SHELL EFFECTS AT HIGH EXCITATION ENERGIES*

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It has been known for some time that a plot of the level spacings of rather highly excited compound nuclei (as measured by s-wave neutron resonances) fluctuates rapidly as a function of mass number.[1,2] The fluctuations are evidently connected in some manner with the shell structure of nuclei since prominent maxima of the level spacings appear at neutron numbers of 50, 82, and 126. Rosenzweig[3] has calculated relative shell-model level densities. We are presenting similar calculations based on an even simpler model.

We shall attempt to account for level spacings of even-A compound nuclei by considering the most significant mode of excitation of compound nuclei to be excitations of 0^+ pairs of like nucleons among the levels of a spherical shell-model scheme. For our calculations we assume that there are a minimum number of unpaired particles (i.e., two in odd-odd or even-even compound nuclei with nonzero angular momentum). One can make a simple count of the number of possible different ways of arranging the two unpaired nucleons in a shell scheme and still obtain a J^π consistent with the spin and parity of the target nucleus and an incident s-wave neutron. This count, which has been carried out only for odd-odd compound nuclei between $N=50$ and 82, is roughly proportional to $2J+1$ (as long as J is not too large), in qualitative agreement with the predictions of a simple Fermi gas model.[4] We therefore define a level spacing $D_0 = 2(2I+1)D_{\text{obs}} \equiv 1/\rho_0$ which should be approximately independent of all but pair excitations of nucleons; I is the spin of the target nucleus and D_{obs} is the observed spacing including all s-wave channels.

If one assumes, for the moment, that a major shell (only partially filled) is isolated, it is easy to calculate the number of possible ways of arranging indistinguishable pairs in the shell as a function of an excitation energy which is measured in units of S, an average spacing for pairs between subshells of a major shell. These simple pair excitations alone are not enough to account for the observed level densities. However, there are two (complex pair) modes of excitation available: Because of the large separation energy (about 2 Mev per nucleon[5]) between major shells, one and only one pair can be excited into a higher major shell. As an alternative, one or possibly more pairs may be brought up from the filled subshells below the unfilled major shell, as will be discussed later. These complex pair excitations can occur in a number of different ways (index i) which leaves an energy $E'-E_i$ for the simple pair excitations. It is clearly possible to calculate the number of possible configurations at a given nuclear excitation energy, E' above the lowest energy state with J^π, by sharing the available energy in all possible ways between simple and complex pair excitations. If the distribution of the number of simple pair excitations, $r(E'-E_i)$, is calculated from a shell model, and r is assumed independent of i, then the total configuration density at E' is the sum over all possible values of E_i. If, for simplicity, one assumes $E_i = nS'$, where $n=0, 1, \ldots,$ then

$$1/D_0 = \rho_0(E') = \sum_{n=0}^{E'/S'} r(E'-nS') = \int_0^{E'} r(E'-E)\,dE/S'. \quad (1)$$

The maximum energy, M, which can be absorbed by simple pair excitation in the partially filled shells, is roughly equal to or less than the excitation energy, B, of the compound nuclei when $35<N<90$ and between 120 and 130 if the pair spacing, S', is about 0.5 Mev. For these cases Eq. (1) becomes

$$\rho_0(B \gtrsim M) = 1/D_0 = n_Z n_N/S'. \quad (2)$$

n_Z and n_N are the number of arrangements of the indistinguishable proton pairs and neutron pairs, respectively, within their major unfilled shells.

The results of these calculations based on Eq. (2) are shown in Fig. 1(a) for odd-odd compound nuclei. The predicted spacings near the rare earths are much too low (dashed curve) due to the fact that $M \gg B$ for these compound nuclei so that Eq. (2) breaks down very badly. The higher calculated points in this region are based on an approximate correction which assumes a triangular distribution for r, and that $B=6$ Mev, $S'=50$ kev, and $S=500$ kev. In Fig. 1(b) the predictions of Eq. (2) are compared to some of the even-even compound nuclei. It is seen that the general trend of the experimental results and in some cases their detailed behavior are correctly predicted.

Sample counts have shown that the general trend of the predicted spacings is unaffected by assuming four rather than two unpaired nucleons. In any case, this number must be small since the pairing energies[6] [about $23(2j+1)A^{-1}$ Mev] limit it. The effect of this restriction on the number of possible configurations is now being investigated in detail by Rosenzweig.[7] However, sample calculations (based on the simplest cases) indicate that a count of all the energetically possible "broken pair" configurations predicts too low a spacing and, hence, we must assume that the stability of most of these configurations does not permit them to be observed as sharp resonances. Consequently, we are unable (in general) to estimate the value of S' from the shell model, since some of the observed resonances may be due to an unknown fraction of the broken pair configurations. In addition, we may also observe configurations arising from the excitation of pairs out of the filled subshells below the shell of interest. Since the conventional shell-model diagram is only a semiempirical summary of information about excitations above the Fermi level, we cannot predict the energetics of this last mode of excitation. It is rather surprising that a single value of S' (50 kev) yields as good agreement between calculated and measured values as that in Fig. 1.

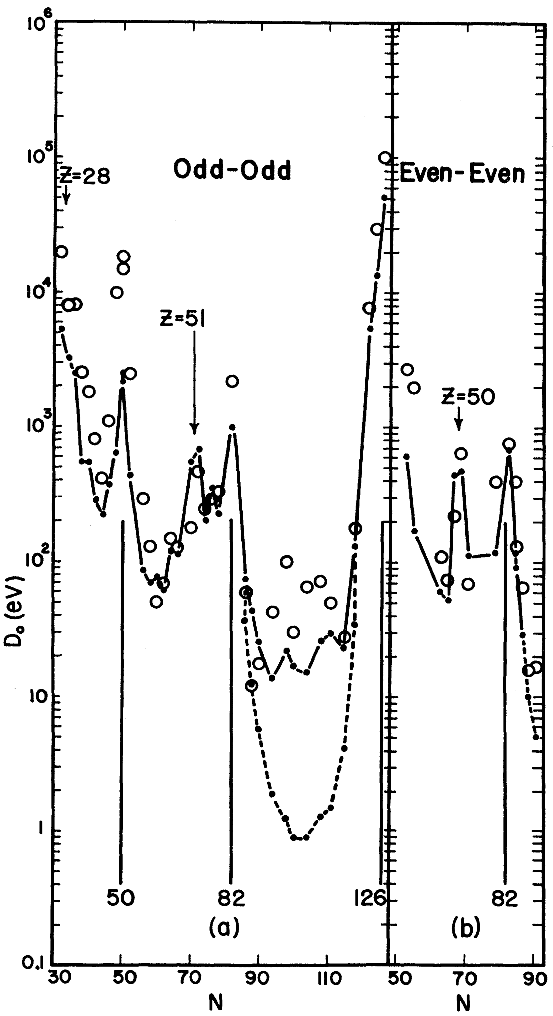

FIG. 1. Calculated and measured values of D_0 are plotted against the number of neutrons N in the target nucleus for even-even and odd-odd compound nuclei. Experimental points are shown as circles. Calculated points are shown as small solid dots connected by straight solid lines except that the predictions of Eq. (2), where it is invalid, are connected by dashed lines. Experimental errors are not indicated. The best known spacings (in general the smallest) are uncertain by ± 20 % or more; the most uncertain points are those near $N=50$ where the errors are of the order of a factor of two or more in either direction. The same value of S' (50 kev) has been used in all calculations.

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