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Population of 0^+ Excited States in ^{238}Pu and ^{240}Pu by Single-Neutron Transfer Reactions*

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Studies of the single-particle transfer reactions ^{239}Pu$(d,p)^{240}$Pu and ^{239}Pu$(d,t)^{238}$Pu indicate that—contrary to theoretical expectations—the ratios of the cross sections for population of the $K^\pi = 0^+$ excited states to those for population of the ground states are of the same order of magnitude as the ratios for the equivalent (p,t) reactions.

The 0^+ excited states in deformed nuclei have lately attracted much experimental interest, and they have been observed in rare-earth[1-3] and actinide nuclei.[4,5] Throughout the latter region of the nuclide chart, the (p,t) reaction has been found to excite the 0^+ states with uniform and fairly high cross sections.

There has also been a great deal of theoretical discussion[6-8] of the character of these 0_1^+ states. Griffin, Jackson, and Volkov[6] proposed a model in which the pairings between oblate and prolate levels were unequal, and van Rij and Kahana[7] believed that this model could explain the high ratio of the cross sections for (p,t) reactions to the $K^\pi = 0^+$ excited states to the cross sections for reactions to the ground state. As pointed out in Ref. 7, one prediction of this model is that the excited $K^\pi = 0^+$ states should be weakly excited in (d,p) and (d,t) reactions. Our purpose in this paper is to test this prediction. It should be noted that Chasman,[8] who based his calculations on experimentally determined single-particle energies, could not agree with their calculated energies for the excited $K^\pi = 0^+$ states, and found that states of the predicted character would be at much higher excitation.

Reference 7 also argued that the experiments of Bjørnholm, Dubois, and Elbek,[9] who studied single-particle transfer reactions to states in ^{234}U, gave evidence for weak single-particle excitation of excited $K^\pi = 0^+$ states. However, since the configuration of the target nucleus prevented excitation of low-spin states in these experiments, this evidence was not definitive.

To learn more about the structure of these states and about their possible two-quasiparticle components, we have studied their population by use of single-neutron transfer reactions on ^{239}Pu, which is a more suitable target because its low spin ($I = \frac{1}{2}$) and its $\frac{1}{2}^+[631]$ configuration would lead to higher cross sections for populating states with I = 0, 2, and 4. Also, the reaction[5] ^{242}Pu$(p,t)^{240}$Pu and the reaction[10] ^{240}Pu$(p,t)^{238}$Pu had been studied, and several $K^\pi = 0^+$ excited states in each final nucleus had been identified through studies either of these reactions or of radioactive decay.

The experimental results are shown in Figs. 1 and 2. The ^{239}Pu$(d,p)^{240}$Pu spectrum from the present work and the ^{242}Pu$(p,t)^{240}$Pu spectrum given by Maher *et al.*[5] are plotted on the same energy scale in Fig. 1. Our ^{239}Pu$(d,t)^{238}$Pu spectrum and the ^{240}Pu$(p,t)^{238}$Pu spectrum given by Schiffer *et al.*[10] are similarly plotted in Fig. 2. All four spectra were taken with an Enge split-pole spectrograph at the Argonne FN tandem Van de Graaff. A 12-MeV deuteron beam was used for the present work, and data for each target were taken at two or three angles and were analyzed by use of the computer program AUTOFIT.

The cross sections and energies of the rotational states observed in the different $K^\pi = 0^+$ bands in ^{240}Pu and ^{238}Pu are listed in Tables I and II, respectively. The energies and cross sec-

