made definitely; (2) the broad bump in the region from 16 to 27 MeV is contributed to by at least $E2$ (or $E0$) excitation at ~22 MeV and $E3$ excitation at ~19 MeV, the sum rule of which both are nearly exhausted.

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Pion-Nucleus Scattering in an Isobar-Doorway Model*

L. S. Kisslinger and W. L. Wang
Department of Physics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 12 March 1973)

Elastic pion-nucleus scattering is treated using a separation in Hilbert space via a projection-operator formalism. The basic assumption is that the elastic scattering and reactions due to the π-N resonance proceed through doorway states consisting of a $\Delta(1236)$ resonance in the nucleus. Nonresonant scattering and absorption processes are included by background terms, which are approximately known. The formalism is applied to π^--^{12}C scattering.

Although there are qualitatively successful calculations of pion-nucleus elastic scattering near the energies of the $\Delta(1236)$ π-nucleon resonance ($T_\pi \approx 150$–250 MeV),[1-4] there are many theoretical difficulties with the present theories and no truly quantitative treatment. We propose a new approach which offers a framework in which one can conveniently study most of these difficulties and express the results in terms of physical quantities. This is the isobar-doorway model, in which one treats the states of the π-N resonance (the isobar) in a nucleus as doorways for entrance into all inelastic nuclear states, plus a background of nonresonant scattering and absorption.

As we shall show, the modification of the binding energy when a nucleon is replaced by a resonance appears in an unambiguous way in the present theory. There has been considerable confusion regarding this quantity, which is only remotely related to the shift in the maximum of the π-nucleus cross section as compared to the π-nucleon cross section, plotted against energy.

As a starting point, the π-nucleon states are separated into the resonant $J=\frac{3}{2}$, $T=\frac{3}{2}$ state [the $\Delta(1236)$] and all other (nonresonant) states. This leads to a separation of the states of the π-nucleus system into subspaces. We accomplish

this via the projection-operator formalism, as has been found quite useful in nuclear reaction theory.[5] The three subspaces which we define here are (1) the P space, where a pion is in a scattering state and the nucleus is in its ground state, (2) the D space of states of one isobar in the nucleus (isobar compound states), and (3) the remaining subspace Q of all other inelastic states. The projection operators P, D, and Q project onto these subspaces, respectively.

Following standard techniques, the Schrödinger equation, $(E-H)|\psi\rangle=0$, can be separated into three coupled equations. By reduction, the Schrödinger equation for the elastic channel is

$$[E-\tilde{H}_{PP}-\tilde{H}_{PD}(E-\tilde{H}_{DD})^{-1}\tilde{H}_{DP}]|P\psi\rangle=0, \quad (1)$$

with the effective Hamiltonian $\tilde{H}=H+HQ(E-H)^{-1}QH$, and $\tilde{H}_{PD}=P\tilde{H}D$, $H_{QQ}=QHQ$, etc. Equation (1) is an exact consequence of the Schrödinger equation. This equation is considerably simplified with the assumption that $H_{PQ}=0$, the basic assumption of our theory. This is the *doorway hypothesis*, which allows no inelastic process to directly couple to the π-nucleus elastic channel through nonresonant interactions.

It is most useful to derive the optical potential corresponding to this model, since the potential provides us with the nonasymptotic part of the pion wave function needed in calculations of inelastic reactions. It also allows one to include Coulomb distortion, background scattering, and absorption conveniently. The optical potential corresponding to Eq. (1) is

$$V=V^{NR}+\sum_D \frac{\langle\Phi_0|H_{PD}|\varphi_D\rangle\langle\varphi_D|H_{DP}|\Phi_0\rangle}{E-E_D-\langle\varphi_D|H_{DQ}(E-H_{QQ})^{-1}H_{QD}|\varphi_D\rangle}, \quad (2)$$

where V^{NR} is the background potential contained in H_{PP}, $|\Phi_0\rangle$ is the nuclear ground state. The isobar compound states (the doorway states) $|\varphi_0\rangle$ and their energy E_D are defined by $(E_D-\tilde{H}_{DD})|\varphi_D\rangle=0$. Note that the doorways are coupled through the Q space, and that they are stable to π decay in the absence of the H_{PD} and H_{QD} interactions. V^{NR} is the potential without the π-N resonance.

