PION PRODUCTION AND THE PION-PION INTERACTION*

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In the preceding Letter[1] the cross sections for $\pi^- + p \to \pi^- + \pi^+ + n$ at incident energies between 260 and 430 Mev were presented. In this note we wish to discuss the theoretical interpretation of these results.

One may first attempt to understand this process using the well-known pion-nucleon interaction. This has been done previously[2] within the framework of the static model, which should remain applicable for the low energies involved here. In this model the incident pion interacts directly with the nucleon, and the nucleon "shakes off" the final pions. This approach yields a cross section for $\pi^- + p \to \pi^- + \pi^+ + n$ which is less than 0.1 mb for $E_{\text{inc}} \lesssim 400$ Mev. This small cross section results from (a) the relatively small, nonresonant, cross section for scattering of the initial and final pions on the nucleon (the incident energy is more than 100 Mev above the position of the 3-3 resonance, and the mean kinetic energy of the final-state pions is more than 50 Mev below the resonance energy); (b) the weak pseudovector coupling ($f^2 \approx 0.08$) which produces the extra meson; and (c) the small amount of phase space available to the outgoing P-wave pions. Thus the direct pion-nucleon interaction makes a negligible contribution to pion production in the 250-450 Mev region.

We have therefore attempted to fit the results of the preceding Letter using a direct pion-pion interaction. The contribution from the interaction of the incident pion with a pion in the nucleon cloud has been computed using the impulse approximation. The sizable cross section at low energies suggests an S-wave π-π interaction, and the rapid rise of the cross section with energy requires a P-wave interaction (all angular momenta are in the π-π center-of-mass system). The S-wave interaction has been described by phase shifts satisfying $[\nu/(\nu+1)]^{1/2} \times \cot \delta_0^I = 1/a_I$ with the isotopic spin $I=0$ and 2, ν the square of the π-π c.m. momentum, and $a_0 = 5\lambda$, $a_2 = 2\lambda$.[3] The P-wave cross section has been computed from the low-energy form $[\nu^3/(\nu+1)]^{1/2} \cot \delta_1^2 = 1/a_1$. A fit to the observed total cross section was obtained with $|\lambda| \approx 1/10$ and $|a_1| \approx 1/5$ (units $\mu_\pi = 1$); the result is shown in Fig. 2 of the preceding Letter. The P-wave phase shift obtained here is consistent with that found by Frazer and Fulco[4] in their investigation of nucleon structure.

The calculation was performed assuming that only the one-meson exchange term contributes. The meson-nucleon rescattering corrections can be computed by incorporating a π-π interaction into the formalism of reference 2. In the one-meson approximation which was used in reference 2, this adds an additional inhomogeneous term to the integral equations obtained there. These new equations can be solved using the same methods as in the earlier work. If the production amplitude is represented by D_0, the correction due to the rescattering of meson k is $\sim [ie^{i\delta_{33}(k)} \sin \delta_{33}(k)] D_0$, assuming that only pion-nucleon scattering in the $T = J = \frac{3}{2}$-state is important. There should then not be large corrections in the present energy region. As in the case of photomeson production,[5] this is because the pion production takes place far from the nucleon. At higher energies, since the momentum transfer is of the order of the incident c.m. momentum, the mean radius of interaction becomes smaller and the rescattering corrections become correspondingly larger.

Over the energy range considered here the ratio of S-wave to P-wave contributions varies from 15 to 1/3; they become equal at $E_{\text{inc}} \approx 400$ Mev. The present experiment does indicate a sharp break within this region. For a given partial wave in the π-π system, the present model predicts a pion angular distribution in the over-all center-of-mass system favoring forward angles, and a pion energy distribution tending slightly toward higher energies than predicted by phase space. Both the angular distribution and the energy spectrum[6] of the π^+ are consistent with this at 317 and 427 Mev, but differ markedly at 371 Mev. This abrupt change can be interpreted as a manifestation of the rapid change in the S/P ratio and the resulting the S-P interference. It does not seem possible to make quantitative estimates of these effects since the intermediate pion is far off the mass shell. Its "mass" varies with the angle and energy of the emitted pions, and the dependence of the cross section on this mass is difficult to estimate. The "free"

π-π scattering cross section can be determined only by the extrapolation procedures recently suggested by Goebel and Chew and Low.[7]

The angular- and energy-correlations in the π-π c.m. system should also be sensitive to the S/P ratio: a fore-aft asymmetry should appear within this energy range, and higher c.m. energies should be favored as the P-wave interaction becomes dominant. An observation of both final pions is required to determine the relevant angle and energy. As discussed in the previous paragraph, it is difficult to make a reliable estimate of these effects using the present phenomenological approach.

