satisfied already,

$$\delta\Psi = \int d^3x\, \delta g_{rs}(x)\delta\Psi/\delta g_{rs}(x)$$
$$= -i\int d^3x\, a^u(x)\mathcal{H}_u(x)\Psi - 2i\int dS_r\, a^u(x) g_{us}\pi^{rs}\Psi.$$

The first term vanishes if the constraint (1) is satisfied; the second is an integral over the surface at infinity which vanishes only if $a^u(x)$ is localized. We therefore have to construct functionals $\Psi\{g_{rs}(x)\}$ which are invariant only under localized transformations.

The problem is solved by transforming g_{rs} into a form which satisfies certain transversality conditions:

$$g_{rs}(x) = y^a{}_{,r} y^b{}_{,s} g^T{}_{ab}(y).$$

If $y^a(x)$ are three independent functions such that the differences $y^a - x^a$ are localized and $g^T{}_{ab}$ satisfies suitable conditions, then we know that Ψ is independent of y^a. By suitable conditions we mean that $g^T{}_{ab}$ has only three independent components and the functions y^a are uniquely determined by $g_{rs}(x)$ together with these conditions. It turns out that the well-known harmonic coordinate conditions fulfill these criteria.

It is convenient to introduce the contravariant densities h^{rs}, related to g_{rs} by $h^{rs}g_{st} = (-g)^{1/2}\delta^r{}_t$. Then we define the "transverse gravitational potentials"

$$h^{Tab}(y) = (\det y^c{}_{,t})^{-1} y^a{}_{,r} y^b{}_{,s} h^{rs}(x), \qquad (2)$$

where $y^a(x)$ is the regular solution of the generalized Laplace equation

$$(h^{rs}y_{,r})_{,s} = 0 \qquad (3)$$

which satisfies the asymptotic condition (localization)

$$\lim_{|x|\to\infty} (y^a - x^a) = 0\ .$$

The functions y^a and consequently also the transverse potentials are uniquely determined by $g_{rs}(x)$. The constraints (1) now tell us that $\Psi = \Psi\{h^{Tab}(y)\}$. The transversality condition, which follows from (2) and (3), is

$$h^{Tab}{}_{,b} = 0, \qquad (4)$$

so there are only three independent transverse components. They may be exhibited explicitly by performing a Fourier transformation and resolving the transform of h^{Tab} parallel and perpendicular to the wave vector k_a: we define

$$\gamma^{ij}(k) = \int d^3y\, \exp(-ik\cdot y) e^i{}_a e^j{}_b [h^{Tab}(y) + \delta^{ab}], \qquad (5)$$

where $e^i{}_a(k)$ is an orthonormal triad of vectors (the scalar product here is Euclidean) such that $k_a = |k|e^3{}_a$. Then, by (4) and (5), the only nonvanishing components of γ^{ij} are γ^{11}, γ^{22}, $\gamma^{12} = \gamma^{21}$. [In addition, the reality condition $\gamma^{ij}(-k) = \gamma^{*ij}(k)$ is to be imposed.] Finally, the wave functional may be written as $\Psi\{\gamma^{ij}(k)\}$.

The transverse potentials which have been defined here reduce in the linearized theory to those which were obtained by Arnowitt and Deser,[3] except that their γ^{ij} was traceless on account of the eighth constraint. It remains to be seen whether Dirac's $\mathcal{H}_L$ condition implies a similar restriction in the full theory.

[1]Equation (11) of this paper should read

$$[\mathcal{H}_u(x), \mathcal{H}_L(y)] = i\mathcal{H}_L(x)\partial/\partial x^u \delta^{(3)}(x-y).$$

[2]P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 333 (1958).

[3]R. Arnowitt and S. Deser, Phys. Rev. 113, 745 (1959).

EFFECT OF NUCLEAR ELECTRIC DIPOLE MOMENTS ON NUCLEAR SPIN RELAXATION IN GASES. P. A. Franken and H. S. Boyne [Phys. Rev. Letters 2, 422 (1959)].

In this Letter it was estimated that careful measurements of nuclear spin relaxation times in noble gases such as He^3 and Xe^{129} could reveal the existence of nuclear electric dipole moments as small as 10^{-3} or 10^{-4} nuclear magneton. (One nuclear magneton $= eh/2Mc = e\times 10^{-14}$ cm.) Indeed, we interpreted the already available measurements on these gases to indicate upper limits of order 10^{-1} to 10^{-2} nuclear magneton.

Professor E. M. Purcell has brought to our attention that we have made a serious error in our implicit assumption that the velocity changes due to collisions in the gas are uncorrelated. Whereas an atom does experience a random walk in its displacements it is not true that the changes in velocity due to collisions are random. For example, if an atom is moving in the x direction it is more than 50 % probable that the next collision will decrease the x component of velocity. Were this not true the atoms in the gas would rapidly depart from the Maxwell-Boltzmann velocity distribution.

Since we implicitly assumed that the changes of velocity are uncorrelated, it becomes apparent that our calculation must be modified. Purcell has found, and we entirely agree, that our calculation is valid only when $\omega\tau_c \geq 1$, where ω is the Larmor frequency of the nuclear magnetic moment in the applied magnetic field and τ_c is the mean time between collisions.

In the event that $\omega\tau_c < 1$, Purcell finds, and we entirely agree, that the relaxation time we computed [Eq. (2) of the Letter] must be multiplied by the approximate factor $[1/\omega\tau_c]^2$.[1] Therefore the estimated maximum electric dipole moment must be multiplied by the approximate factor $[1/\omega\tau_c]$, which is of order 10^3 for the experiments discussed in the Letter.

It seems entirely possible that experiments could be done with He and Xe under conditions where $\omega\tau_c \cong 1$, in which case the formulas of the Letter are applicable. For magnetic fields of ~20 000 gauss and room temperature we estimate that gas pressures of ~1/20 atmosphere would be satisfactory. We see no objection to compressing or even liquifying the gases for short periods of time in order to perform the necessary spin polarization measurements. We estimate that upper limits of $\sim 10^{-2}$ nuclear magneton could still be established, but the experiments are difficult. Such upper limits would still be an improvement over what can be obtained by previously discussed methods.[2]

We wish to express our gratitude to Professor Purcell for bringing this correction to our attention, and for several valuable discussions.

---

[1]Professor Purcell has performed an exact calculation of this factor which will be submitted to the Physical Review.

[2]E. M. Purcell and N. F. Ramsey, Phys. Rev. 78, 807 (1950).