Using the closure approximation for (2), one may write the optical-model potential as

$$V=V^{NR}+\mathfrak{N}\alpha(E)\tilde{t}_{33}(\vec{k}',\vec{k};E)F_{00}(\vec{k}',\vec{k}), \quad (3)$$

where

$$\alpha(E)=\frac{[E-\bar{E}_D+\frac{1}{2}i\Gamma_{\pi N}(E)]}{[E-\bar{E}_D+\frac{1}{2}i\Gamma_{\text{in}}(E)]},$$

$F_{00}(\vec{k}',\vec{k})$ the nuclear form factor, $\Gamma_{\pi N}(E)$ and $\Gamma_{\text{in}}(E)$ the $\Delta(1236)$ decay and inelastic widths, respectively, $\tilde{t}_{33}$ the resonance amplitude evaluated in the medium, and $\bar{E}_D$ an average doorway state energy. The pion momentum in the π-nucleon (π-nucleus) center-of-mass system is indicated by $\vec{k}$ ($\vec{k}$). The numerical factor $\mathfrak{N}$ is the $N+Z/3$ ($Z+N/3$) for π^- (π^+) scattering. In obtaining (3) we have assumed that the *interaction* is unmodified in the medium, *not* the impulse approximation.

Note that interactions in the D space shift $\bar{E}_D$ from the unperturbed energy $E_0\equiv M(1236)-M(\text{nucleon})+$the nuclear ground-state energy. The *total* energy shift in the denominator of (2), which we refer to as δE, equals $\bar{E}_D-E_0$ plus the additional energy shift from coupling to Q space. This quantity δE is the difference between the Δ and the nucleon effective binding. The resonance energy is therefore at $E_R=E_0+\delta E$. We take $\tilde{t}$ to be

$$\tilde{t}_{33}(\vec{k}',\vec{k};E)=\frac{\Gamma_{\pi N}(E)/2}{E-M(1236)-\delta E+i\Gamma_{\pi N}(E)/2}\hat{k}'\cdot\hat{k}. \quad (4)$$

This form assumes that the width is not directly modified by multiple interactions; it is not a basic approximation and is not critical for the conclusions of the present work. With Eqs. (3) and (4) we obtain an optical-model potential of the usual gradient form[6]:

$$V(r)=b_0(E)\rho(r)-\alpha(E)b_1(E)\nabla\cdot\rho(r)\nabla, \quad (5)$$

where b_0 and b_1 are related to π-nucleon scattering amplitudes, with a modified $\tilde{t}_{33}$ as given by Eq. (4), and ρ is the nuclear density.

The optical potential which we obtain is not a first-order potential, but contains some multiple-scattering corrections to all orders. However, the simple form (5) follows from the closure approximation and from Eq. (4). Without closure, the nuclear density $\rho(r)$ in (5) will be replaced by a nonlocal operator. A more detailed treatment, using a better treatment of the off-shell behavior and of the width and form factor, will result in further modification. Also, the transformation to the π-nucleus system will result in some modification in what we have called the nonresonant potential.[1] Note that with the form (5) numerical calculations can be done with the standard optical-potential computer code, ABACUS-M.[1] Before we present the results, let us turn to the T-matrix formalism. Within the same approximations

as discussed above, the T matrix can be written

$$T(\vec{k}', \vec{k}) = T^{NR}(\vec{k}', \vec{k}) + \frac{\frac{1}{2}\mathfrak{N}\Gamma_{\pi N}(E)\hat{k}' \cdot \hat{k} F_{00}(\vec{k}', \vec{k})}{E - M(1236) - \delta E + \frac{1}{2}i[\Gamma_{el}(E) + \Gamma_{in}(E)]}, \quad (6)$$