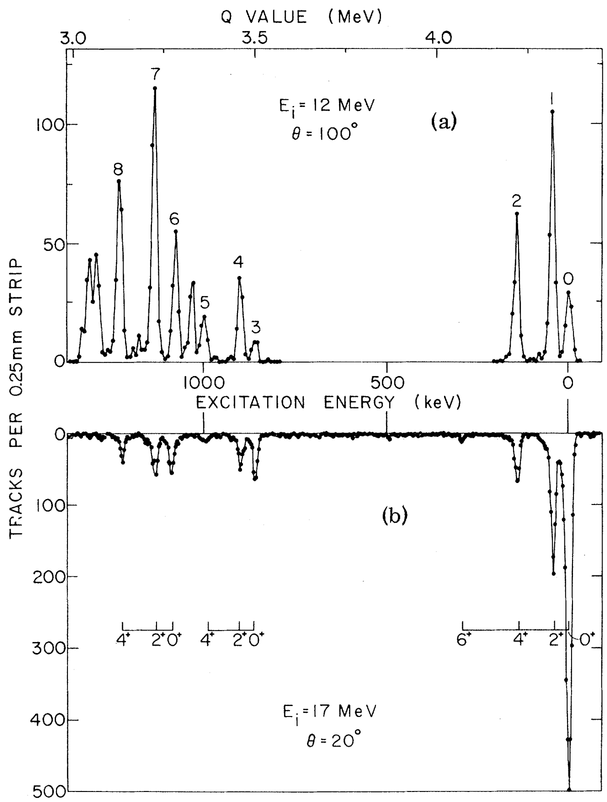

FIG. 1. Comparison of spectra observed in (a) the reaction ^{239}Pu(d,p) and (b) the reaction ^{242}Pu(p,t) to final states in ^{240}Pu. The (p,t) data are from Ref. 5.

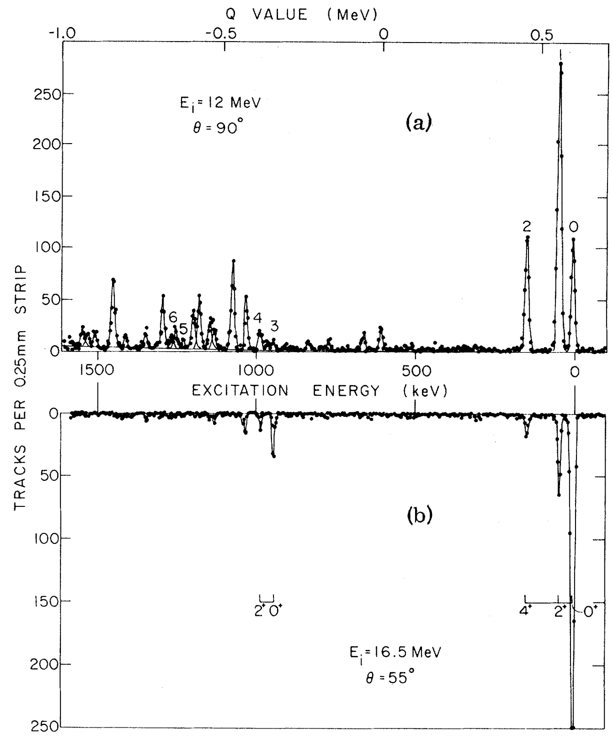

FIG. 2. Comparison of spectra observed in (a) the reaction ^{239}Pu(d,t) and (b) the reaction ^{240}Pu(p,t) to final states in ^{238}Pu. The (p,t) data are from Ref. 10.

tions obtained from the corresponding (p,t) reactions are also included for comparison.

In the reaction ^{239}Pu$(d,p)^{240}$Pu, two excited $K^\pi = 0^+$ rotational bands appear to be populated. Levels 4, 5, and 6 fit the energies of the $I=0$, 2, and 4 levels of the first excited $K^\pi = 0^+$ band observed[5] in the reaction ^{242}Pu$(p,t)^{240}$Pu. We will define the population ratio $(\sigma_{d,p}{}^{\text{exc}}/\sigma_{d,p}{}^{\text{gnd}})_I$ as the ratio of the

TABLE I. Excitation energies E_x and population ratios observed for the $K^\pi = 0^+$ bands in ^{240}Pu from the reaction ^{239}Pu(d,p). Here $\sigma_{d,p}{}^{\text{exc}}$ is the Q-corrected cross section for the transition to the spin-I member of the excited-state band and $\sigma_{d,p}{}^{\text{gnd}}$ is the cross section for the transition to the spin-I member of the ground-state band. The ground-state Q value for this reaction ^{239}Pu$(d,p)^{240}$Pu is $Q_0 = 4.300 \pm 0.010$ MeV.

State No.	E_x (keV)	I	Population ratio $(\sigma_{d,p}{}^{\text{exc}}/\sigma_{d,p}{}^{\text{gnd}})_I$ 100°	125°	Angle-averaged ratio	(p,t) ratio[a]
1	0	0	1	1	1	1
2	42	2	1	1	1	1
3	142	4	1	1	1	1
4	862	0	0.180 ± 0.043	0.171 ± 0.039	0.176 ± 0.041	0.15
5	902	2	0.251 ± 0.033	0.243 ± 0.030	0.247 ± 0.032	0.30
6	1005	4	0.378 ± 0.046	0.272 ± 0.043	0.275 ± 0.045	0.27
7	1091[b]	0	0.889 ± 0.129[b]	0.918 ± 0.124[b]	0.904 ± 0.016[b]	0.10
8	1137	2	0.759 ± 0.070	0.741 ± 0.064	0.750 ± 0.067	0.80
9	1227	4	1.006 ± 0.113	1.111 ± 0.117	1.059 ± 0.115	0.74

[a]Values from Ref. 5. [b]Doublet, as explained in the text.

