Both his results and ours with cold neutrons reveal the hindered rotation but there are marked differences concerning the other features. His measurements do not show the sharp peaks that we have observed, particularly those corresponding to the small energy changes. Instead of the latter, he observes a general broadening of the elastic peak, which is somewhat smaller than that expected from diffusive motions. This result is in sharp contrast to ours, which shows no evidence of broadening related to diffusive motions.

*Guest scientist from University of Freiburg, Freiburg, Germany.

†Guest scientist from Finland Institute of Technology, Helsinki, Finland.

[1]Eisenhauer, Pelah, Hughes, and Palevsky, Phys. Rev. 109, 1046 (1958).

[2]G. H. Vineyard, Phys. Rev. 110, 999 (1958).

[3]L. Van Hove, Phys. Rev. 95, 249 (1954).

[4]M. Magat, Ann. Physik 6, 108 (1936).

[5]C. H. Townes (private communication).

[6]B. N. Brockhouse, Phys. Rev. Letters 2, 287 (1959).

SCATTERING OF 50- to 140-Mev PHOTONS BY PROTONS AND DEUTERONS*

L. G. Hyman,† R. Ely,‡ D. H. Frisch,|| and M. A. Wahlig**
Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received June 18, 1959)

The Compton scattering of photons by protons has been studied in several experiments[1-4] below π-meson photothreshold, and in two experiments[5,6] above. We report here further work below meson threshold using both protons and deuterons as targets.

The techniques of identifying and measuring the energy of the bremsstrahlung x-rays from the M.I.T. synchrotron after they are scattered by thin targets are essentially as reported in Pugh et al.[1] The many changes which have been made in the synchrotron itself, giving it greater stability and intensity, have been described in recent M.I.T. progress reports.[7] The many changes in construction and performance of the photon telescope, energy-measuring liquid scintillator, and especially electronics, are being reported elsewhere.[8]

In Fig. 1 experimental and theoretical cross sections for scattering of photons by hydrogen are plotted as a function of lab energy at a c.m. angle of 90°. The cross sections are given per unit solid angle in the c.m. system for direct comparison with the Illinois points[5] and those of other groups.[2-4] Our results are given as a shaded area within which the cross section must lie in order to give, after appropriate folding, a fit to a smooth curve through our observed data. A discussion of this technique, which of necessity assumes a fairly smooth cross section, is given by Pugh et al.[1] The statistical standard deviation is represented by the height of the shaded area,

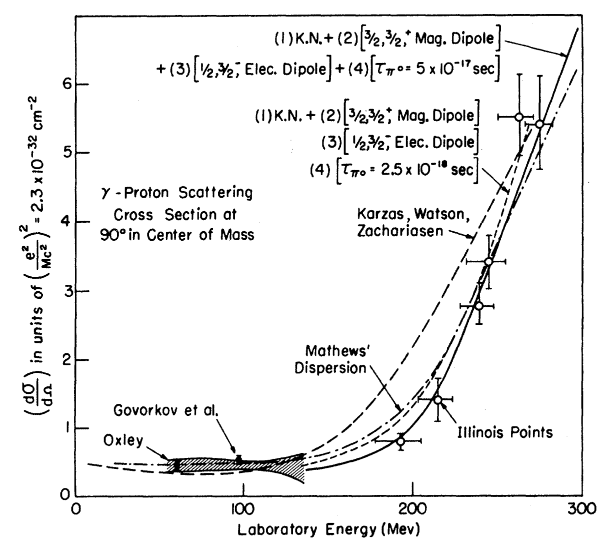

FIG. 1. Differential cross sections for scattering of 50-280 Mev photons by hydrogen at 90° c.m.

but statistical errors are thereby exaggerated somewhat, since the data were taken in channels of approximately 10 Mev, but the cross section at most energies is averaged by the counter response over an energy band of about 20 Mev. On the other hand, a possible 10% error in absolute normalization is not displayed. We see in Fig. 1 that these new data, and the results of the other experiments at this angle, are all in good agreement below meson threshold and fit in smoothly with the Illinois data.

The dashed lines in Fig. 1 are the predictions of previous calculations. As has been pointed out before,[9] the Mathews dispersion theory curve[10] is clearly too high in the 160-200 Mev region.

The meson-theoretic prediction, Karzas et al.,[11] falls somewhat too low in the region below meson threshold and rises far above the data in the 200-Mev region.

We calculated the solid curve using for each polarization the coherent addition of four amplitudes:

(1) The Klein-Nishina amplitude for scattering by a point Dirac particle. Note that this is not the Powell[12] (Dirac particle with added Pauli moment) amplitude[13]; that is, in this energy region we will describe the anomalous electromagnetic properties of the proton in terms of its excited states.