where Γ_{el} is the total elastic width, T^{NR} is the background scattering T matrix corresponding to V^{NR}. In this preliminary study, the off-shell distortions in the resonant term have been neglected; this removes the question of the off-shell behavior, such as the one assumed in Eq. (4). The distortion is approximately incorporated by including a factor $\exp(2i\delta_l^{NR})$ (due to V^{NR}) for each partial wave l in the resonant term. We feel, however, that with the parameter $\Gamma_{tot} = \Gamma_{el} + \Gamma_{in}$ yet undetermined in Eq. (6), such a refinement may not be informative.

In the following we discuss two calculations which have been performed in this model. In the optical-potential formalism, we assume $\alpha(E) = 1$ in Eq. (5) and study the modification due to δE; this investigation is meaningful since the important feature of the parameter b_1 is not very sensitive to the value of $\Gamma_{in}(E)$. The result is best demonstrated in Fig. 1, which shows the parameter $b_1(E)$ as obtained with and without an energy shift $\delta E = 11$ MeV. It is interesting to see that such an energy shift gives the modified values of $b_1(E)$ quite close to the "best-fit" parameters.[7] The differential cross sections using the optical potentials are shown as solid and dash-dotted lines in Fig. 2. The optical potential corresponding to $\delta E = 11$ MeV gives a marked improvement at all energies.

It should be noted that $\delta E = +11$ MeV corresponds to the $\Delta(1236)$ being approximately unbound. The lack of binding of the baryon resonance could be explained by the absence of a direct interaction and the presence of only an exchange interaction.[8] The interpretation is that the direct interaction is diminished for particles which are unstable under strong interactions. The Pauli principle also tends to move the resonance up,[9] but for finite nuclei the effect is much smaller than for uniform nuclear matter; and at the energy of the resonance might even be negli-

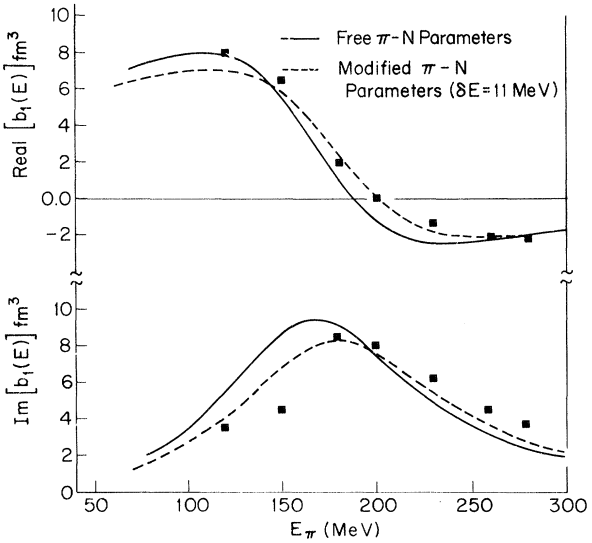

FIG. 1. Modification of the parameter $b_1(E)$ in the optical potential, Eq. (4), due to the shift of resonance energy, δE. Solid line, obtained directly from the free π-nucleon scattering amplitudes; dashed line, for the π-nucleon scattering amplitudes in the nuclear medium, as modified by an energy shift $\delta E = 11$ MeV. Squares, best-fit parameters.

