find a very close agreement between the resulting electron energies and the expected appearance potentials of O^- and C^-. In Table I we have listed these energies and the appearance potentials for negative ions derived from CO if the dissociation energy of CO is taken as 11.11 ev and the affinity of O as 1.45 ev.[4] For $C^- + O^+$ we have listed values taken from the theses of Lagegren[5] and Petrocelli[6] separately. In CO, as in H_2, we find at least one very broad peak below the first one that appears from free electron capture. In the present case the ion formed would have to be CO^- if our ideas are correct. A mass spectrographic analysis is under way to elucidate these processes further.

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DIPOLE STATE IN NUCLEI*

G. E. Brown† and M. Bolsterli
School of Physics, University of Minnesota, Minneapolis, Minnesota
(Received October 28, 1959)

In order to explain the unexpectedly high (γ, p) cross section in heavy nuclei, various authors[1,2] have proposed that these protons arise mainly from a direct process. The close relationship of this process to the shell model and optical model has been elucidated by Wilkinson,[3] who points out that the initial state of the nucleus is quite well described by the shell model. The proton involved in the direct process can then be considered as being initially in an eigenstate in the shell-model well. Upon absorption of the dipole gamma ray, the proton makes a transition to either a bound level in the well or one in the continuum. Because this state is not stationary, it is given a width $\Gamma + 2W$, where W is the absorption in the optical-model well at the relevant excitation and describes the absorption of the single-particle excitation into compound states and Γ is the width for escape. The proportion of fast protons that escapes is then $\Gamma/(\Gamma + 2W)$. The picture is very appealing, in that it produces the observed order of magnitude of fast particles, which is several orders of magnitude greater than the statistical description predicts. The relation of this description to one in terms of compound states of the system has been given in detail.[4] In reference 4 it is made clear that the highly excited levels discussed by Wilkinson are really combinations of thousands or millions of compound states which, however, act coherently as a single-particle state for some processes. We will refer to these groups of compound states as single-particle excitations.

The positions of the single-particle excitations can be found directly through other processes. Recently, (d, p) experiments using poor resolution[5,6] have determined the positions of single-particle excitations lying between zero bombarding energy and the binding energy of the last neutron. The spacings of these excitations seem to be in sharp conflict with those required by Wilkinson. For example, in Ti^{49}, the $f_{7/2}$ and $g_{9/2}$ levels are only about 4 Mev apart. However, just the transition between these two levels is an appreciable part of the giant dipole resonance in this nucleus, which comes at an energy of about 15 Mev. It is true, of course, that one must add a pairing energy to the 4 Mev before making the comparison, because in the absorption of the gamma ray a pair is generally broken. However, this is only one or two Mev. It seems, therefore, that the transition between single-particle excitations should occur at an energy of only about half that of the giant dipole resonance.

We should like to point out in this note that these two energies cannot be compared directly, since, in the dipole absorption, a hole is formed in the nucleus. Since the process is a dipole one, the excited particle and hole are strongly correlated in angle; i.e., their angular momentum must be coupled to form a 1- state, assuming the original nucleus to be in a 0+ state. Because many particle-hole states can be formed, and because these states are almost degenerate in energy, the particle-hole interaction can have a profound effect in redistributing dipole transition strength.

We shall demonstrate these effects by using a schematic model, the mathematics of which is suggested by the Copenhagen work on pairing interactions.[7] Major numerical approximations are made in going from the actual situation to this rough schematic model. The model does, however, exhibit how coherent effects are able to push the dipole transitions to much higher energies than one would, at first sight, think possible.

We consider first protons in a potential well, and will indicate the extension to the case of protons and neutrons in a nucleus later. The main contributions to the absorption come from closed shells. We shall, therefore, specialize our discussion to nuclei with double closed shells, neglecting the influence of the few valence nucleons. We neglect spin, and consider only transitions from l to $l+1$. Choosing our "vacuum" as the initial nucleus, we see that the gamma ray creates a particle-hole pair through the process shown in Fig. 1. With axes oriented so that the Hamiltonian describing the interaction with radiation is

$$H_I = e(2\pi\hbar\omega)Z, \tag{1}$$

the particle-hole state formed in absorption of the gamma ray is

$$\varphi_i(\vec{r}_p, \vec{r}_h) = (-)^{l_i}[Y_{l_i}(\theta_h, \varphi_h)Y_{l_i+1}(\theta_p, \varphi_p)]_0^1 R_{l_i}(r_h)R_{l_i+1}(r_p), \tag{2}$$

where

$$[Y_{l_i}Y_{l_i+1}]_0^1 = \sum_m C(l_i, l_i+1, 1; m, -m, 0)Y_{l_i}^{\ m}Y_{l_i+1}^{\ -m}.$$

