(1973).
[6]A. Bodek, Ph.D. thesis, Massachusetts Institute of Technology, 1973 (unpublished).
[7]E. M. Riordan, Ph.D. thesis, Massachusetts Institute of Technology, 1973 (unpublished).
[8]E. M. Riordan *et al.*, to be published.
[9]J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971).
[10]O. Nachtman, Phys. Rev. D 5, 686 (1972).
[11]J. W. Moffat and V. G. Snell, Phys. Rev. D 3, 2848 (1971).
[12]G. Domokos *et al.*, Phys. Rev. D 3, 119 (1971).
[13]S. D. Frell and T. D. Lee, Phys. Rev. D 5, 1738 (1972).
[14]E. D. Bloom and F. J. Gilman, Phys. Rev. D 4, 2901 (1971).
[15]P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B28, 240 (1971).

Observations on $\bar{p}d$ Annihilations at Rest into Two Pions

L. Gray, Theo. Papadopoulou, E. Simopoulou, and A. Vayaki
Nuclear Research Center Democritos, Aghia Paraskevi Attikis, Athens, Greece

and

T. Kalogeropoulos and J. Roy
*Department of Physics, Syracuse University, Syracuse, New York 13210**
(Received 9 February 1973)

The ratio $(\bar{p}d \to \pi^-\pi^0 p_s)/(\bar{p}d \to \pi^-\pi^+ n)$ at rest, with spectator momenta $\lesssim 300$ MeV/c, has been measured and found to be 0.68 ± 0.07. This implies that $(75 \pm 8)\%$ of the annihilations in deuterium into two pions come from odd $\bar{N}N$ orbital angular momenta, in disagreement with the S-state–dominance hypothesis. It has also been observed that the $\bar{p}d \to \pi^-\pi^0 p_s$ production rate depends on the spectator momentum, which suggests energy-sensitive $\bar{N}N$ phenomena near threshold.

A fraction of our $\bar{p}d$ film, obtained using the 30-in. Brookhaven National Laboratory deuterium bubble chamber exposed to a stopping antiproton beam, has been analyzed to study the reactions

$$\bar{p}d \to \pi^+\pi^- n_s, \quad (1)$$

$$\bar{p}d \to \pi^-\pi^0 p_s. \quad (2)$$

One part of the film, containing 2.5×10^5 annihilations, has been double scanned for these reactions concurrently, using the following acceptance criteria: (a) events with a single negative track having a projected length >15 cm and a projected momentum >500 MeV/c in all three views (*one-prong*); (b) events with two tracks, the negative one satisfying the criteria as in (a) and the positive being a stopping proton (*two-prong*); (c) events with two tracks, both of them satisfying the criteria as in (a) and, in addition, having a projected opening angle >164° (*collinear*). These criteria have been chosen so that the efficiencies for detecting Reactions (1) and (2) are independent of the spectator momenta up to ~300 MeV/c, while they reduce substantially the measuring effort. In addition, another part of the film was scanned and measured for one- and two-prong events without the momentum cut. These events are included in the analysis and are also used to normalize the high-momentum spectra obtained with the criteria (a) and (b) to the total number of one- and two-prong events. Reactions (1) and (2) have been identified on the basis of the measured momenta as follows.

Reaction (1).—In Fig. 1(a) the invariant mass squared (M^2) of the collinear tracks assumed to be pions is displayed versus the missing momentum $p_m = |\vec{p}_+ + \vec{p}_-|$. The signal for Reaction (1), centered at high M^2 and low (spectator) missing momentum, is well separated. The background is negligible for $M^2 > 2.95$ GeV^2 [Fig. 1(b)] and these events (305) are considered to belong to Reaction (1).