TABLE II. Excitation energies E_x and cross-section ratios $\sigma_{d,t}{}^{\text{exc}}/\sigma_{d,t}{}^{\text{gnd}}$ observed for the $K^\pi = 0^+$ bands in ^{238}Pu from the reaction ^{239}Pu(d,t). Here $\sigma_{d,t}{}^{\text{exc}}$ is the Q-corrected cross section for the transition to the spin-I member of the excited-state band and $\sigma_{d,t}{}^{\text{gnd}}$ is the cross section for the transition to the spin-I member of the ground-state band. The ground-state Q value for this reaction ^{239}Pu$(d,t)^{238}$Pu is $Q_0 = 0.604 \pm 0.010$ MeV.

State No.	E_x (keV)	I	Population ratio $(\sigma_{d,t}{}^{\text{exc}}/\sigma_{d,t}{}^{\text{gnd}})_I$ 90°	120°	140°	Angle-averaged ratio	(p,t) ratio[a]
1	0	0	1	1	1	1	1
2	44	2	1	1	1	1	1
3	146	4	1	1	1	1	1
4	943	0	0.054 ± 0.014	0.051 ± 0.010	0.018 ± 0.003	0.034 ± 0.008	0.131
5	984	2	0.044 ± 0.008	0.058 ± 0.007	0.058 ± 0.003	0.053 ± 0.006	0.193
6	1228	0	0.022 ± 0.007	0.023 ± 0.008	0.009 ± 0.003	0.018 ± 0.007	≤ 0.01
7	1264	2	0.018 ± 0.004	0.033 ± 0.008	0.011 ± 0.002	0.021 ± 0.005	≤ 0.01

[a]Values from Ref. 10.

cross section for populating an excited state, when corrected for Q dependence, to the cross section for populating the state of the same spin I in the ground-state rotational band. The value of this ratio averaged over all angles is $(\sigma_{d,p}{}^{862}/\sigma_{d,p}{}^{0})_0 = 0.176$ for the reaction ^{239}Pu$(d,p)^{240}$Pu and $(\sigma_{p,t}{}^{862}/\sigma_{p,t}{}^{0})_0 = 0.15$ for the reaction ^{242}Pu$(p,t)^{240}$Pu. The excitation energies of levels 8 and 9 also fit the energies of the $I=2$ and 4 states of the second $K^\pi = 0^+$ band observed in the reaction ^{242}Pu$(p,t)^{240}$Pu. Similarly, for the reactions to the $I^\pi = 2^+$ state (level 8) and to the state of corresponding spin in the ground-state band, the population ratio is 0.75 in the present (d,p) study, while the corresponding ratio in the (p,t) studies is 0.80.

However, the energy of level 7 is 15 keV lower than that of the second $I^\pi = 0^+$ excited state seen in the (p,t) studies, and the population ratio for the reactions to level 7 and to the $I^\pi = 0^+$ ground state is 0.90, while the corresponding ratio measured in the (p,t) studies is 0.10. Because of this energy shift, we believe that level 7 is a doublet, one component of which is the second $I^\pi = 0^+$ excited state. If this is true, then for both $K^\pi = 0^+$ excited vibrational bands, the population ratios are about the same for the (d,p) and (p,t) reactions to ^{240}Pu. That is, the population of the state with $K^\pi = 0^+$, $I=0$, $E_x = 862$ keV is about 18% of that of the ground-state band; and the population of the state with $K^\pi = 0^+$, $I=0$, $E_x = 1091$ keV is about 10% of that of the ground state.

Since (to first order) the (d,p) reaction can populate only states in which one component is the $\frac{1}{2}^+[631]$ neutron (which is unpaired in the target), we must be populating the $\{\frac{1}{2}^+[631]\}^2$ part of each state.[2] Also, in this case the only two-quasiparticle component that can contribute to these states is the $\{\frac{1}{2}^+[631] + \frac{1}{2}^+[620]\}$ state, whose excitation energy is ~2 MeV so it would not mix strongly in either band. Moreover, the signature of this state shows no evidence for a strong admixture.