The lower suffixes p and h refer to particle and hole, the R's are radial wave functions, and the C's are Clebsch-Gordan coefficients. The phase factor $(-1)^{l_i}$ is put in for convenience to make all quantities positive. In medium and heavy nuclei, a large number of particle-hole states can be formed. The particle-hole interaction will mix these, and to find the perturbed eigenstates, we must solve the secular equation. In realistic situations, this is quite complicated, e.g., see Elliott and Flowers[8] where this is carried out for O^{16}. We shall, therefore, make several approximations which give us a schematic model where we can display the qualitative features explicitly. We shall return later to a discussion of the approximations. First, we shall use zero-range forces, i.e., we take the particle-hole interaction to be

$$V(\vec{r}_p - \vec{r}_h) = V_0\delta(\vec{r}_p - \vec{r}_h),$$

with V_0 positive. (If the particle-particle force is attractive, the particle-hole one is repulsive.) Now when Y_{l_i} and Y_{l_i+1} have the same argument,

$$[Y_{l_i}Y_{l_i+1}]_0^1 = (-)^{l_i}[(l_i+1)/4\pi]^{1/2}Y_1^{\ 0}, \tag{3}$$

so that the diagonal elements of our secular matrix are

$$\epsilon_i + (l_i+1)(V_0/4\pi)\int_0^\infty R_{l_i}^{\ 2}R_{l_i+1}^{\ 2}r^2dr,$$

where the ϵ_i are the unperturbed energies, i.e., the energies of the dipole excitations Wilkinson is considering. The off-diagonal elements are

$$(l_i+1)^{1/2}(l_j+1)^{1/2}(V_0/4\pi)\int_0^\infty R_{l_i}R_{l_i+1}R_{l_j}R_{l_j+1}r^2dr.$$

We next make the further approximation of set-

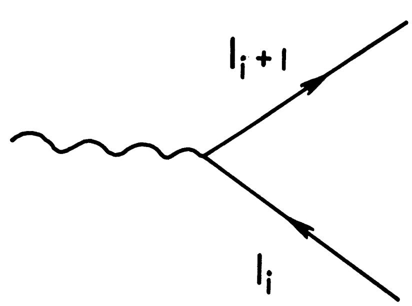

FIG. 1. Diagram representing the process by which a gamma ray creates a particle-hole pair.

ting the radial integrals equal, i.e., we set

$$(V_0/4\pi)\int_0^\infty R_{l_i}{}^2 R_{l_i+1}{}^2 r^2 dr = (V_0/4\pi)\int_0^\infty R_{l_i} R_{l_i+1} R_{l_j} R_{l_j+1} r^2 dr = G, \tag{4}$$

giving us a secular equation

$$0 = \begin{vmatrix} \epsilon_1 + (l_1+1)G - \lambda & (l_1+1)^{1/2}(l_2+1)^{1/2}G & (l_1+1)^{1/2}(l_3+1)^{1/2}G & \cdots \\ (l_1+1)^{1/2}(l_2+1)^{1/2}G & \epsilon_2 + (l_2+1)G - \lambda & (l_2+1)^{1/2}(l_3+1)^{1/2}G & \cdots \\ (l_1+1)^{1/2}(l_3+1)^{1/2}G & (l_2+1)^{1/2}(l_3+1)^{1/2}G & \epsilon_3 + (l_3+1)G - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix}$$

By collecting powers of $(\epsilon_i - \lambda)$ one can easily show that this is equivalent to the equation

$$\prod_{i=1}^{n} (\epsilon_i - \lambda) + \sum_{j=1}^{n} \prod_{i=1(i\neq j)}^{n} (\epsilon_i - \lambda)(l_j+1)G = 0, \tag{6}$$

which in turn is equivalent to

$$\sum_{j=1}^{n} (l_j+1)/(\lambda - \epsilon_j) = 1/G. \tag{7}$$

The solutions of Eq. (7) can be obtained by plotting the left- and right-hand sides graphically as in Fig. 2, where the ×'s show the solutions λ_n. It is seen that the uppermost eigenvalue is pushed up a large amount. Let us now ask what happens when the ϵ_i become degenerate. Then, it easily is found that the solutions λ_i are

$$\lambda_1 = \lambda_2 = \cdots = \lambda_{n-1} = \epsilon, \qquad \lambda_n = \epsilon + \sum_{i=1}^{n} (l_i+1)G, \tag{8}$$

where ϵ is the common value of the ϵ_i. Denoting

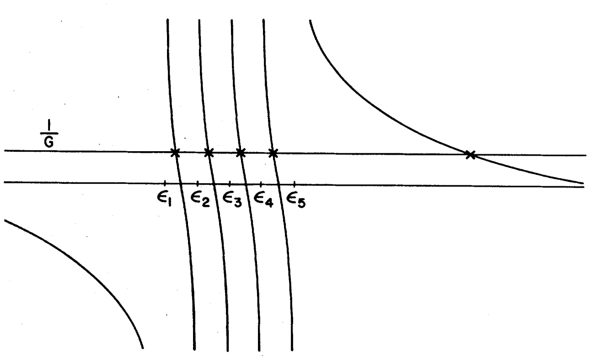

FIG. 2. Graphical solution of Eq. (7).

