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TRANSFORMATION OF MUONS INTO ELECTRONS*

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It is an outstanding puzzle of particle physics that muons do not decay electromagnetically into electrons without the emission of neutrino pairs, even though all quantum numbers of muon and electron are the same.[1] Processes of this sort which could have been observed include $\mu\to e+\gamma$ decay,[2] $\mu\to e$ via absorption of virtual photons in a mesonic atom,[3] $\mu\to 3e$ via internal conversion,[4] muonium decay into photons,[5] etc. The absence of such transformations does not constitute a paradox, there being no compelling reason why muons should transform into electrons, but it seems a mystery that processes which are allowed energetically and in every other known respect do not occur. In this note we shall indicate how for a certain class of interactions which might be expected to induce $\mu-e$ transformations, the similarity of the μ and e actually forbids the transformations.

Let us first consider why $\mu\to e$ transformations do not take place as strong and electromagnetic interactions; for example, why is $\mu\to e+\gamma$ decay not about as fast as $\Sigma^0\to\Lambda^0+\gamma$ decay? Let us suppose that the muon and electron are described by a local Lagrangian $\mathcal{L}$ involving only the fields $\psi_e, \psi_\mu, A^\lambda$ where ψ_e and ψ_μ can be chosen to have the same parity. Now, the simplest mechanisms that might yield fast $\mu\to e$ transformations are contact terms in $\mathcal{L}$,

$$\mathcal{L}_1=-\rho\bar{\psi}_e\psi_\mu-\text{H.c.};\quad \mathcal{L}_2=-\xi\bar{\psi}_e\gamma\cdot D\psi_\mu-\text{H.c.},\qquad(1)$$

where the derivative ∂_λ and the photon field A_λ enter in the gauge-invariant combination, $D_\lambda=\partial_\lambda-ieA_\lambda$. These two interactions exhaust all possible parity-conserving, renormalizable, gauge- and Lorentz-invariant interactions among photons, muons, and electrons that can be added to the usual Dirac Lagrangian in order to generate $\mu\to e$ transformations. Furthermore, because the μ and e appear to differ only in their rest mass, it seems not unreasonable to regard them as "different states of the same particle." In this case, since the bare Lagrangian is known to contain $\bar{\psi}_\mu\psi_\mu$ and $\bar{\psi}_e\psi_e$ terms of the type 1, one might also expect "off-diagonal" $\bar{\psi}_\mu\psi_e$ terms. However, since quantum electrodynamics has been successful without the introduction of nonminimal electromagnetic interactions or other nonrenormalizable interactions, one is also inclined to exclude "off-diagonal" terms of this kind, at least from the fundamental Lagrangian.

Suppose we now attempt to calculate, as an example, the lowest order $\mu-e-\gamma$ vertex. There are five diagrams that add to give a nonvanishing function of the lepton momenta,

$$\Gamma_{e\mu}{}^{\lambda}=e\rho[\gamma^{\lambda}S(p_{\mu},m_{e})+S(p_{e},m_{\mu})\gamma^{\lambda}] +e\xi[\gamma^{\lambda}+\gamma^{\lambda}S(p_{\mu},m_{e})ip_{\mu}\cdot\gamma +ip_{e}\cdot\gamma S(p_{e},m_{\mu})\gamma^{\lambda}], \quad (2)$$

where $S(p,m)=(i\gamma\cdot p-m)/(p^2+m^2)$. However, when we put the lepton momenta on their mass shells (i.e., set $p_\mu\cdot\gamma=im_\mu$, $p_e\cdot\gamma=im_e$), we find a striking cancellation; the sum of the first two, and of the last three terms becomes zero.

We shall now show that the cancellation of the matrix element for $\mu\to e$ transformations takes place to all orders in ρ, ξ and e provided that $|\xi|<1$, and with any number of real or virtual photons emitted or absorbed. Thus it is impossible to induce $\mu\to e$ transformations without introducing either new fields, or more complicated (and nonrenormalizable) interactions such as

$$\bar\psi_e\sigma_{\lambda\eta}\psi_\mu F^{\lambda\eta}.$$

Our proof is based on the fact that there exists a new set of fields, ψ_e', ψ_μ' which are linear combinations of ψ_e and ψ_μ such that the total Lagrangian when written in terms of the ψ' becomes just the sum of two uncoupled Dirac Lagrangians for ψ_e' and ψ_μ' separately. To see this let us introduce a matrix notation, writing

$$\psi=\begin{pmatrix}\psi_\mu\\ \psi_e\end{pmatrix}.$$

Then the Lagrangian obtained by adding $\mathcal{L}_1+\mathcal{L}_2$ to the usual Dirac Lagrangian may be written

$$\mathcal{L}=-\bar\psi\gamma\cdot D\mathcal{A}\psi-\bar\psi\mathcal{B}\psi+\mathcal{L}_m, \quad (3)$$

where $\mathcal{L}_m=-\frac{1}{4}F_{\lambda\eta}F^{\lambda\eta}$, and

$$\mathcal{A}=\begin{pmatrix}1 & \xi^*\\ \xi & 1\end{pmatrix},\quad \mathcal{B}=\begin{pmatrix}m_\mu & \rho^*\\ \rho & m_e\end{pmatrix}. \quad (4)$$

