If we take $\theta = \theta_l = 450°K$,

$$A_v = 0.4\, A_v^{cl}\, \theta_l / T, \qquad (10)$$

in agreement with theory.

The value of the surface contribution is much smaller than has previously been supposed.[5] It has been shown[6] that A_s is given by the expression

$$A_s = (1 - p) \times \tfrac{3}{4}(v/c), \qquad (11)$$

where p is the fraction of electrons specularly reflected from the surface, and v is an average of the Fermi velocity of the electrons. If it is assumed that there is diffuse scattering of the electrons ($p = 0$), Eq. (11) gives $A_s = 0.0027$ when one takes $v = 1.55 \times 10^8$ cm sec^{-1}. The present data show that this is far too large and hence that we must suppose $p \sim 1$ to obtain agreement with theory, i.e., the scattering of the electrons by the surface at optical frequencies is predominantly specular. This behavior is to be contrasted with that at microwave frequencies, where the scattering has been shown[7] to be essentially diffuse. No explanation of this discrepancy can be advanced.

Measurements on silver and gold are currently being made and will be reported in the near future.

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NEW METHOD FOR THE EVALUATION OF h/e FROM THE QUANTUM LIMIT OF THE CONTINUOUS X-RAY SPECTRUM

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Planck's constant, h, is one of the least accurately determined of the physical constants. This fact is all the more surprising, as there exists a quite accurate method among the so-called "h/e experiments" contributing most to its determination. This method is the determination of the short-wavelength limit (SWL) of the continuous x-ray spectrum.[1-3] Unfortunately, this method suffers from a serious systematic error of hitherto unknown source.[4,5] Cohen and DuMond[4] and Bearden and Thomsen[5] suspected the origin of the discrepancy between this and all other determinations of h to lie in the ignorance of the exact shape of the SWL, and in the associated inability for identifying the true position of the threshold.

It will be shown, in this method, how these twenty year old discrepancies may be resolved by means of the isochromat measurements which we have made. We will also show that the SWL experiments agree with the majority of the other experiments. This problem is also important for the new determinations of some of the input data for planned readjustments of the atomic constants.[6]

The investigations of the SWL are describable in terms of the measurements of isochromats, i.e., the voltage applied to the x-ray tube is varied in small steps, and the intensity for a fixed wavelength setting of a monochromator is measured. For the small wavelength range of interest, this physical situation leads to a scale replica of the spectral intensity curve in the vicinity of the SWL. The problem is then to find out the exact position of the threshold voltage belonging to the wavelength setting of the monochromator. Up to now, the usual procedure has been to identify this position with that of the "point of maximum bending" (maximum of the second derivative) of the observed isochromat.[4,5,7]

The implicit model, in Sommerfeld's theory of bremsstrahlung, which has previously applied, is that of an encounter of an electron with a stripped, isolated nucleus in the so-called free-

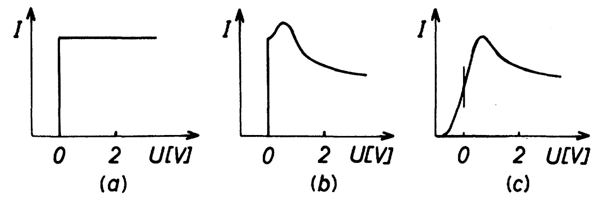

FIG. 1. Ideal W isochromats. (The abscissa is the difference between tube voltage and threshold voltage; nominal voltage ≈ 1250 v.) (a) According to Sommerfeld's theory of bremsstrahlung; (b) allowance made for the band structure of tungsten ($T = 0°$K); (c) allowance made for the finite temperature of the anticathode.

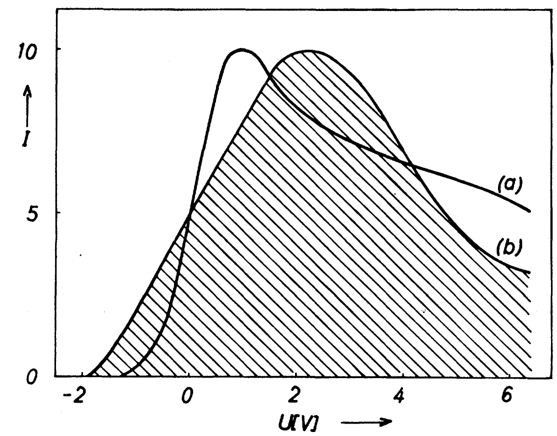

FIG. 2. Theoretical (a) and experimental (b) isochromat, normalized to same height of maximum. (Abscissa as in Fig. 1.)