the perturbed eigenstates by χ_i, the nth state is

$$\chi_n = [\sum (l_i+1)]^{-1/2} \sum (l_j+1)^{1/2} \varphi_j. \tag{9}$$

Therefore, this state is pushed up through coherent effects from all of the degenerate levels. Since a large number of particle-hole states participate and since each term in the sum in Eq. (8) is of the order of one Mev, the nth level is pushed up by several Mev. Now, the dipole transition amplitude to any one of the unperturbed states φ_i is just

$$T_i = \int \varphi_i(\vec{r}_p, \vec{r}_h)\delta(\vec{r}_p - \vec{r}_h) H_I(\vec{r}_p) d^3r_p d^3r_h$$
$$= [(l_i+1)/3]^{1/2} \int_0^\infty R_{l_i} R_{l_i+1} r^3 dr. \tag{10}$$

Assuming, again, that the radial integrals are all equal, it is clear from Eq. (9) and the orthogonality of the φ_i that the uppermost level carries, in this approximation, all of the dipole transition strength. That is, through the particle-hole interaction, the lower levels have been denuded of their dipole transition strength and this has been transferred into the uppermost level. We propose, therefore, to call this level the "dipole level."

These arguments can be generalized to cover the transitions l_i to $l_i - 1$, as well, by changing l_i to $l_i - 1$ in the relevant matrix elements of the secular matrix and in T_i. However, whereas the approximation of setting all radial integrals equal does not seem unreasonable in the transitions l_i to $l_i + 1$ where the important transitions are "nodeless to nodeless" ones, it would not seem to be justified in the transitions l_i to $l_i - 1$. However, in heavy nuclei (e.g., see Wilkinson's calculations for Sn and Pb in reference 3) almost all of the oscillator strength is contained in the transitions $l_i + 1$.

To take account of effects due to the neutrons in the nucleus, we consider first the case of light nuclei and neglect Coulomb forces, so that isotopic spin can be considered to be a good quantum number. Then the dipole interaction Hamiltonian, after removal of center-of-mass coordinates, is

$$H_I = (e/2)(2\pi\hbar\omega)Z\tau_3, \tag{11}$$

for a nucleus with equal number of protons and neutrons; i.e., the effective charge on the proton is $e/2$, that on the neutron, $-e/2$. Since the Hamiltonian is the third component of a vector in isotopic spin space, only $T=1$ states will be formed from applying it to the $T=0$ ground state. The particle-hole states will now be

$$(2)^{-1/2}[\varphi_i^{\,+}(\vec{r}_p, \vec{r}_h) - \varphi_i^{\,-}(\vec{r}_p, \vec{r}_h)], \tag{12}$$

where the + indicates that the first part of the wave function refers to a proton particle-hole state, and the - indicates that the other part refers to a neutron particle-hole state. In the $T=0$ state, which cannot be formed by absorption of the gamma ray, the - sign would be replaced by a + sign.

Now the question of what happens to the $T=0$ and $T=1$ levels depends on the isotopic spin dependence of the force. A repulsive particle-hole interaction containing no isotopic spin dependence will push the $T=0$ level up in energy, leaving the $T=1$ level unchanged, whereas a force of character $\vec{\tau}_i \cdot \vec{\tau}_j$ which is repulsive for like particles will push the $T=1$ level up. The Rosenfeld mixture used by Elliott and Flowers[8] is of the latter type. A more detailed discussion of effects of a force such as the Rosenfeld mixture would require introduction of spin, but there is no doubt that the isotopic spin dependence is such as to push the $T=1$ levels up.

In heavy nuclei, where isotopic spin is no longer a good quantum number, neutron and proton excitations can be treated independently. Whereas it is true that the neutron particle-hole states are created with opposite phase from the proton particle-hole ones because of the τ_3 in the interaction Hamiltonian, a force of the $\vec{\tau}_i \cdot \vec{\tau}_j$ character will give opposite signs when evaluated between particle-hole states of like particles and particle-hole states of unlike particles so that the matrix elements in the secular matrix will again tend to be all positive.

We have made radical approximations in this schematic model, but the qualitative features should be given correctly, at least for heavy nuclei. Unfortunately, detailed calculations without our simplifying assumptions are difficult to make here, and these have been carried out only for the case of O^{16}. Here, the work of Elliott and Flowers does show that the dipole transitions occur at a high energy as a result of the particle-hole interactions, but the dipole strength is large not only in the top level, but in both of the two highest levels, although the lower levels are denuded as we would predict. Nothing in our model would predict that two levels should be pushed up.

