a muon decay interaction identical to Eq. (2) except for the absence of the direct vector coupling. The two versions of the spin-zero theory are identical for $g_1^{(e)}=0$, and in general differ only by a direct self-interaction of $J_\lambda^{(e)}$. The choice between these formalisms depends upon whether one regards first-order or second-order field equations as the more basic. [It is only the Kemmer theory that may be developed according to the canonical formulation of Schwinger.[6]]

For the discussion of $\mu\to e+\gamma$, we shall assume $g_2^{(e)}=0$ and neglect finite contributions proportional to the electron mass. The matrix element for this mode becomes (q = photon momentum)

$$\frac{e}{96\pi^2}\frac{g_1^{(\mu)}g_1^{(e)}}{m^2}\bar{\psi}_{(e)}\frac{1-\gamma_5}{2}\left\{\tfrac{1}{2}m^{(\mu)}\sigma_{\alpha\beta}F^{\alpha\beta}-\gamma_\mu A_\mu q^2\left[\ln\left(\frac{K^2}{m^2}\right)+c+O\left(\frac{q^2}{m^2}\right)\right]\right\}\psi_{(\mu)},$$

where K is a cutoff to the divergent momentum integration, and c is a finite constant which depends upon the manner in which the cutoff is introduced. The decay interaction contains a finite, point-like, "moment" interaction, and a divergent "charge" interaction which does not contribute to the free-particle decay. The branching ratio for the decay of a free muon into the $e+\gamma$ channel is

$$\rho=\alpha/24\pi\approx10^{-4},$$

which is ~40 times greater than the current experimental upper limit.[7] In contrast to the result with a vector intermediary,[8-10] the result is both convergent and unambiguous.

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SOME GRAVITATIONAL WAVES*

Asher Peres
Department of Physics, Israel Institute of Technology, Haifa, Israel
(Received October 19, 1959)

The few hitherto known wave-like exact solutions of the Einstein gravitational equations represented either plane[1,2] or cylindrical[3] waves. We here intend to derive a new class of solutions, displaying a lesser degree of symmetry, and thus more generality.

Let us consider the metric

$$ds^2=-dx^2-dy^2-dz^2+dt^2-2f(x,y,z+t)(dz+dt)^2.$$

One can easily show that $R_{zz}=R_{zt}=R_{tt}=f_{xx}+f_{yy}$, and other components vanish. If f is a harmonic function of x and y, this metric thus satisfies Einstein's equations in vacuo, whatever may be its dependence on $(z+t)$.

A convenient tetrad of orthonormal vectors is

$$h_{(1)}{}^k=(\cos a,\ \sin a,\ 0,0),$$
$$h_{(2)}{}^k=(-\sin a,\ \cos a,\ 0,0),$$
$$h_{(3)}{}^k=(0,0,1-f,f),$$
$$h_{(4)}{}^k=(0,0,-f,\ 1+f),$$

where $\tan(2a)=(f_{xy}/f_{xx})$. The only nonvanishing independent physical components[4] of the curvature tensor are

$$\sigma=R_{(m1n1)}=-R_{(m2n2)}=(f_{xx}{}^2+f_{xy}{}^2)^{1/2},$$

where m and n take the values 3 and 4 only. Our metric thus belongs to the second class of Petrov's classification.[4]

Let us examine two examples. If

$$f = (x^2 - y^2)\sin(z + t),$$

one has $\sigma = 2\sin(z + t)$. This is a plane monochromatic wave.

Another example is

$$f = xy(x^2 + y^2)^{-2}\exp[b^2 - (z + t)^2]^{-2} \text{ for } |z + t| < b,$$

$$f = 0 \text{ for } |z + t| \geq b.$$

(Note that f has derivatives of all orders at $|z + t| = b$.) This is a wave packet travelling with unit velocity in the negative z direction. It contains the singular segment $x = y = 0$, $-b - t < z < b - t$. The question of the existence of stable regular gravitational wave packets is still open.

If $f_{xx} + f_{yy} \neq 0$, one still has $g^{mn}R_{mn} = 0$ and $g^{mn}R_{mr}R_{ns} = 0$. The source of f can then be interpreted as a null electromagnetic field.[5]

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ERRATUM

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In Fig. 2(b) the transition energy between the 10+ and 8+ levels of Th^{232} should read 273 kev instead of 373 kev.