We shall only make use of the Hermiticity of $\mathcal{A}$ and $\mathcal{B}$, and the positivity of $\mathcal{A}$ (for $|\xi|<1$); our proof will work for any number of fields. The condition that $\mathcal{A}$ is positive is required to make the canonical anticommutation relations consistent and so is not really a new assumption. If we introduce the fields ψ' by writing

$$\psi=S\psi',\quad (\mathrm{Det}S\neq 0) \quad (5)$$

then clearly $\mathcal{L}$ may be rewritten as

$$\mathcal{L}=-\bar\psi'\gamma\cdot DS^\dagger\mathcal{A}S\psi'-\bar\psi'S^\dagger\mathcal{B}S\psi'+\mathcal{L}_m. \quad (6)$$

It is well known[6] that even though $\mathcal{A}$ and $\mathcal{B}$ may not commute, we can always choose S such that $S^\dagger\mathcal{A}S=1$ and $S^\dagger\mathcal{B}S$ is diagonal. When we choose S in this fashion, $\mathcal{L}$ becomes

$$\mathcal{L}=-\bar\psi_{e'}(\gamma\cdot D+m_{e'})\psi_{e'} -\bar\psi_{\mu'}(\gamma\cdot D+m_{\mu'})\psi_{\mu'}+\mathcal{L}_m, \quad (7)$$

where $m_{e'}$ and $m_{\mu'}$ are given by the eigenvalues of $S^\dagger\mathcal{B}S$. The field equations and commutation rules for ψ' are the usual equations for uncoupled leptons, whether deduced from (7) or from (3) and (5).

By inspection we see that the theory described by (7) possesses two stable leptons, which of course we call electron and muon, and is invariant under separate gauge transformations on $\psi_{e'}$ and $\psi_{\mu'}$, which means that transitions between e' and μ' are forbidden. However, it may not be clear why we obtain a cancellation when [as in (2)] we compute the Feynman diagrams for $\mu\to e$ transformations from (3), rather than the $\mu'\to e'$ transformations. When we compute using (3), what we obtain is the Fourier transform of a quantity

$$\Gamma_{iJ}{}^{\lambda_1\cdots\lambda_n}(x,y,z_1,\cdots z_n)=\langle T\{\psi_i(x)\bar\psi_J(y),A^{\lambda_1}(z_1)\cdots A^{\lambda_n}(z_n)\}\rangle_0,\quad (i,J=\mu,e), \quad (8)$$

whereas using (7) we obtain a quantity Γ_{iJ}' defined correspondingly in terms of ψ_i' and ψ_J'. Using (5) we have

$$\Gamma=S\Gamma'S^\dagger, \quad (9)$$

and since $\Gamma_{e\mu}'=0$,

$$\Gamma_{e\mu}=S_{ee}S_{\mu e}{}^{*}\Gamma_{ee}'+S_{e\mu}S_{\mu\mu}{}^{*}\Gamma_{\mu\mu}', \tag{10}$$

and, as in (3), this is not zero. But when we compute matrix elements from (10), what we calculate is[7]

$$\int d^4x d^4y\, \bar{u}_e(x)\left[\left(\gamma\cdot\frac{\partial}{\partial x}+m_e{}^R\right)\Gamma_{e\mu}{}^{\lambda_1\cdots\lambda_n}(x,y,z_1,\cdots z_n)\left(\gamma\cdot\frac{\partial}{\partial y}+m_\mu{}^R\right)\right]u_\mu(y), \tag{11}$$

where $m_e{}^R$, $m_\mu{}^R$ are renormalized masses. Substitution of (10) into (11) gives zero for both terms, since Γ_{ee}' has no singularity on the muon mass shell, so that we can integrate by parts and use

$$\left(\gamma\cdot\frac{\partial}{\partial y}+m_\mu{}^R\right)u_\mu(y)=0, \tag{12}$$

and similarly $\Gamma_{\mu\mu}'$ has no singularity on the electron mass shell. Actually it may be seen from a very general theorem[8] on the particle interpretation of field theory that the matrix element $\langle e\,|\,T\{A^{\lambda_1}\cdots A^{\lambda_n}\}\,|\,\mu\rangle$ is given correctly in general (except for field renormalization constants) by applying the reduction formula (11) to either $\Gamma_{e\mu}'$ or $\Gamma_{e\mu}$ or even to the diagonal elements of Γ' or Γ.

If the μ and e had anomalous magnetic moments of nonelectromagnetic origin, the theorem would not in general hold, and the existence of the terms (1) would then lead to $\mu-e$ transitions.