In this case of a light nucleus, there are, however, several features which are not at all well described in our picture, and may be responsible for this. First, spin-flip transitions are relatively important here, because of the low l involved, and the unperturbed energies ϵ_i for these transitions are quite different from those of the non-spin-flip transitions. (We have neglected spin in our model, but it is clear that when generalized to include spin, it will describe only the non-spin-flip transitions well, because only these are nearly degenerate in energy.) Furthermore, the transitions l_i to $l_i - 1$ are relatively important in oxygen. Neither of these features is present in the medium and heavy weight nuclei, where almost all of the oscillator strength is carried by the l_i to $l_i + 1$ transitions involving no spin flip.

Some general qualitative features come out of the work of Elliott and Flowers which are present in our model. In particular, it is true that the state which is pushed highest up in energy is the most symmetric state, in terms of the particle and hole, i.e., that is the state in which expansion of the χ_i in terms of the φ_i such as in Eq. (10) involves mostly + signs with our choice of phases for the φ_i. Since the matrix elements of the secular matrix are all positive, clearly this state will lie highest in energy. This is analogous to the most symmetric state lying lowest in energy when attractive forces are present, as was noted long ago by Wigner and collaborators.[9] The more symmetric the state is, the more dipole strength it carries.

We believe that our schematic model indicates that an increasing regularity will occur in going towards heavier nuclei, so that the coherent effects can become strong enough to shift the dipole transitions up several Mev. The schematic model is, of course, no substitute for detailed calculations, but indicates the possibility of these coherent effects in a simple way.

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†Permanent address: Mathematical Physics Department, University of Birmingham, Birmingham, England.

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ANGULAR DISTRIBUTION OF NEUTRONS FOLLOWING THE NUCLEAR CAPTURE OF POLARIZED MUONS

A. Astbury, I. M. Blair, M. Hussain, M. A. R. Kemp, H. Muirhead, and R. G. P. Voss*
Nuclear Physics Research Laboratory, University of Liverpool, Liverpool, England
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Measurements[1,2] of the total capture rates of negative muons in complex nuclei have indicated that the magnitudes of the squares of the coupling constants must be about the same size as those in the processes of β and μ decay. Previous results have not shown, however, whether parity conservation is violated in the interaction, and, until recently,[3] no evidence has existed concerning the relative signs of the coupling constants.

In order to examine some of these problems the authors have attempted to measure the angular distribution of the neutron about the direction of muon spin in the process

$$\mu^- + p \rightarrow n + \nu.$$

An asymmetry in this distribution would provide clear evidence of parity nonconservation. In addition, various authors[4-7] have shown that the value of the asymmetry coefficient α in the expression for the angular distribution,

$$D(\theta) = 1 + \alpha\, \vec{\sigma}\cdot\vec{p}/|\vec{p}|, \tag{1}$$

is dependent upon the relative signs and magnitudes of the coupling constants (here $\vec{\sigma}$ represents the muon spin vector, and $\vec{p}$ the neutron momentum).

The experimental arrangement is shown in Fig. 1. The negative meson beam, of momentum 190 Mev/c, from the Liverpool synchrocyclotron was moderated by an appropriate amount of polythene so that the muons were brought to rest in the target. Pions stopped in the absorber between counters 2 and 3. The target was S^{32}; this was chosen in order to provide a sufficient number of neutrons, and to avoid complete depolarization of the muon beam.[8]

The arrival of a muon was signalled by the coincidence sequence $23\bar{4}$, and the emission of a neutron by $\bar{1}\bar{3}\bar{4}5$ (referred to hereafter as "start" and "stop" events, respectively). Counter 5 was of a type developed by Brooks,[9] and was used in order to discriminate against the γ rays emitted following muon capture. This discrimination is essential, since the rate of emission of γ rays is comparable to that of neutrons.[2] The counter had an average efficiency for the detection of neutrons in the energy range of 4 - 15 Mev of

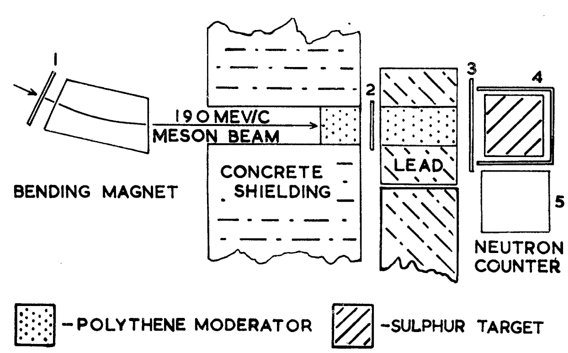

FIG. 1. Experimental arrangement.