free transition. The resulting form of the ideal isochromat for monochromatic electrons and a thin anticathode is shown in Fig. 1(a), where the isochromat is considered measured with infinitely good resolution. The physical situation described by Sommerfeld's theory is not quite accurate, because any real anticathode is made of a metallic sheet. One has to take into account the special properties of the solid state. The electrons having generated an energetic quantum in the neighborhood of the SWL must be located in the nonoccupied part of the conductivity band. The distribution of these free positions determines the energy distribution of the continuous spectrum at the SWL and therefore also the shape of the isochromat. Now we make the first assumption that the transition probabilities to the final states of the electrons are equal for all free positions in the nonoccupied regions of the conductivity band. Then the distribution of these positions gives directly the shape of the ideal isochromat. This is carried out in Fig. 1(b) for the band structure of tungsten by Manning and Chodorow.[8] The Fermi edge of W corresponds to the threshold voltage. It should be emphasized here that our final result depends only to a very small degree on the details of the band structure. In Fig. 1(c) allowance is made for the high temperature of the anticathode during our measurements resulting in a broadening of the Fermi edge. We make now the second essential assumption that we obtain the thin anticathode spectrum within about the first ten volts from threshold even for the really thick anticathode used for the measurements. This assumption, though surprising at first sight, is supported quite well by the distribution of the characteristic energy losses of the electrons in W.[9,10] With this conception the measured isochromat differs from that of Fig. 1(c) only due to the finite resolving power of the experimental apparatus. We obtain the "instrumental window" of our experiment by folding up numerically the following influences: (1) the monochromator pass-band; (2) the fluctuations of the x-ray tube voltage; (3) the energy distribution of the emitted electrons. Now this instrumental window is folded with the isochromat of Fig. 1(c). Having considered all influences correctly we should obtain a curve in conformity with our measured isochromat. Fig. 2(a) shows the calculated isochromat and (b) the measured one. The resemblance is quite satisfactory. The remaining deviation of the rise to the maximum has its origin probably in the limited accuracy of the calculation of the resolving power.

For our problem the most important information in Fig. 2 is that concerning the threshold voltage position. It is simply given by the abscissa belonging to the ordinate with half the value of the maximum. The consequences of this result are contained in Table I (corresponding to Table VII of reference 4). It shows the conparable SWL experiments since 1950. Comparable in this sense means measurements with W anticathodes and W equipotential cathodes. The discrepancy of the last two columns is the deviation of the threshold voltage in the SWL experiments from the threshold voltage required by the bulk of the other experiments, first for the previous evaluation method and then for the new conception. The sum of discrepancies in the last column disappears below the error limits

Table I. X-ray quantum limit determinations.

Experiment	Nominal voltage (volts)	Resolution[a] (volts)	Discrepancy (volts) old	Discrepancy (volts) new
BS[b]	8050	0.75	-1.4	+1.5
BS[b]	19600	1.7	-3.4	-1.4
BJW[c]	6112	0.55	-0.7	+0.4
BJW[c]	10168	0.95	-0.8	+0.4
		Sum of discrepancies	-6.3	+0.9
FHD[d]	24500	12	-4.3	+3.4

[a]Full half-width.
[b]J. A. Bearden and G. Schwarz, Phys. Rev. 79, 674 (1950).
[c]Bearden, Johnson, and Watts, Phys. Rev. 81, 70 (1951).
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of the measurements. Due to the bad resolution, the newest investigation in the lowest line does not fulfill our second assumption and therefore is not taken into account. It should be mentioned that values of the last column contain also a small correction for the mean velocity of the emitted electrons as first recognized by Bearden and Thomsen.[5]

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THEORY OF THE VECTOR INTERACTION WITH A CONSERVED CURRENT AND THE BETA DECAY OF Na^{24}-Al^{24} NUCLEI

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In the theory of the vector interaction with a conserved current the observed deviations from the $\Delta T=0$ selection rule in Fermi transitions have to be explained only in terms of isotopic spin impurities.[1] However, in the conventional theory exchange mesonic currents may also induce Fermi transitions with $\Delta T \neq 0$. Recently an attempt[2] has been made to estimate the contribution of isotopic spin impurities introduced by the Coulomb interaction between the protons. The relevant Coulomb matrix elements have been calculated with the wave functions given by the j-j coupling shell model. A comparison between the calculated Fermi matrix element and the experimental one was performed in the case of the β^- decay of the $T=1$, $J=4^+$ state of Na^{24} to the state $T=0$, $J=4^+$ of Mg^{24}. The experiments[3] on Na^{24} agree only with a value of M_F smaller than 10^{-3} while the theoretical estimate yields a value of about 1.3×10^{-2}. The two following interpretations are possible: (a) By using j-j coupling shell model wave functions the Coulomb matrix ele-