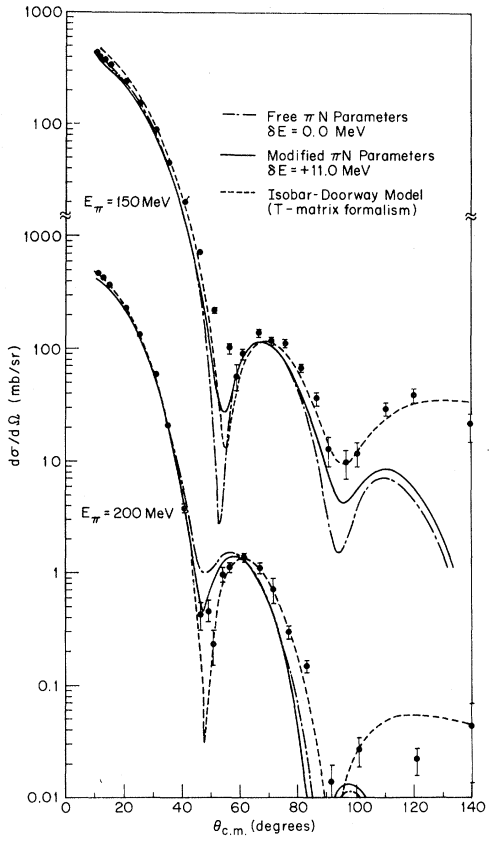

FIG. 2. Example of $\pi^- + {}^{12}C$ differential cross section at two energies near the (3, 3) resonance. The data are from Ref. 10. Dash-dotted and solid lines, results of the optical-model calculations using free $b_1(E)$ parameters and the modified $b_1(E)$ parameters as shown in Fig. 1, respectively. Dashed line, result of the T-matrix formulation of Eq. (6).

gible (see Bethe, Ref. 3).

We next show the results of our T-matrix calculation. The energy shift is taken to be 11 MeV; this quantity has only a small effect in the T-matrix calculation. One can choose the width Γ_{tot} to reproduce the experimental total cross sections.[10] With such a choice of Γ_{tot}, the T matrix is completely defined. The resulting differential cross sections are shown as the dashed lines in Fig. 2. The agreement is generally quite good, except that there is a discrepancy near the first minimum where the interference between nonresonant and resonant amplitudes is *very* important. The distortion of the pion wave function, as neglected in (6), may give some improvement.

For a detailed study of the differential cross section, especially at the backward angles for both the optical-potential and T-matrix calculations, one needs an improved treatment of the off-shell t matrix, the form factors, the transformation to the π-nucleus system, and so forth. These can all conveniently be included within the framework of the present theory.

It might be worthwhile to point out some features related to the interpretation of the (3, 3) resonance in the nucleus. The maximum in the total cross section is at about 150 MeV. The zero of the real part of our T matrix vanishes at about 175 MeV, while the "resonance energy" is 200 MeV. Thus our T-matrix formulation allows us to extract this many-body energy, $\bar{E}_D$, in the presence of an important background. For a more accurate estimate of this energy, and for consistency with the optical-potential calculation, the distortion due to V^{NR} must be included in the resonant part of the T matrix.

We would like briefly to compare the methods used here to other work on π-nucleus scattering. Other recent work makes use of optical potentials,[1] the Glauber approximation,[2] the concept of the index of refraction,[3] or deals directly with a multiple-scattering series.[4] Since the most important new information one will learn will involve corrections to the impulse approximation, and will deal with large-angle scattering, the methods of this paper should be considerably superior to the Glauber approximation. Also, for dealing with finite nuclei, the present method has a much sounder theoretical basis than a theory making use of an index of refraction for a uniform medium. Here one should note the strong disagreement between our results for the difference between the $\Delta(1236)$ and nucleon binding, δE, in comparison with the result of Barshay, Rostokin, and Vagradov.[3] One cannot directly connect the energy at which the cross section is maximum with the binding of the resonance. As we have shown, these are only weakly related (see Locher, Steinmann, and Straumann[2]). Even the zero of Re(T) cannot be directly used to estimate the Δ binding since this is strongly modified by the background contribution. The clear separation of background from resonance is an essential feature which enables us to extract information about the dynamics of the baryon resonance in nuclei.

The present work will be most useful in providing an alternative optical potential, which includes dynamic effects difficult to include in the optical models derived from a multiple-scattering approach. We would like to emphasize here that the present theory is able to handle the concept of binding even for a particle that is so unstable that it cannot propagate very far in nuclei. Although it is difficult to calculate the binding energy, modified form factor, and widths needed for the potential derived here—so that a potential derived from a multiple-scattering series might prove to be more accurate—it will be most informative to fit the parameters of the present theory to data, since they provide important new insight on the nature of the strong interactions. Moreover, a detailed microscopic calculation is possible, and could result in a truly quantitative treatment.