According to the model proposed here, the P-wave, $T=1$, π-π interaction should dominate for $E_{\text{inc}} > 400$ Mev. It follows that the charged-to-neutral production ratio $(\pi^- + p \rightarrow \pi^- + \pi^+ + n)/(\pi^- + p \rightarrow \pi^- + \pi^0 + p)$ is predicted to be 2, the square of the ratio of the pion-nucleon coupling constants for charged and neutral meson emission. This interaction also yields no production of $2\pi^0$. For $\pi^+ + p$, only the reaction $\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$ is permitted by the $T=1$ interaction. As a result, the inelastic cross section for $\pi^+ + p$ should be 1/3 that for $\pi^- + p$; experimentally,[8,9] for $E_{\text{inc}} \approx 500$ Mev, this ratio is about 1/4. Since the probability of finding two pions in the nucleon cloud is small, this model also predicts that double-pion production should be unlikely compared to single-pion production.

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[2]L. S. Rodberg, Phys. Rev. 106, 1090 (1957); E. Kazes, Phys. Rev. 107, 1131 (1957).

[3]This relation is suggested by the dispersion-theoretic approach of G. F. Chew and S. Mandelstam, University of California Radiation Laboratory Report UCRL-8728 (unpublished).

[4]W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959). The π-π energies involved in our problem are too small to obtain any indication of the resonance which they require. At higher energies there is some indication of a peak in the inelastic cross section at the expected energy (reference 8). See also G. Takeda, Phys. Rev. 100, 440 (1955).

[5]G. F. Chew and R. E. Low, Phys. Rev. 101, 1579 (1956).

[6]V. Perez-Mendez (private communication).

[7]C. Goebel, Phys. Rev. Letters 1, 337 (1958); G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959). As these authors point out, the sharp diffraction peak observed in elastic pion-nucleon scattering provides another indication of a strong π-π interaction.

[8]Blevins, Block, and Leitner, Phys. Rev. 112, 1287 (1958).

[9]R. R. Crittenden et al., Phys. Rev. Letters 2, 121 (1959).

CONSEQUENCES OF ATOMIC CONVERSION FOR THE INTERPRETATION OF EXPERIMENTS ON THE SPIN-DEPENDENCE OF MUON ABSORPTION*

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The ground state of a μ-mesonic atom of nonzero nuclear spin I is split by the hyperfine interaction into two states, of total angular momenta $F=I\pm\frac{1}{2}$. It was pointed out by Bernstein, Lee, Yang, and Primakoff[1] (hereinafter referred to as BLYP) that (a) the lifetimes of the members of this doublet may, as a consequence of the incoherence of the two states, be distinct, and that (b) these lifetimes will actually differ if the muon absorption rate depends on F, i.e., if the relevant weak interaction is at least partly spin-dependent. The total muon disappearance rate is the sum of the absorption and the decay rates, and the latter is indeed to an excellent approximation independent of F. BLYP estimated the fractional difference δ between the two lifetimes on the basis of a simple nuclear model,[2] and suggested an experimental test for the spin-dependence of muon absorption: Demonstrate that the rate of appearance of decay electrons (originating from μ^- bound in a monoisotopic target with $I\neq 0$) does not, as a function of time, follow a simple exponential. The logarithm of the decay curve should indeed exhibit (in the sense defined below) a positive curvature proportional to δ^2. This quadratic dependence would of course prevent one, as BLYP emphasized, from establishing which member of the doublet absorbs faster, i.e., from settling a question of particular theoretical significance.