The $\pi^\pm$ momentum distribution of the $\pi^+\pi^- n$ events is shown in Fig. 1(d) and is fitted well by a Gaussian. The center of the peak (919 MeV/c) is in good agreement with the expected position for Reaction (1) at rest. The width (±53 MeV/c) is the combination of the measurement error and the uncertainty introduced by the unseen neutron. This uncertainty is ±17 MeV/c for a spectatorlike distribution and consequently the width is essentially due to the measurement errors. The missing momentum of the $\pi^+\pi^-$ (neutron momentum) is shown in Fig. 1(c). It fits

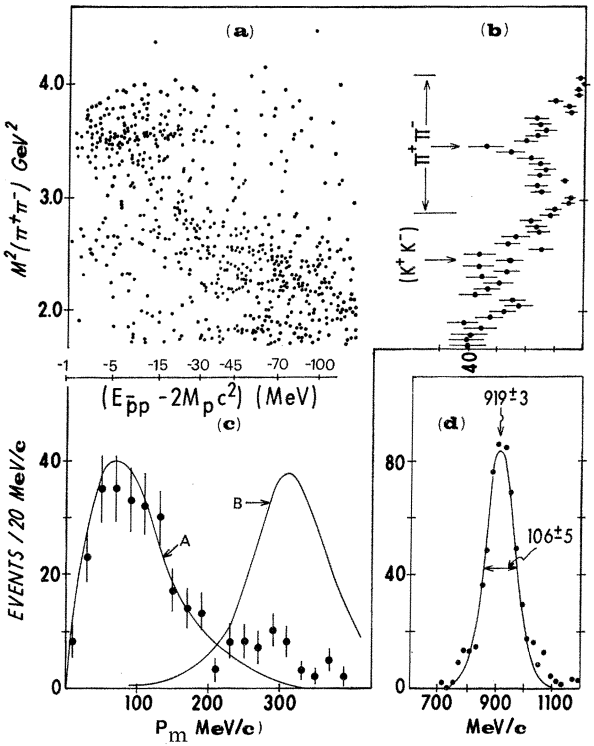

FIG. 1. (a) Collinear events. (b) Projection of (a). (c) Projection of (a) with $M^2(\pi^+\pi^-) > 2.95$ GeV^2. Curves A and B are Hulthén-like predictions including measurement errors for at-rest (A) and 300-MeV/c in-flight (B) annihilations. (d) $\pi^{\pm}$ momentum spectra for the $\pi^+\pi^-$ events ($M^2 > 2.95$ GeV^2).

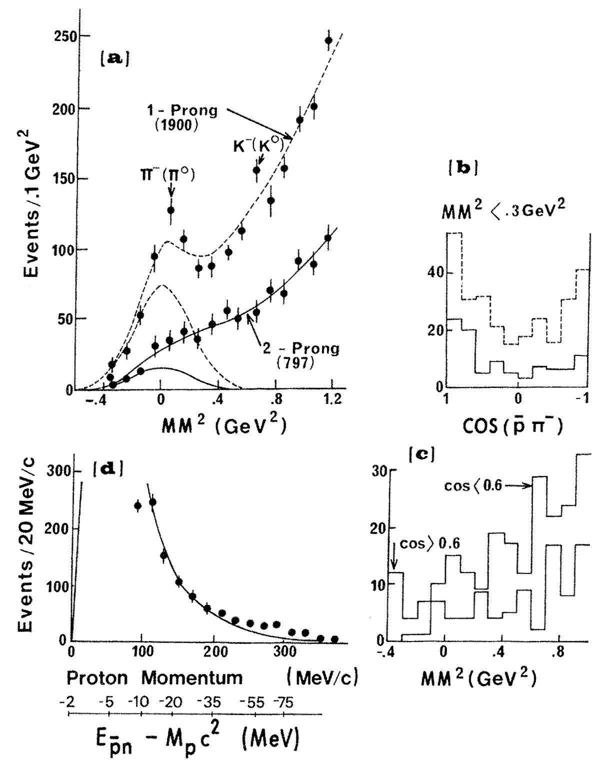

FIG. 2. Dashed lines, one-prong, and solid lines, two-prong events. (a) Missing-mass-squared spectra. Curves are fits to multiple π^0 reactions (see text). The Gaussian curves represent the amount of Reactions (1) and (2) found by the fit.

reasonably well a spectator distribution below 200 MeV/c if measurement errors are taken into account and assuming at-rest annihilations. The angular distribution of the π^- with respect to the beam is symmetric around 90° as expected for at-rest annihilations. Therefore these 305 ± 17 *events belong to Reaction (1) at rest.*