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Evidence for Recoil Effects in Heavy-Ion Transfer Reactions*

D. G. Kovar, B. G. Harvey, F. D. Becchetti, J. Mahoney, D. L. Hendrie, H. Homeyer,† W. von Oertzen,‡ and M. A. Nagarajan

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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Data are presented for single-proton transfer reactions induced by ^{12}C and ^{16}O ions on ^{208}Pb at high bombarding energies. Analysis using a no-recoil distorted-wave Born approximation (DWBA) yields spectroscopic factors in marked disagreement with light-ion results. Analysis using DWBA theory including first-order recoil corrections indicates that the discrepancies are largely, if not entirely, the result of ignoring recoil effects.

At present it is the uncertainties in reaction theories for heavy ions which have, to a large extent thus far, limited the spectroscopic information obtainable from heavy-ion transfer reactions.[1,2] These uncertainties mainly concern the question of the validity of some of the approximations used in the distorted-wave Born approximation (DWBA),[3-5] in particular the "no-recoil" approximation. The DWBA transition amplitude for a transfer reaction

$$(c_1+p)+c_2 \rightarrow c_1+(c_2+p) \tag{1}$$

can be written as (with a_1 for c_1+p and a_2 for c_2+p)[4]

$$T=\int d^3r_i \int d^3r_f\, \chi^{(-)*}(\vec{k}_f, \vec{r}_f) \times \langle a_2 c_1 | V | a_1 c_2 \rangle \chi^{(+)}(\vec{k}_i, \vec{r}_i), \tag{2}$$

where the χ are the distorted waves and the matrix element is the form factor. Computation of the six-dimensional integral is lengthy[5] and is usually reduced to a simpler three-dimensional integral using the no-recoil approximation, i.e.,[4]

$$\vec{r}_i=\vec{r}-(m_p/M_{a_1})\vec{r}_1 \approx \vec{r}, \tag{3}$$

$$\vec{r}_f=(M_{c_2}/M_{a_2})\vec{r}+(m_p/M_{a_2})\vec{r}_1 \approx (M_{c_2}/M_{a_2})\vec{r}, \tag{4}$$

where $\vec{r}$ is the vector between the cores c_1 and c_2, and $\vec{r}_1$ and $\vec{r}_2$ are the vectors connecting the transferred particle p in a_1 and a_2 to the cores c_1 and c_2, respectively. The validity of this approximation is questionable for many reactions.[3,5] In particular it can be shown that, because they are vectors, the inclusion of the recoil terms [i.e., the neglected terms of order $(m_p/M_c)\vec{r}_1$] has the effect of introducing additional angular-momentum transfer which may significantly change the calculated cross section.[5,6] In the present study we show that the contributions arising from the recoil terms are important and that their effects can be predicted by a simple first-order treatment.

The reactions (^{12}C, ^{11}B) at 78 MeV and (^{16}O, ^{15}N) at 104 and 140 MeV on ^{208}Pb were studied using the magnetic spectrometer at the Lawrence Berkeley Laboratory 88-in. cyclotron.[7] Sample spectra for (^{16}O, ^{15}N) are shown in Fig. 1. The proton single-particle states in ^{209}Bi, which dominate the spectra, are populated with different relative intensities at the different ^{16}O bombarding energies. As noted previously,[8] the $j=l+\frac{1}{2}$ ($\equiv j_>$) final states are populated more strongly than the $j=l-\frac{1}{2}$ ($\equiv j_<$) states in the single-proton transfer reactions induced by either ^{12}C or ^{16}O. In the 140-MeV ^{16}O reaction this feature is much less pronounced than at the lower bombarding energy.

The differential cross sections extracted were analyzed with a no-recoil DWBA[4,9] using finite-