In the reaction ^{239}Pu$(d,t)^{238}$Pu, two excited $K^\pi = 0^+$ rotational bands are also populated. One of these—the one that includes the state with $K^\pi = 0^+$, $I=0$, $E_x = 943$ keV—was seen to be populated in the reaction ^{240}Pu$(p,t)^{238}$Pu.[10] The other band, which includes the state with $K^\pi = 0^+$, $I=0$, $E_x = 1228$ keV, was not observed in the (p,t) reaction, but was assigned in studies of the electron-capture decay of ^{238}Am.[11]

In the lowest excited $K^\pi = 0^+$ band, the energies of levels 4 and 5 at 943 and 984 keV, respectively, agree within 1 keV with the energies of the $I=0$ and $I=2$ states seen in the reaction ^{240}Pu$(p,t)^{238}$Pu.[7] The angle-averaged population ratio for the (d,t) reactions populating the $I=0$ state is 0.034, while the corresponding ratio for the (p,t) reaction is 0.131.

Levels 6 and 7 at 1228 and 1264 keV, respectively, have the same excitation energies as the $I=0$ and $I=2$ states observed by Ahmad *et al.*[11] in ^{238}Pu. The inability to observe population of these states in the (p,t) studies[10] sets an upper limit of 1% of the ground-state strength. In the (d,t) studies we find that the angle-averaged value of the population ratio for transitions to the $I^\pi = 0^+$ state is 0.018. Therefore, the two excited $K^\pi = 0^+$ bands are seen to have 3.4 and 2% of the ground-state strength in the (d,p) reaction, but 13.1 and ≤ 1% of the ground-state strength in the (p,t) reaction. Thus, these two reactions show no marked similarity in their ratios of excited-state to ground-state strengths.

In conclusion, the excited $K^\pi = 0^+$ states in both ^{240}Pu and ^{238}Pu are seen to be populated in single-particle transfer reactions. For the ^{240}Pu final nucleus, the ratio of the strengths for population of the excited $K^{\pi}=0^{+}$ bands to those for the ground-state bands is the same for the reaction ^{239}Pu(d,p) as for ^{242}Pu(p,t). For the ^{238}Pu final nucleus, the corresponding ratio for the reaction ^{239}Pu(d,t) is only $\frac{1}{4}$ of that for the reaction ^{240}Pu(p,t).

Since the relative population of the $K^{\pi}=0^{+}$ states is as large for the (d,p) reaction as it is for the (p,t), and since the relative population in the (d,t) reaction is still a quarter of that in the (p,t), it appears that the prediction of van Rij and Kahana[7] is not upheld in these two cases. However, it should be noted that we have not taken two-step processes into account, and these could be responsible for the strengths seen in both reactions.

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Qualitative Theory of Pion Scattering by Nuclei*

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The requirement of unitarity, together with the theory of the refractive index, puts a limit on the wave number of a pion (of given energy) in nuclear matter (of given density). A paradox in the conventional theory of pion scattering by complex nuclei is thereby avoided.

The standard theory of the scattering of pions by complex nuclei is based on the Kisslinger potential.[1] Several numerical calculations have been carried out[2-4] with this potential, and agreement with experiment has generally been satisfactory. There is also an extensive literature on modifications of this potential and other approaches.

The Kisslinger potential has, however, a very puzzling feature, as follows. Using Ref. 2a, Eqs. (20) to (22), and neglecting the Coulomb potential, we find for the wave number of a pion in nuclear matter of *constant* density[5] ρ

$$k^2 = k_0^2(1+b_0\rho)/(1-b_1\rho), \tag{1}$$

where k_0 is the free-space wave number, ρ the density of nuclear matter, $k_0^2=\omega^2-\mu^2$, and b_0 and b_1 are parameters describing the s and p scattering of a pion by a nucleon. These are determined by Auerbach *et al.* from the scattering of π by *free* nucleons, with the result (at 80-MeV lab energy)

$$b_1 = 6.5+1.8i \text{ F}^3. \tag{2}$$

Disregarding the imaginary part, k^2 becomes infinite at $\rho=0.16$ F^{-3}, i.e., just about the density of normal nuclear matter, and becomes negative for higher density. This is a quite unphysical behavior.

The paradox is solved by two considerations, viz. (a) unitarity and (b) the Pauli principle. To discuss this problem, we start from the general theory of the refractive index which gives

$$k^2-k_0^2 = 4\pi\rho f(k,0), \tag{3}$$