Reaction (2).—In Fig. 2(a) the $(MM)^2 = (M_d + M_{\bar{p}} - \omega_\pi - \omega_s)^2 - (\vec{p}_\pi + \vec{p}_s)^2$ distributions for the one- and two-prong events are shown below 1.2 GeV, where ω_π, ω_s are the π^- and p energy and $\vec{p}_\pi$, $\vec{p}_s$ their momenta. We found by comparing the spectra with and without the projected momentum cut that the spectrum below 1.2 GeV^2 is not affected by our criteria. In the case of the one-prong event the momentum of the unseen proton ($\vec{p}_s$) is set equal to zero. This approximation increases $\Delta(MM)^2$ but, as found on the basis of a spectator-like distribution, its contribution is negligible in comparison to the measurement error on the negative track. The signal at $m_{\pi^0}{}^2$ is clear in the one-prong, but *its presence is uncertain in the two-prong events.*

We have performed fits on these distributions assuming that they are generated by $\pi^- n\pi^0$ ($n = 1, 2, 3, 4$). The $n = 1$ contribution [Reaction (2)] has been found using a Gaussian centered at $m_{\pi^0}{}^2$ and a width of ± 0.20 GeV^2 which has been determined from the known π^- momentum error. The contributions for $n = 2, 3, 4$ have been found using phase-space distributions in which the π^--momentum measurement errors have been taken into account. In the interval $-0.4 < (MM)^2 < 1.2$ GeV^2, the results of these fits give 347 ± 19 (18%), 738 ± 38 (39%), 586 ± 60 (31%), and 227 ± 27 (12%) events for one-prong and $n = 1, 2, 3, 4$, respectively. The corresponding numbers for two-prong events are 78 ± 19 (10%), 407 ± 71 (51%), 191 ± 124 (24%), and 119 ± 83 (15%). Other fits have been performed as well with variable width, different $(MM)^2$ intervals, and second-order polynomials as background. All these fits are consistent with the results presented above, particularly regarding the number of $\pi^-\pi^0$ events and the width. Furthermore the ratios $\pi^-2\pi^0 : \pi^-3\pi^0 : \pi^-4\pi^0 = 1:2:2.7$ are reasonable as compared with the statistical-model predictions of 1:1.7:1.

The fitted width to the $\pi^-\pi^0$ events is $\pm 0.20 \pm 0.02$ GeV^2, in agreement with the calculated one from the collinear events at rest. The width would become ± 0.38 and ± 0.55 GeV^2 if annihilations in flight—200 and 300 MeV/c, respectively—were treated as being at rest. The angular distributions of the π^- with respect to the beam for the one- and two-prong events with $(MM)^2 < 0.3$ GeV^2 are displayed in Fig. 2(b). The one-prong events are symmetric about 90° while we observe an asymmetry 0.32 ± 0.16 for the two-prong resulting from an enhancement at $\cos\theta_{\bar{p}\pi^-} > 0.6$. The $(MM)^2$ for events with $\cos\theta_{\bar{p}\pi^-} > 0.6$ and $\cos\theta_{\bar{p}\pi^-} < 0.6$ are displayed in Fig. 2(c). It is obvious that the $\pi^-\pi^0 p_s$ two-prong events are not connected to the observed asymmetry. We thus conclude that the $\pi^-\pi^0$ events are coming from annihilations at rest.

Charge independence relates[1] the $\bar{p}d \to 2\pi N$ branching ratios via

$$f(\pi^+\pi^- n) = \tfrac{1}{2} f(\pi^-\pi^0 p) + 2f(2\pi^0 n). \qquad (3)$$

Statistics of the pions and parity conservation imply that $I(\bar{N}N) = 0$ corresponds to $L(\bar{N}N) = \text{odd}$ and $I = 1$ to $L = \text{even}$. Since the $2\pi^0$ come from $I = 0$ ($L = \text{odd}$), Eq. (3) implies $f(\pi^-\pi^0 p)/f(\pi^+\pi^- n) = 2$ for S-capture dominance. From our data, after correcting for relative $(\pi^+\pi^-/\pi^-\pi^0)$ scanning (1.10 ± 0.03) and processing (1.08) efficiencies for Reactions (1) and (2), we find 245 ± 25 $\pi^-\pi^0 p$ and 362 ± 25 $\pi^+\pi^- n$ events corresponding to the same number of $\bar{p}d$ annihilations (the observed $\pi^-\pi^0 p$ have been scaled down to the first sample). Therefore

