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μ-MESON DECAY

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It was originally suggested by Yukawa[1] that the muon decay may be mediated by an unstable boson with weak couplings to lepton fields. More recently,[2,3] a charged, spin-one boson has been proposed in order to generate the V-A form of the weak decay interactions. It has already been pointed out[4] that a charged scalar intermediary can give rise to the desired couplings, providing an alternative but generally equivalent theory of the weak interactions. This applies when the first-order Kemmer formalism for spinless bosons is employed, but not with the use of the more conventional Klein-Gordon second-order theory.

The field equations in the Kemmer theory may be written

$$\varphi_\lambda = m^{-1}\partial_\lambda\varphi + m^{-2}(g_1^{(\mu)}J_\lambda^{(\mu)} + g_1^{(e)}J_\lambda^{(e)}),$$

$$\varphi = m^{-1}\partial_\lambda\varphi_\lambda + m^{-2}(g_2^{(\mu)}J^{(\mu)} + g_2^{(e)}J^{(e)}), \quad (1)$$

when both scalar and vector interactions are present, where

$$J_\lambda^{(e)} = \bar{\psi}_{(e)}\gamma_\lambda\frac{1-\gamma_5}{2}\psi_{(\nu)} + \text{c.c.},$$

$$J^{(e)} = \bar{\psi}_{(e)}\frac{1-\gamma_5}{2}\psi_{(\nu)} + \text{c.c.}$$

Only in the absence of any vector interaction is φ_μ simply proportional to $\partial_\mu\varphi$. The induced normal muon decay interaction, to lowest order in $g_1^{(e)}$ and $g_2^{(e)}$, is

$$m^{-2}g_1^{(\mu)}g_1^{(e)}\int d^4x\,J_\lambda^{(e)}(x)J_\lambda^{(\mu)}(x)$$

$$+\left(g_2^{(\mu)} + g_1^{(\mu)}\frac{m^{(\mu)}}{m}\right)\left(g_2^{(e)} + g_1^{(e)}\frac{m^{(e)}}{m}\right)$$

$$\times\int d^4x\,d^4x'\,J^{(e)}(x)D_F(x-x',m)J^{(\mu)}(x'). \quad (2)$$

The first term is a purely local V-A interaction, the second a nonlocal S-P interaction. If $g_2^{(e)} = 0$, the scalar admixture for muon decay becomes $m^{(\mu)}m^{(e)}/m^2$ – less than 10^{-4} for nucleonic mass of the intermediary.[5] Alternatively, if either or both of the factors $[g_2^{(\mu)} + g_1^{(\mu)}(m^{(\mu)}/m)]$ and $[g_2^{(e)} + g_1^{(e)}(m^{(e)}/m)]$ are set equal to zero, the normal μ-decay interaction (to this order in g) becomes a purely local one.

Using the Klein-Gordon formalism, we obtain a muon decay interaction identical to Eq. (2) except for the absence of the direct vector coupling. The two versions of the spin-zero theory are identical for $g_1^{(e)}=0$, and in general differ only by a direct self-interaction of $J_\lambda^{(e)}$. The choice between these formalisms depends upon whether one regards first-order or second-order field equations as the more basic. [It is only the Kemmer theory that may be developed according to the canonical formulation of Schwinger.[6]]

For the discussion of $\mu \to e+\gamma$, we shall assume $g_2^{(e)}=0$ and neglect finite contributions proportional to the electron mass. The matrix element for this mode becomes (q = photon momentum)

$$\frac{e}{96\pi^2}\frac{g_1^{(\mu)}g_1^{(e)}}{m^2}\bar{\psi}_{(e)}\frac{1-\gamma_5}{2}\left\{\tfrac{1}{2}m^{(\mu)}\sigma_{\alpha\beta}F^{\alpha\beta}-\gamma_\mu A_\mu q^2\left[\ln\left(\frac{K^2}{m^2}\right)+c+O\left(\frac{q^2}{m^2}\right)\right]\right\}\psi_{(\mu)},$$

where K is a cutoff to the divergent momentum integration, and c is a finite constant which depends upon the manner in which the cutoff is introduced. The decay interaction contains a finite, point-like, "moment" interaction, and a divergent "charge" interaction which does not contribute to the free-particle decay. The branching ratio for the decay of a free muon into the $e+\gamma$ channel is

$$\rho=\alpha/24\pi\approx 10^{-4},$$

which is ~40 times greater than the current experimental upper limit.[7] In contrast to the result with a vector intermediary,[8-10] the result is both convergent and unambiguous.

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SOME GRAVITATIONAL WAVES*

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The few hitherto known wave-like exact solutions of the Einstein gravitational equations represented either plane[1,2] or cylindrical[3] waves. We here intend to derive a new class of solutions, displaying a lesser degree of symmetry, and thus more generality.

Let us consider the metric

$$ds^2=-dx^2-dy^2-dz^2+dt^2-2f(x,y,z+t)(dz+dt)^2.$$

One can easily show that $R_{zz}=R_{zt}=R_{tt}=f_{xx}+f_{yy}$, and other components vanish. If f is a harmonic function of x and y, this metric thus satisfies Einstein's equations in vacuo, whatever may be its dependence on $(z+t)$.

A convenient tetrad of orthonormal vectors is

$$h_{(1)}{}^k=(\cos a,\ \sin a,\ 0,0),$$
$$h_{(2)}{}^k=(-\sin a,\ \cos a,\ 0,0),$$
$$h_{(3)}{}^k=(0,0,1-f,f),$$
$$h_{(4)}{}^k=(0,0,-f,\ 1+f),$$

where $\tan(2a)=(f_{xy}/f_{xx})$. The only nonvanishing independent physical components[4] of the curvature tensor are

$$\sigma=R_{(m1n1)}=-R_{(m2n2)}=(f_{xx}{}^2+f_{xy}{}^2)^{1/2},$$

where m and n take the values 3 and 4 only. Our