SUPERFLUIDITY OF NUCLEAR MATTER*

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(Received August 17, 1959)

The possible existence of an energy gap in the spectra of nuclei, analogous to the energy gap in the theory of superconductivity, has been the subject of great interest recently.[1,2] A criterion for the superfluidity of infinite nuclear matter has been given in the form of a variational principle.[2] In this Letter we report on computations which are an improvement over those in CMS in three regards; the effective mass approximation has been removed, more realistic two-body potentials have been employed, and superior trial functions have been obtained. With these modifications we find that the criterion is satisfied, and nuclear matter is superfluid. That is to say, the lowest state of nuclear matter is a superfluid state separated from the first excited state by an energy gap. In this ground state, which is unobtainable by perturbation theory (and lower in energy than the state described by Brueckner et al.), nucleons are strongly correlated. The correlation function $g(r)$ satisfies an equation[2] which may be expressed as the criterion for the extremum of a quantity $\lambda(0)$. Thus the criterion for superfluidity may be given[2] in the form

$$\frac{\lambda(0)}{4\pi} = \frac{\hbar^2}{m^*}\int_0^\infty |k^2 - k_F^2|\,|g(k)|^2\,dk + \int_0^\infty v(r)\,|g(r)|^2\,dr < 0, \quad (1)$$

where $g(r)$ is a trial function with Fourier sine transform $g(k)$, $v(r)$ is the true two-body potential, $\hbar k_F$ is the Fermi momentum, and m^* is the effective mass at the Fermi surface. If the effective mass approximation is not made, then Eq. (1) is replaced with

$$\frac{\lambda(0)}{4\pi} = 2\int_0^\infty |e(k) - e(k_F)|\,|g(k)|^2\,dk + \int_0^\infty v(r)\,|g(r)|^2\,dr < 0, \quad (2)$$

where $e(k)$ is the effective one-body potential for a particle in infinite matter.

The arguments of CMS suggest a trial function of the form

$$g(r) = \ln\beta r\ \varphi(r), \quad r < 1/\beta$$
$$= 0, \quad r \geq 1/\beta \quad (3)$$

where $\varphi(r)$ is arbitrary except that it approaches $\sin k_F r$ as $r \to \infty$, and β is a variational parameter. In this case Eq. (2) yields for the leading term, in the limit as $\beta \to 0$,

$$\frac{\lambda(0)}{4\pi(\ln\beta)^2} = 2\int_0^\infty |e(k) - e(k_F)|\,u^2(k)\,dk + \int_0^\infty v(r)\varphi^2(r)\,dr, \quad (4)$$

where $u(r) = \sin k_F r - \phi(r)$, and $u(k)$ is the Fourier sine transform of $u(r)$.

In the limit that $k_F \to 0$ the basic approximations of the theory probably break down. In this limit we can, however, develop mathematical results pertaining to the variational principle which give insight at nonzero values of k_F. If $k_F \to 0$ then in the effective mass approximation the variational principle is minimized by a solution of the Schrodinger equation,

$$\left[-\frac{\hbar^2}{m}\frac{d^2}{dr^2} + v(r)\right]\psi_{\mathrm{Sch}}(r) = \frac{\hbar^2 k_F^2}{m}\psi_{\mathrm{Sch}}(r). \quad (5)$$

Combined with the results of CMS, given above, this suggests a trial function with

$$\varphi(r)=\psi_{\mathrm{Sch}}(r), \quad r<r_0$$
$$=\sin k_F r, \quad r_0<r \tag{6}$$

where ψ_{Sch} is an appropriately normalized solution of the Schrödinger equation and r_0 is chosen so that $\phi(r)$ and $\phi'(r)$ are continuous at r_0. If we write

$$e(k)=\frac{\hbar^2k^2}{2m}+V(k), \tag{7}$$

then Eq. (4) may be written, after using the relation between kinetic and potential energy for a solution of Eq. (5),

$$\frac{\lambda(0)}{4\pi(\ln\beta)^2}=I_L(\text{long range})+I_D(\text{dispersion})+I_P(\text{Pauli}),$$

where

$$I_L=\int_{r_0}^{\infty}v(r)\sin^2k_Fr\,dr,$$

$$I_D=2\int_0^{\infty}V(k)u^2(k)dk-2V(k_F)\int_0^{r_0}u^2(r)dr,$$

$$I_P=4\int_0^{k_F}[e(k)-e(k_F)]u^2(k)\,dk. \tag{8}$$

In this form the equation is particularly suited for numerical computation since $u(r)$ is nonzero only over a finite domain; $V(k)$ is essentially nonzero over only a finite domain, thus requiring the evaluation of the Fourier sine transform of $u(r)$ for only a finite domain of k values, and in contrast to Eq. (2) none of the integrands have points of singularity. Numerical computations have been made employing a high-speed digital computer and using the Gammel-Thaler potential for ^1S states[3] and a variety of choices for $V(k)$ and k_F. For k_F we have used 1.34 f^{-1} and 1.40 f^{-1} appropriate to the observed density in heavy nuclei, corresponding to the equations for nuclear radii $R=1.13A^{1/3}$ f and $R=1.08A^{1/3}$ f. [1 fermi (f) $\equiv 10^{-13}$ cm.] We have also examined $k_F=1.48$ f^{-1} which corresponds to a density slightly higher than that observed in finite nuclei but perhaps close to the density of infinite nuclear matter. For the single-body potential in infinite nuclear matter $V(k)$, we have used the values given by Brueckner and Gammel,[4,5] as well as an equation of the form

$$V(k)=-V_0/(1+\alpha k^2), \tag{9}$$

where we have chosen the values of V_0 and α by the following two criteria: (*i*) $V(k_F)$ shall equal the binding energy per nucleon (15.5 Mev) plus the kinetic energy at the Fermi surface ($\hbar^2k_F^2/2m$); (*ii*) the effective mass at the Fermi surface is $m^*=0.67m$, i.e.,

$$\left.\frac{de(k)}{dk}\right|_{k_F}=\frac{\hbar^2k_F}{m^*}. \tag{10}$$

The failure of $V(k)$ as obtained by Brueckner and Gammel to satisfy criterion (*i*) is the reason for our choosing a potential of the form of Eq. (9). We find the parameters listed in Table I.