$$f(\pi^-\pi^0 p)/f(\pi^+\pi^- n) = 0.68 \pm 0.07. \qquad (4)$$

This is clearly in contradiction to the decade-long S-capture–dominance hypothesis.[2] Notice from Figs. 1(c) and 2(d) that the ratio (4) has been determined for annihilations with spectator momenta $\lesssim 300$ MeV/c. The same conclusion has been reached also by Devons *et al.*[3] from the ratio $(\bar{p}p \to 2\pi^0)/(\bar{p}p \to \pi^+\pi^-)$. The difficulties inherent in the study of $2\pi^0$, as well as the use of branching ratios for determining $2\pi^0$ and $\pi^+\pi^-$ independently, may generate some skepticism of this result which has been expressed as follows:

$$[(\bar{p}p)_{I=0(L=\text{odd})}{}^{\text{H}} \to 2\pi]/[(\bar{p}p)^{\text{H}} \to 2\pi]_{\text{all}} = 0.40(\pm 20\%). \qquad (5)$$

Using (3) and the observed number of $\pi^+\pi^-$, $\pi^-\pi^0$ events, we find 120 ± 15 $2\pi^0$ events and, following Ref. 3,

$$[(\bar{p}p)_{I=0(L=\text{odd})}{}^{\text{D}} \to 2\pi]/[(\bar{p}p)^{\text{D}} \to 2\pi]_{\text{all}} = 0.75 \pm 0.08 \qquad (6)$$

which not only confirms the conclusions reached in Ref. 3 but shows that the majority of the 2π in deuterium come from odd $\bar{N}N$ angular momenta.

The result (6) is larger than (5) and this may be a reflection of instrumental uncertainties in the result (5), or the increase may be real, due to either the dependence of the capture probability on spectator momentum[4] or to resonances (bound states) near threshold.[5] We have examined the following aspects of our data in search of effects on spectator momentum or equivalently on $\bar{N}N$ energy [see scales in Fig. 1(c) and 2(d)]. (a) We have estimated that $(\bar{p}d \to \pi^+\pi^- n_s)/(\bar{p}d \to \text{all} + n_s) = (42 \pm 12) \times 10^{-4}$, which compares well within the statistical errors to $(\bar{p}p \to \pi^+\pi^-)/(\bar{p}p \to \text{all}) = (32 \pm 3) \times 10^{-4}$.[6] (b) The neutron spectrum of (1), Fig. 1(c), fits reasonably well to a spectatorlike distribution, including measurement errors, but the resolution is comparable to the width of the distribution. (c) When the spectrum of the two-prong events is extrapolated below $\lesssim 80$ MeV/c using a Hulthén-like distribution the number of one-prong events observed is underestimated by 28% after taking into account scanning and processing efficiencies. *This result indicates a preference for low* ($\lesssim 80$ MeV/c) *spectator momenta.* (d) The production ratios of (2) among one- and two-prong events with $(MM)^2 < 1.2$ GeV^2 are 0.18 ± 0.01 and 0.10 ± 0.02, respectively. These ratios differ by $\sim 4\sigma$, indicating that *the production of (2) decreases with increasing spectator momentum or equivalently decreasing $\bar{N}N$ energy.* In view of the uncertainties regarding the "observation" of (2) this effect is more striking than these numbers suggest and similar to the one discussed in (c).

These effects are so sensitive to spectator momentum that they might be difficult to understand in terms of kinematical effects in deuterium.[4] On the other hand, they can easily be understood in terms of narrow resonances or bound states.[7] We find, on the basis of the branching ratios given in (d) and following Bogdanova, Dal'karov, and Shapiro,[8] that $\Gamma \lesssim 10$ MeV and $B \lesssim 7$ MeV and only solutions with $L(\bar{N}N) = 0$ are possible, implying a narrow $\bar{N}N$ bound state in ${}^3S_1(I=1)$. If, however, the number of the two-prong $\pi^-\pi^0 p$ events is overestimated, then higher L values and smaller upper limits on the width and binding energy are possible.

One of us (T.E.K.) is grateful to President P. Demopoulos for his support and encouragement during his sabbatical at Democritos. Without his support this work might not have been possible. We are also grateful to R. Bizzarri, G. Moneti, and T. Filippas for interesting comments.

