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EXPERIMENTS ON THE HYDRODYNAMIC STABILITY OF HELIUM II BETWEEN ROTATING CYLINDERS*

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The hydrodynamic stability of the flow of liquid helium II between rotating cylinders has been examined theoretically by Chandrasekhar and Donnelly.[1] They showed that if one begins with a two-fluid set of equations of motion containing a linear coupling force (mutual friction) between the normal and superfluid components, there should be two critical velocities at which instability begins. The first of these to appear as the speed of rotation is increased is clearly associated with the motion of the superfluid component, the second with the normal fluid component of the liquid. The particular choice of the form of the coupling term determines the exact location of the two critical velocities.

In order to test this theory, a rotating cylinder viscometer has been built and has proved to be a very sensitive instrument for studying stability in ordinary liquids.[2,3] In particular, the onset of instability can be located within very small speed intervals and agrees with theory to within 1%.[2] The instrument, as used for the measurements reported here, is substantially the same as in reference 2 except that the null position is determined by means of a special interferometer which will be described in detail elsewhere.

This Letter reports preliminary measurements with helium II. In these experiments the deflection of the outer cylinder is denoted as ϕ and the period of rotation of the inner cylinder as P, so that in laminar flow the product ϕP is a constant proportional to the viscosity η of the liquid. The constant $C = \eta/\phi P$ is determined by calibration in air. In all cases reported here the cylinders have radii $R_1 = 1.9$ cm and $R_2 = 2.0$ cm. In Fig. 1 (a) is shown the result of measurements on liquid nitrogen which has been kept from boiling by a slight overpressure. The critical velocity is marked by an abrupt increase in ϕP at $P_c = 3.46$ sec. The slight rise in effective viscosity just before the critical period of rotation is observed in all ordinary liquids.[2] It amounts to less than 1% and cannot be seen in the helium measurements because of the small torques being measured.

The results at three different temperatures in liquid helium are shown in Figs. 1 (b), (c), and (d). It can be seen that the results are much different than in (a), particularly at the lowest temperatures. There appear to be three regions of interest. At long periods of rotation the viscosity is shear-independent. The value of viscosity corresponding to the horizontal line agrees satisfactorily with values obtained by Heikkila and Hollis Hallett[4] for the case where torque is measured on an inner cylinder instead of the present arrangement. The values of ϕP begin to rise at the speed denoted by P_s and tend to level off; the values of ϕP in this region are not very reproducible and may depend upon the rate of increase of speed of the motor. The amount of rise in effective viscosity varies with temperature, being greatest at the lowest temperature. This suggests that the superfluid component is involved. At the speed indicated by P_n there is a sudden change of slope in the curve corresponding to that observed at P_c in (a). At P_n characteristic fluctuations of the mirror on the suspension are seen just as in ordinary liquids. The appearance of these curves is in many ways similar to the damping vs amplitude curves discussed by Donnelly and Hollis Hallett[5] for oscillating systems in helium II.

The preliminary results shown in Fig. 1 are consistent with the prediction of Chandrasekhar and Donnelly that there are two critical velocities in this experiment. The rise in ϕP at P_s is considered to be due to cellular motion associated with instability in the superfluid and the rise at P_n with cellular motion associated with instability in the normal fluid. The experiment is being continued at different temperatures and with different cylinder sizes. We hope also to increase the sensitivity of the viscometer so as to give greater accuracy at low speeds. It should then be possible to make quantitative comparisons with the theory.

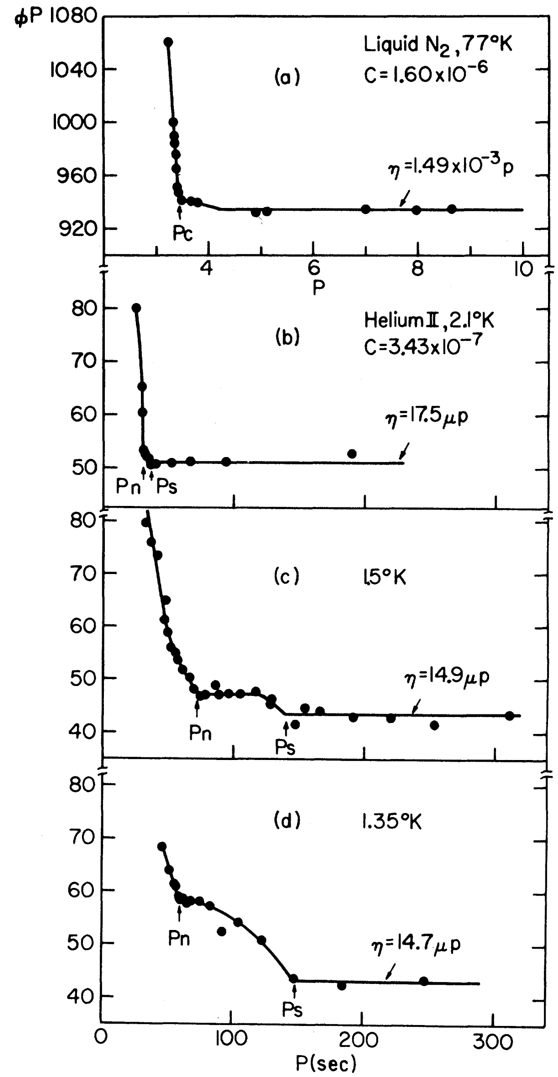

FIG. 1. The variation of ϕP, which is proportional to the effective viscosity, with period of rotation P of the inner cylinder. (a) Liquid nitrogen at 77°K; (b), (c), and (d) liquid helium II at three different temperatures. P_s and P_n denote the critical periods of rotation for instability due to the superfluid and normal fluid components.

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[5] R. J. Donnelly and A. C. Hollis Hallett, Ann. Phys. 3, 320 (1958).