The results of the calculation are given in Table II where it may be seen that for both choices of $V(k)$ and $k_F=1.34$ f^{-1} there is superfluidity,[6] while at $k_F=1.48$ f^{-1} there is no superfluidity. For $k_F=1.40$ f^{-1} there is superfluidity with the potential of Eq. (9), but not with that of Brueckner and Gammel. This is a variational calculation so that in the cases where the values of $\lambda(0)$ are very close to zero, one can expect that a superior trial function might well yield superfluidity.

It should be recalled[2] that the hard core in the two-body potential is extremely important in the calculation since in its absence there is superfluidity for <u>any</u> attractive potential.[7] It can also be seen in Table II that a potential with a positive phase shift at momentum $\hbar k_F$ does not in general suffice for superfluidity.[8] The small values of I_P show why a trial function of the form we have chosen is so successful.

We have investigated the validity of the effective mass approximation by computing $\lambda(0)$ at $k_F=1.40$ f^{-1} and with $m^*=0.67m$. In the same arbitrary

Table I. Parameters for the single-body potential in infinite nuclear matter [Eq. (9)].

k_F (f^{-1})	V_0 (Mev)	α (f^{-2})
1.34	80.71	0.2992
1.40	87.41	0.2815
1.48	96.35	0.2646

Table II. Values of the various terms in Eq. (8) in arbitrary units, for different values of k_F and two different single-body potentials in infinite matter.

	I_L	I_D	I_P	$I_L+I_D+I_P$
$k_F=1.34$; V of Eq. (9)	-55.4	21.0	2.4	-32.0
$k_F=1.34$; V of B and G[a]	-55.4	43.3	2.6	- 9.5
$k_F=1.40$; V of Eq. (9)	-43.9	22.7	3.3	-17.9
$k_F=1.40$; V of B and G[a]	-43.9	44.2	3.5	+ 3.8
$k_F=1.48$; V of Eq. (9)	-28.8	25.1	4.9	+ 1.2
$k_F=1.48$; V of B and G[a]	-28.8	45.6	4.8	+21.6

[a]See references 4 and 5.

units employed in Table II, we find $I_L=-43.87$, $I_D=79.79$, and $I_P=2.85$. Although the Pauli term is estimated satisfactorily in this manner, it can be seen that the dispersion term is grossly overestimated. This is because the hard core in the two-body potential introduces very high Fourier components into the correlation function, thus making the dispersion term sensitive to the nature of the single-body potential at high momenta.

Evaluation of the energy gap and the two-body correlation function, with a discussion of the implication of these results in the theory of nuclear structure, will be the basis of a future communication. The existence of an energy gap in finite nuclei, of the type demonstrated here to exist in infinite nuclear matter, is strongly suggested by these computations, in agreement with experimental evidence.

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*Supported in part by the National Science Foundation and the Office of Ordnance Research.

[1]Bohr, Mottelson, and Pines, Phys. Rev. 110, 936 (1958); C. De Dominicis and P. C. Martin, Bull. Am. Phys. Soc. 3, 224 (1958); Bogoliubov, Tolmachev, and Shirkov, A New Method in the Theory of Superconductivity (Consultants Bureau, New York, 1958); V. G. Soloviev, Nuovo cimento 10, 1022 (1958); V. J. Emery, Nuclear Phys. 12, 69 (1959).

[2]Cooper, Mills, and Sessler, Phys. Rev. 114, 1377 (1959), hereafter called CMS.

[3]J. L. Gammel and R. M. Thaler, Phys. Rev. 107, 1337 (1957).

[4]K. A. Brueckner and J. L. Gammel, Phys. Rev. 109, 1040 (1958).

[5]For purposes of numerical computation we have fitted the curve of Brueckner and Gammel with

$$V(k/k_F)=[-111+39.1(k/k_F)^2-8.26(k/k_F)^3]\ \text{Mev}, \quad k/k_F\leq 2.4$$

$$=0, \quad k/k_F>2.4.$$

[6]This may be contrasted with the results of Emery who finds that the Fourier component of the two-body potential at the Fermi momentum must be negative for "Cooper singularities" in the Bethe-Goldstone equation. The Bethe-Goldstone equation is, however, different from the basic equations of our theory [CMS, Eqs. (18) and (19)] so that a direct comparison is not possible. Certainly an energy gap exists in our theory despite the fact that all the Fourier components of the two-body potential are infinite.

[7]De Dominicis and Martin replace the two-body potential with an effective potential which is nonsingular and everywhere attractive, and thus obtain superfluidity for nuclear matter.

[8]Soloviev finds as a criterion for superfluidity that the two-body potential be "essentially attractive" at energies corresponding to the Fermi momentum. This differs from our criterion in that it ignores the contributions of the Pauli and dispersion terms, I_P and I_D.