s T_c(1-I\chi_N{}^0)^{1/2}} \qquad (10)$$

(γ is Euler's constant). This change in coupling constant occurs over a range of energies of order T. For the AM state the interaction is given by (1b) and the reduction of $\chi_{zz}{}^0$ causes a reduction of the first term. The net effect is that $\delta\lambda_1$ is attractive and 3 times larger than (10) when written in terms of Δ, normalized so that the second-order terms in the free energies are equal.

The resulting perturbation in the free energies can be estimated by first perturbing the gap equation,

$$1=\int_0^{\omega_s}d\epsilon E^{-1}[\lambda+\delta\lambda(E)]\tanh(\tfrac{1}{2}\beta E), \qquad (11)$$

since the terms in this equation of order Δ^2 must have come from functionally differentiating the fourth-order terms in the free energy.[13] For the BW state,

$$(\Delta F^s)_{\rm BW}=\frac{0.1\Delta^4}{2\lambda\omega_s T_c(1-I\chi_N{}^0)^{1/2}}. \qquad (12)$$

In the AM state,

$$(\Delta F^s)_{\rm AM}=-3(\Delta F^s)_{\rm BW}. \qquad (13)$$

The fourth-order terms in the weak-coupling theory (ΔF^0) are such that

$$(\Delta F^0)_{\rm AM}-(\Delta F^0)_{\rm BW}\equiv(\Delta F^0)_{\rm AM\text{-}BW}$$
$$=\tfrac{7}{80}\zeta(3)\Delta^4/(\pi T_c)^3. \qquad (14)$$

Therefore, if we plot the free energy versus the ratio $(\Delta F^s)_{\rm BW}/(\Delta F^0)_{\rm AM\text{-}BW}$, we find that the two states cross and the AM state becomes stable when the ratio is greater than $\frac{1}{4}$. We have been able to show that the AM state is the most stable unitary state in this regime from general invariance arguments[14] for the fourth-order free energy. The state identified by BW as the AM state is never the lowest energy state. For He^3 near the melting curve, $\omega_s=0.3°$K, $1-I\chi_N{}^0=0.05$, $T_c=3.0$ mK, and $\lambda=\frac{1}{5}$, we obtain

$$\frac{(\Delta F^s)_{\rm BW}}{(\Delta F^0)_{\rm AM\text{-}BW}}\simeq\frac{8T_c}{\lambda\omega_s(1-I\chi_N{}^0)^{1/2}}=1.8$$

which is a factor of 7.5 times that needed to stabilize the AM state. If we had set $1-I\chi_N{}^0$ equal to the enhancement of the susceptibility over its value obtained using the Fermi-liquid mass,[9] i.e., $1-I\chi_N{}^0\simeq0.25$, the above estimate would have given 0.8. Recently, Webb *et al.*[15] have measured the discontinuity in the specific heat, which is a factor of 1.2 larger than BCS theory predicts. This enhancement is equivalent to a reduction of the fourth-order terms in the free energy. If we attribute this reduction entirely to the spin-fluctuation effect calculated here (BW being the same as BCS), we find a value of 0.6 for $(\Delta F^s)_{\rm BW}/(\Delta F^s)_{\rm AM\text{-}BW}$ compared to the 0.8 obtained above. This number is still sufficient to stabilize the AM state.

If the spin-fluctuation effects are strong, it is surprising that the B transition ever takes place (interpreting it, as we would like to, as an anisotropic to isotropic transition). It seems possible that in the "B" state χ has been considerably enhanced analogously to what occurs in a superconductor in the presence of magnetic impurities. In any case, the above arguments suggest that the "B" and "A" transitions should converge towards each other as the pressure decreases and spin fluctuations become less pronounced.

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Two-Roton Raman Scattering in He^3-He^4 Solutions

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The two-roton Raman spectra of superfluid solutions of He^3 and He^4 are measured at a temperature of 1.3 K for molar concentrations of He^3 up to 31%. At 31% He^3 concentration, the shift in the roton energy is inferred from these spectra to be $+0.3 \pm 0.5$ K, in striking contrast to the -3.5-K shift inferred from previous measurements of normal-fluid density. The width of the two-roton peak is resolved and is a measure of the He^3-roton interaction.

In spite of extensive study of the roton excitation in superfluid helium, many facets of its behavior remain to be explained. In particular, the effect of He^3 impurities on the phonon-roton dispersion relation of superfluid helium has been studied both experimentally and theoretically.[1-5] It has recently been predicted[4,5] that the addition of a few percent He^3 would drastically modify the phonon-roton curve by essentially hybridizing the phonon-roton branch and the He^3 quasiparticle branch. Experiments involving fourth-sound propagation,[1] ion mobilities,[2] and an "oscillating disk-stack" measurement of normal-component density[3] have been used to infer a significant decrease in the roton energy Δ in superfluid solutions of He^3 and He^4. Specifically, Δ was found to decrease from 8.5 to 5 K when the molar concentrations of He^3 was increased from 0 to 30%. These experiments, however, do not investigate the roton directly, but measure quantities such as normal-fluid density. It is, therefore, particularly desirable to study the roton in these solutions with a more direct probe. Since neutron scattering is difficult in strong solutions of He^3 because of the large nuclear absorption cross section, inelastic light scattering is a unique choice for such a study. Since the light has a small wave vector compared to that of the roton, light couples only to pairs of rotons with nearly equal and opposite wave vector. The high density of states near the roton minimum produces a sharp peak in the spectrum of the scattered light at a frequency shift corresponding to the energy of the roton pair.[6] We report here measurements of two-roton Raman spectrum in He^3-He^4 solutions containing up to 31% molar concentration of He^3. In striking contrast to the work referred to above, we infer an increase of 0.6 ± 1 K in the energy necessary to create a pair of rotons. An increase in the two-roton linewidth due to the He^3-roton interaction was also measured and was found to be approximately linear with He^3 concentration.

The experimental apparatus has been described previously,[7] except that for these experiments the sample cell was made from beryllium-copper. An argon-ion laser beam at 5145 Å with power of 100 to 200 mW is focused through indium-sealed 0°-sapphire windows into the sample cell. Depolarized scattering at 90° to the incident beam is collected with $f/3$ optics, analyzed with a 0.75-m double-grating spectrometer, and detected with a cooled Channeltron photomultiplier. The noise in the spectra is predominantly statistical arising from the low level of scattered light (~10 counts per second per resolution interval). The possibility that laser heating might cause "heat flush" effects and thereby decrease the concentration of He^3 in the laser beam was checked by observing that the spectrum of scattered light is independent of laser power when the sample cell is maintained at a fixed temperature.

In Fig. 1 is shown the Stokes-shifted Raman spectrum of superfluid helium for several concentrations of He^3. These spectra were taken at a temperature of 1.30 K with an instrumental resolution [half width at half-maximum (HWHM)] of 0.75 cm^{-1}. The peak at a frequency shift of approximately 17.5 K is due to scattering from