(2) The induced magnetic dipole resonance amplitude for scattering by the $T=3/2$, $J=3/2^+$ first excited state of the proton, following Austern,[14] Feld,[15] and Yamaguchi.[16] Phase shifts taken from the Chew-Low-Chiu-Lomon[17] analysis of the π-nucleon interactions were used in the single-level approximation.[11]

(3) The induced electric dipole resonance amplitude for scattering by the $T=1/2$, $J=3/2^-$ second excited state of the proton.[18-21] This crude but honest single-level calculation was patterned after Feld's "atomic model" calculation of (2) above.

(4) The amplitude shown by Low[22] to result from the special coupling of a pair of photons to a proton through the decay of a virtual π^0 meson, which thus is a function of the (unknown) π^0 mean life τ_{π^0}.

The sum of amplitudes (1), (2), and (4) gives approximately the same result (not shown) as the Mathews dispersion theory, which was evaluated prior to knowledge of the $1/2$, $3/2^-$ resonance, provided that $\tau_{\pi^0} \geq 2\times10^{-18}$ sec, but does not give a very good fit to the data. This discrepancy is removed by addition of amplitude (3), resulting in the excellent fit given by the solid line in Fig.1.

The γ-d cross section, completely in the laboratory system, is given along with the γ-p cross section in Fig. 2(a). Note that our detector has insufficient energy resolution ot make possible the separation of coherent elastic scattering by the whole deuteron from incoherent quasi-elastic scattering by the constituent proton or neutron. At 90° lab and 100 Mev one expects[23] roughly half elastic and half inelastic. The dashed curve is constructed from Mathews' nucleon amplitudes[10] in the impulse approximation,[24] using a Hulthén wave function matched to electron scattering by deuterium.[25] Judging by Mathews' curve in Fig.1, we expect the calculation to give an upper limit to the deuteron cross section. But the observed deuteron cross section seems to be about 1.6 ± 0.3 times the hydrogen cross section over the whole energy range. Since the deuterium run was bracketed by two halves of the hydrogen run, under identical conditions, we find no way to explain this high ratio.

The cross sections for H and D at 50° are given in Fig. 2(b). The sharp rise at energies below

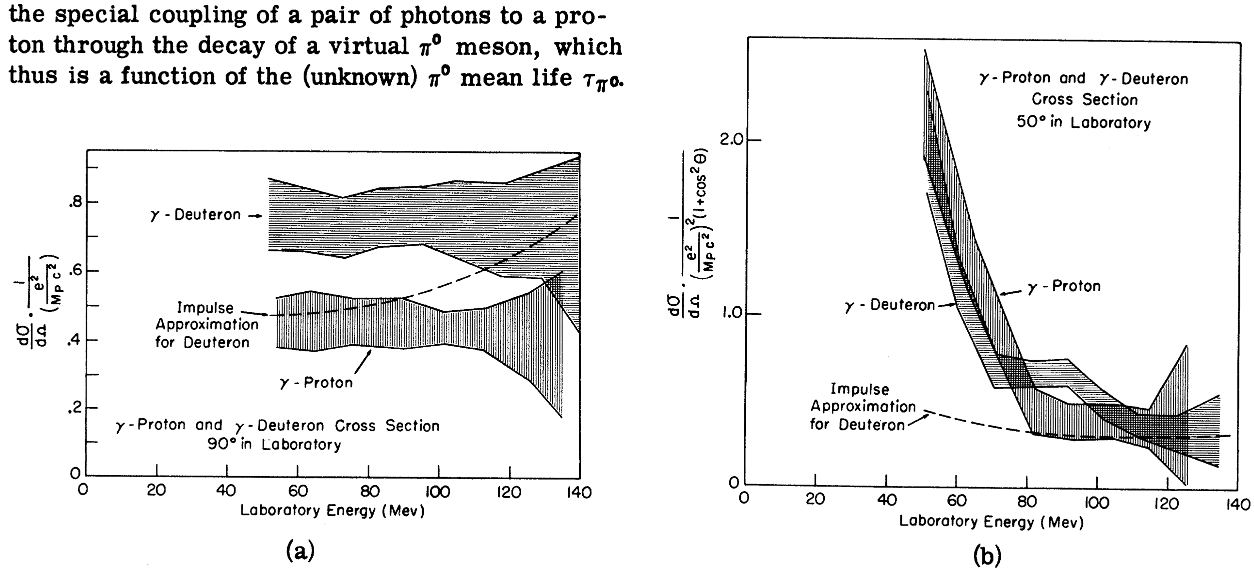

FIG. 2. (a) Differential cross sections for scattering of 50-140 Mev photons by hydrogen and deuterium at 90° lab. (b) Differential cross sections for scattering of 50-140 Mev photons by hydrogen and deuterium at 50° lab.