If the original Lagrangian contained weak interactions, given for example (symbolically) by

$$\mathcal{L}_\beta=-\bar{\psi}_\nu O\psi_i\bar{\psi}_n O\psi_p\beta_i-\text{H.c.}, \tag{13}$$

$$\mathcal{L}_{\bar{\nu}\nu}=-\bar{\psi}_J O\psi_i\bar{\psi}_\nu O\psi_\nu n_{Ji}, \tag{14}$$

then $\mathcal{L}_\beta$ and $\mathcal{L}_{\bar{\nu}\nu}$ can also be written in the same way, in terms of ψ', β', n', where

$$\beta'=\beta S,\quad n'=S^\dagger n S. \tag{15}$$

If the μ and e had other weak Fermi-type interactions such as $\bar{\psi}_\mu\psi_\mu\bar{\psi}_e\psi_e$, the transformation to the new fields μ', e' would induce terms which give $\mu'\to e'$ transitions. These would be comparable in strength to ordinary weak interactions.

The above discussion indicates that we may be able to define "fundamental" lepton fields ψ such that (a) the free fields and electromagnetic interactions are invariant under the interchange of ψ_e and ψ_μ; (b) the weak interactions break this symmetry by involving only one (say ψ_μ) of these two fields; (c) there are no off-diagonal derivative-type interactions. This is accomplished by setting

$$\xi=0,\quad \rho\ \text{real},\quad m_e=m_\mu=m,$$

$$\beta\simeq(1,0),\quad \eta\simeq\begin{pmatrix}1&0\\0&0\end{pmatrix}. \tag{16}$$

It then follows that the matrix S is independent of ρ and m, and given by

$$S=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}, \tag{17}$$

so that the experimentally observable parameters become

$$\beta'\sim\frac{1}{\sqrt{2}}(1,1),\quad n'=\begin{pmatrix}1&1\\1&1\end{pmatrix}\frac{1}{2}. \tag{18}$$

Thus this hypothesis is in agreement with the experimental fact that β decay and μ absorption have about the same unrenormalized coupling constants. (We can also obtain $m_e'\ll m_\mu'$ by assuming $\rho\simeq m$.) It is in any case possible to define ψ_e and ψ_μ so that the asymmetry between them is induced by weak interactions and not by the free field, rather than vice versa, provided that the original Lagrangian is of the form (4) + (13) + (14).

We may also apply these considerations to the case of weak contact terms, diagonal or off-diagonal. Here we may not assume invariance under space inversion, and hence must also introduce terms of form

$$\mathcal{L}_3\sim\bar{\psi}\gamma_5\mathfrak{D}\psi+\bar{\psi}D\cdot\gamma\gamma_5\mathfrak{C}\psi. \tag{19}$$

The total Lagrangian is then

$$\mathcal{L}=-\bar{\psi}[D\cdot\gamma\mathfrak{A}+\mathfrak{B}+D\cdot\gamma\gamma_5\mathfrak{C}-i\gamma_5\mathfrak{D}]\psi+\mathcal{L}_m, \tag{20}$$

where $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}$ are Hermitian and differ from their values in (3) by terms of first order in the weak interactions. It can be shown that, defining ψ' by

$$\psi=(S+T\gamma_5)\psi', \quad [\mathrm{Det}(S+T\gamma_5)\neq 0], \qquad (21)$$

we can choose S and T such that $\mathcal{L}$ is given by (7), to first order in the weak interactions.[9] This is of some "practical" importance, since if there is any structure in μ decay,[10] it may lead to effective interactions such as those discussed above through diagrams in which ν and $\bar{\nu}$ annihilate virtually. But the terms of form (1) and (20) cannot themselves lead to any electromagnetic $\mu\to e$ transitions.[11]

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[1]Unless we are willing to give up the 2-component neutrino theory, we know that $\mu\to e+\nu+\bar{\nu}$.

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[9]It is likely that this theorem is true to all orders in "weak interactions" of the type included in the Lagrangian (20). That is, by a general transformation of the form (21), one can transform $\mathcal{L}$ into a new Lagrangian in which $\mathcal{A}=1$ and $\mathcal{B}=\begin{pmatrix} m_\mu' & 0 \\ 0 & m_e \end{pmatrix}$, with $\mathcal{C}=0$ and $\mathcal{D}=0$.

[10]S. Bludman and A. Klein, Phys. Rev. 108, 550 (1958); T. D. Lee and C. N. Yang, Phys. Rev. 108, 1611 (1958).

[11]This fact has been noted by M. Gell-Mann, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 261, and by N. Cabibbo and R. Gatto (to be published). The proof of Cabibbo and Gatto actually applies directly only to the nonderivative terms of (1) and (20). A very simple general proof is as follows: We have already shown that diagrams containing vertices of type (1) and (2) alone, or (20) and (21) alone, do not lead to $\mu\to e$ transformations. But to lowest order in weak interactions, each diagram contains a single vertex of type (1) or (20).