*Work supported in part by the National Science Foundation.

[1] H. J. Lipkin and S. M. Peshkin, Phys. Rev. Lett. 28, 862 (1972).

[2] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Lett. 3, 61 (1959); P. Anninos *et al.* Phys. Rev. Lett. 20, 402 (1968).

[3] S. Devons, T. Kozlowski, P. Nemethy, S. Shapiro, N. Horwitz, T. Kalogeropoulos, J. Skelly, R. Smith, and H. Uto, Phys. Rev. Lett. 27, 1614 (1971).

[4] R. Bizzarri, CERN Report No. 72-10, 1972 (unpublished), p. 161.

[5] T. E. Kalogeropoulos, CERN Report No. 72-10, 1972 (unpublished), p. 319.

[6] R. Armenteros *et al.*, Phys. Rev. Lett. 17, 344 (1965); C. Baltay *et al.*, Phys. Rev. Lett. 15, 532 (1965).

[7] L. Gray, P. Hagerty, and T. Kalogeropoulos, Phys. Rev. Lett. 26, 1491 (1971).

[8] L. N. Bogdanova, O. D. Dal'karov, and M. S. Shapiro, Phys. Rev. Lett. 28, 1418 (1972).

Intercept of the Pomeranchuk Singularity*

N. N. Khuri
Rockefeller University, New York, New York 10021
(Received 26 March 1973)

For any unitary theory in which the average multiplicity grows slower than any small positive power of s, and in which the Regge-type asymptotic behaviors $\sigma_{tot}(s) \sim s^{\alpha_P(0)-1}$ and $\sigma_{el}(s) \sim s^{2\alpha_P(0)-2}$ hold, modulo powers of $\ln(s)$ for both, I derive a simple, necessary and sufficient condition which guarantees that $\alpha_P(0)=1$. The condition is $\max_n \sigma_n \leq C\sigma_{el} s^\epsilon$ for arbitrarily small $\epsilon > 0$ and large enough s, where σ_n is the actual cross section for producing n hadrons in the final state, σ_{el} is the total integrated elastic cross section, and C is independent of ϵ.

The reason why the Pomeranchuk singularity has an intercept close to unity is one of the central problems of strong interaction dynamics. Recently, there have been several interesting papers that propose answers to this question in the context of certain specific models.[1-3]

The total cross section is given by the trivial relation

$$\sigma_{tot}(s) = \sum_n \sigma_n(s), \tag{1}$$

where σ_n is the cross section for producing n hadrons in the final state. As stressed by Harari,[1] experimentally the various σ_n's vary in a dramatic way over the same range of energy in which $\sigma_{tot}(s)$ is roughly constant [modulo powers of $\ln(s)$]. The problem then is to understand how a sum of highly variable σ_n's produces a σ_{tot} that is energy independent and leads to the condition $\alpha_P(0)=1$.

This paper looks at this problem from a general point of view and *not* in the context of a specific model. General and simple conditions on σ_n are sought in addition to unitarity, which guarantee that $\alpha_P(0)=1$. The conditions found are two: The first states that no matter how the σ_n's vary with s, no $\sigma_n(s)$ should equal or exceed const $\times \sigma_{el}(s)s^\epsilon$, where $\sigma_{el}(s)$ is the integrated total elastic cross section for $ab \to ab$. The second condition demands that the average multiplicity $\langle n\rangle$ increase slower than any positive power of s. More specifically, for large enough s the conditions are given by the following two simple inequalities[4]:

$$\sigma_n(s) \leq C\sigma_{el}(s)\exp[(\ln s)^{1-\epsilon}], \quad n \geq 2, \tag{1a}$$

$$2 \leq \langle n\rangle \leq \exp[(\ln s)^{1-\epsilon}], \tag{1b}$$

where $0<\epsilon<1$, and C is independent of n. I shall show that (1a), (1b), and unitarity do indeed imply that $\alpha_P(0)\equiv 1$.

At the end of this note I shall also discuss (1a) and (1b) in the context of the specific models of Refs. 1–3 and show that both inequalities are indeed trivially a feature of these models. I shall also point out examples of models where (1a) is not satisfied and hence $\alpha_P(0)\neq 1$. In making the comparison with the models a problem arises because none of the models make any statements