80 Mev is very probably not Compton scattering; the most probable origin of this low-energy background will be discussed below. The cross sections above 80 Mev are not very accurate, and we can say only that (1) the hydrogen data above 80 Mev are fitted well by Mathews' dispersion curve and (2) around 80 Mev the deuterium cross section is appreciably greater than the hydrogen cross section but seems to drop off as the energy rises near meson threshold. Because of the complexity of the deuteron problem, especially in this region of momentum transfers, and with the necessity of including all four types of elementary amplitudes listed above, no attempt has been made to interpret the 50° deuterium data.

A brief study was made of the 50° low-energy background, which had been observed previously[1] and ascribed to inner bremsstrahlung accompanying pair production (radiative pair production). But as estimated by Feynman and Gomez,[26] the intensity of inner bremsstrahlung is not sufficient to account for most of this large background. We therefore measured, using a carbon target, the number of electrons coming to the detector from the target, and found that there were enough to produce the observed spectrum of gamma rays at low energies by bremsstrahlung in the rather thick beryllium shield[27] in front of the counter telescope. The dependence of this background on target thickness and machine energy is in agreement with what would be expected from the number of wide-angle pair electrons[28] produced in the target plus the Mott scattering of small-angle electrons produced both in the target and upstream from it.

We conclude: (1) the observed scattering of photons by protons at 90° c.m. over the energy range 0-300 Mev is described well by a model of the proton as a Dirac particle plus strong pion interactions giving the well-known 3/2, $3/2^+$ and 1/2, $3/2^-$ resonances, provided the half-life of the π^0-meson is greater than 10^{-18} second. (2) The scattering of photons by deuterons below meson threshold is more than expected.

We thank D. Luckey, L. Osborne, and the M.I.T. Synchrotron staff for generous help with the experiment; B. Feld, M. Friedman, F. Low, C. Nielsen, F. Villars, and V. Weisskopf for stimulating discussions; and the U.S. Atomic Energy Commission, the Office of Naval Research, the Office of Scientific Research, the National Science Foundation, the General Electric Company, and the Sloan Foundation for their generous support.

*This work is supported in part by funds provided by the U.S. Atomic Energy Commission, the Office of Naval Research, and the Air Force Office of Scientific Research.

†Present address: Harvard University, Cambridge, Massachusetts.

‡General Electric Fellow.

‖Sloan Fellow.

**National Science Foundation Predoctoral Fellow.

[1]Pugh, Gomez, Frisch, and Janes, Phys. Rev. 105, 982 (1957).

[2]Govorkov, Goldanski, Karpuhkin, Katsenko, and Pavlovskaya, Doklady Akad. Nauk. SSSR 111, 988 (1956) [translation: Soviet Phys. Doklady 1, 735 (1956)].

[3]C. L. Oxley, Phys. Rev. 110, 733 (1958).

[4]Alvarez, Crawford, and Stevenson, Phys. Rev. 112, 1267 (1958).

[5]Auerbach, Bernardini, Filosofo, Hanson, Odian, and Yamagoda, Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956(CERN Scientific Information Service, Geneva, 1956),p. 291.

[6]Littauer, DeWire, and Feldman, Bull. Am. Phys. Soc. 4, 253 (1959).

[7]D. Luckey, L. S. Osborne, et al., Massachusetts Institute of Technology Laboratory for Nuclear Science Progress Report, 1958 (unpublished).

[8]L. G. Hyman, Massachusetts Institute of Technology Ph.D. thesis, 1959 (unpublished).

[9]G. Bernardini, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

[10]J. Mathews, California Institute of Technology, Ph.D. thesis, 1957 (unpublished).

[11]Karzas, Watson, and Zachariasen, Phys. Rev. 110, 253 (1958).

[12]J. L. Powell, Phys. Rev. 75, 32 (1949).

[13]R. Gomez and D. Walecka, Phys. Rev. 104, 1479 (1956).

[14]N. Austern, Phys. Rev. 100, 1522 (1955).

[15]B. T. Feld, Ann. Phys. 4, 189 (1958).

[16]See mention in T. Yamagoda, University of Illinois Ph.D. thesis, 1956 (unpublished).

[17]H. Y. Chiu and E. L. Lomon, Ann. Phys. 6, 50 (1959).

[18]R. R. Wilson, Phys. Rev. 110, 1212 (1958).

[19]F. P. Dixon and R. L. Walker, Phys. Rev. Letters 1, 142 (1958).

[20]P. C. Stein, Phys. Rev. Letters 2, 11 (1959).

[21]R. Peierls, Phys. Rev. Letters 1, 174 (1958).

[22]F. E. Low (private communication). This amplitude was taken with the sign indicated by the perturbation theory treatment of π^0-decay. If the opposite sign were taken, the best fit would be for $\tau = \infty$, and the data would set a lower limit of about $\tau \geq 5 \times 10^{-17}$ sec.

[23]R. H. Capps, Phys. Rev. 106, 1031 (1957).

[24]G. Chew and H. Lewis, Phys. Rev. 84, 779 (1951).

[25]J. McIntyre, Phys. Rev. 103, 1464 (1956).

[26]R. P. Feynman and R. Gomez (private communication).

[27]This beryllium shield was several times thicker, in radiation lengths, than similar shields used in other experiments.

[28]R. C. Miller, Phys. Rev. 95, 796 (1954).

A FORMAL OPTICAL MODEL

J. S. Bell and E. J. Squires
Atomic Energy Research Establishment, Harwell, England
(Received June 15, 1959)

A construction is given of an equivalent one-body potential for the elastic scattering of a particle incident on a complex target. We consider explicitly the case that incident and target particles are identical nonrelativistic fermions and allow fully for antisymmetry. The method is inspired by that of Frantz and Mills[1] and removes some defects therefrom; it also removes the basis for their proposed change in phenomenological optical model analysis. Center-of-mass motion is still ignored.

Denoting by $|\bar{\alpha}\rangle$ the scattering state and by $|\bar{0}\rangle$ the target ground state, we define the model wave function as

$$\phi(\vec{r}, t) = \langle \bar{0} | \bar{\psi}(\vec{r}, t) | \bar{\alpha} \rangle, \tag{1}$$

where $\bar{\psi}$ is the Heisenberg field operator of second quantization. For $|\bar{\alpha}\rangle$ we take the state

$$|\bar{\alpha}\rangle = \int_{-\infty}^{t} dt' e^{-iEt'} \bar{\psi}^\dagger(\vec{r}', t') |\bar{0}\rangle, \tag{2}$$

which corresponds to a source of particles of energy E at the point $\vec{r}'$; if r' is sufficiently large only a plane wave actually reaches the target. In writing, with $x \equiv (\vec{r}, t)$,

$$\phi(x) = \int_{-\infty}^{+\infty} dt' e^{-iEt'} G(x, x'), \tag{3}$$

$$G = \langle \bar{0} | T(\bar{\psi}(x), \bar{\psi}^\dagger(x')) | \bar{0} \rangle, \tag{4}$$

we make no error by taking the time-ordered rather than the retarded product, for the surplus contribution depends on the possibility of absorbing a particle at $\vec{r}'$ from the state $|\bar{0}\rangle$ and so vanishes for large r'.

We construct G by a perturbation theory where in zero order the real forces are replaced by a fictitious one-body potential, in general nonlocal,

$$\int d\vec{r} d\vec{r}' \bar{\psi}^\dagger(\vec{r}, 0) U(\vec{r}, \vec{r}') \bar{\psi}(\vec{r}', 0).$$

An S matrix is defined by

$$\frac{\partial}{\partial t} S(t, t') = -iH'(t) S(t, t'),$$

$$S(t', t') = 1, \quad H'(t) = e^{iH_0 t} H' e^{-iH_0 t},$$

where H_0 is the zero-order Hamiltonian and the total Hamiltonian is $H_0 + H'$. In terms of interaction representation operators ψ, the Heisenberg operator $\bar{\psi}$ can be written

$$\bar{\psi}(x) = S^{-1}(t, 0) \psi(x) S(t, 0),$$

and we have the usual expression

$$G = \frac{\langle 0 | S(\infty, t) \psi(x) S(t, t') \psi^\dagger(x') S(t', -\infty) | 0 \rangle}{\langle 0 | S(\infty, -\infty) | 0 \rangle},$$

for $t > t'$, and similarly for $t < t'$, where $|0\rangle$ is the zero-order target ground state—assumed nondegenerate. Expanding above and below in powers of H', we have

$$G = \sum_{n=0}^{\infty} \frac{1}{n!} \int dt_1 \cdots dt_n \langle 0 | T\{H'(t_1) \cdots H'(t_n) \psi(x) \psi^\dagger(x')\} | 0 \rangle \Big/ \sum_{n=0}^{\infty} \frac{1}{n!} \int dt_1 \cdots dt_n \langle 0 | T\{H'(t_1) \cdots H'(t_n)\} | 0 \rangle. \tag{5}$$

We make a diagrammatic analysis of this following Hubbard,[2] using Wick's theorem and regarding $|0\rangle$ as "vacuum." The denominator has the effect simply of cancelling all diagrams in the numerator not linked to the terminal operators $\psi(x)$ and $\psi^\dagger(x')$; G is therefore the sum of linked diagrams only from the numerator. We call "improper" a linked diagram which falls into two disconnected parts on the removal of some particle (as distinct from interaction) line. Reasoning familiar