large. DF stress the agreement of their theory with experiment regarding the relative intensities of the $M1$ and $E2$ transitions between the 2_1 and 2_2 states, in addition to the relative intensities of other $E2$ transitions. However, it might be much more reasonable to consider the effect of single-particle excitations which begin to assume importance at this point.[13] This matter, however, will be discussed elsewhere together with a more detailed account of the context of the present note.

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[2]In this note we consider exclusively states which have even parity; therefore, we will not specify the parity of any state here and in the following.

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[4]Throughout this note $E_n(I)$ is the energy, above the ground state, of a state I_n, n labelling the position, energy-wise, of the particular state in question among all the others with the spin I.

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[8]For the notation β and γ see, e.g., reference 6. k_n, β_n, and C are constant parameters whose magnitudes are adjusted to produce agreement of the spectrum with experiment.

[9]In this calculation we considered only the first two terms of (1); i.e., we set $k_n = 0$ for $n \geq 3$. This is not, however, too bad an approximation because as is easily seen all the n = odd (and/or even) terms act, in so far as the lower states are concerned, in a similar way to produce the desired modification of the WJ spectrum. The two terms in (1) remaining are treated as the perturbations to the WJ Hamiltonian with the states $N \leq 4$ treated explicitly, and those with $N \geq 5$ accounted for by a closure argument. (The definition of N is the same as in reference 6.)

[10]P. H. Stelson and T. K. McGowan, Phys. Rev. 110, 489 (1958); Bull. Am. Phys. Soc. 2, 267 (1957).

[11]S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, No. 16 (1955).

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[13]In the derivation of Eq. (3.2) of DF, they expressed the nuclear shape by taking up to the quadratic term in α to obtain a nonvanishing $M1$ transition probability. This might not be consistent with the assumption that the deformation is described by an ellipsoid. If the latter had included also the 2^4-th pole deformation, DF would have obtained a vanishing $M1$ transition probability. We are indebted to Professor B. R. Mottelson for a discussion on this point.

ANGULAR DISTRIBUTIONS FROM STRIPPING REACTIONS OF LOW Q VALUES*

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Proton groups from the reactions $Li^7(d,p)Li^8$ and $C^{12}(d,p)C^{13*}$ (3.09-Mev state) for a range of deuteron energies between 0.5 and 2.5 Mev have been studied using high-resolution magnetic analysis. In fitting the measured angular distributions for the protons to a simple stripping theory, uncorrected for Coulomb and other perturbing effects, an unusually high degree of agreement was found.

Wilkinson[1] suggested that such agreement might be expected for reactions of fairly low Q values.

It is well known that the deviations of measured stripping (deuteron-induced) reactions from the simple Butler[2] and Born-approximation[3] theories for moderate energies (≥ 10 Mev, say) take a characteristic form. In general the first maximum can be unambiguously matched, but with increasing angle the agreement soon deteriorates. In particular the well-defined first minimum in the theoretical curves is filled in and subsequent maxima are poorly defined. In fact some 20 or so degrees after the first maximum the measured intensity is normally persistently higher than the theoretical prediction. This is attributed to competing compound-nucleus formation. The perturbing effects of nuclear and Coulomb interactions also play their role in distorting the simple comparison.

At low deuteron energies the deviation of measured distributions from simple theoretical ones becomes more serious. This is not unexpected as the effect of compound-nucleus formation and interaction effects are anticipated to be more prominent.

In order to improve agreement between experiment and theory, many modifications have been made to the simpler formalisms. Among these can be mentioned the work of Friedman and Tobocman,[4] Daitch and French,[5] Horowitz and Messiah,[6] Grant,[7] Tobocman,[8] and Tobocman and Kalos.[9] These refinements show that the effect of considering the Coulomb interaction is to displace the angular distributions toward larger angles, to broaden the peaks and fill in the valleys, and to reduce the total cross section. On the other hand, the effect of introducing nuclear interactions is to displace peaks towards smaller angles, to sharpen these peaks, and to reduce the total cross section. The distortion of simple stripping curves which can be achieved is strikingly great. It is unfortunate that when allowing for all possible effects, agreement with experiment is readily obtained, so that it is impossible to say whether the parameters employed to achieve such agreement have any particular physical significance. The conclusion is drawn that in order to avoid having to make hypotheses on the values of these parameters, experiments should be performed at energies well above the Coulomb barrier.

Two examples of the results obtained for the $Li^7(d,p)Li^8$ (ground state) reaction with Q value of -0.188 Mev are given in Fig. 1. These were obtained for incident deuteron energies of 1.9 and 1.5 Mev, respectively.

Another reaction of small Q value has been studied, viz., $C^{12}(d,p)C^{13*}$ (3.09-Mev state). The Q value is -0.367 Mev. An example of the results obtained in an angular distribution, for an incident deuteron energy of 2.1 Mev, is given in Fig. 2.

The theoretical curves associated with the measurements given in these diagrams were obtained from the non-Coulomb, noninteraction stripping theory formalism of Friedman and Tobocman,[4] using for convenience the tables and graphs of Enge and Graue,[10] based on this formalism. Other stripping derivations have also been tried, also with striking agreement.

Excitation functions have been measured for

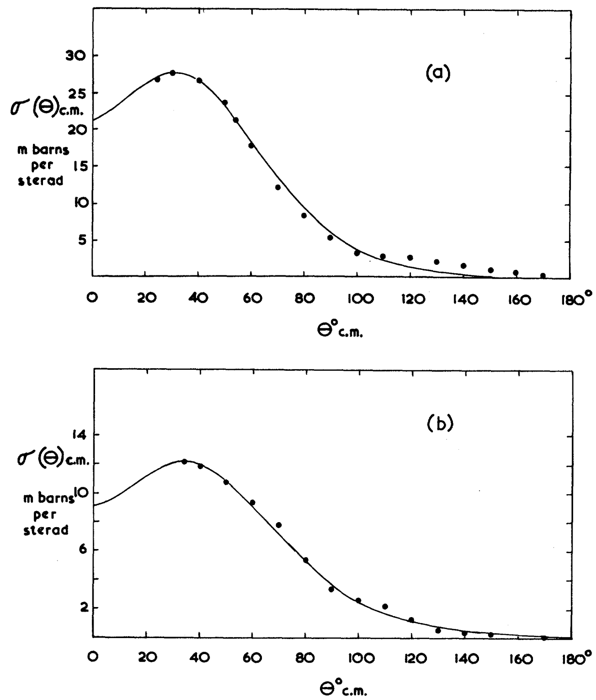

FIG. 1. Angular distribution of protons from the reaction $Li^7(d,p)Li^8$. (a) $E_d = 1.9$ Mev. The points represent experimental observations; the solid curve represents a calculated stripping distribution with $R = 6.0 \times 10^{-13}$ cm and $l = 1$. (b) $E_d = 1.5$ Mev. The points represent experimental observations; the solid curve represents a calculated stripping distribution with $R = 6.5 \times 10^{-13}$ cm, and $l = 1$.

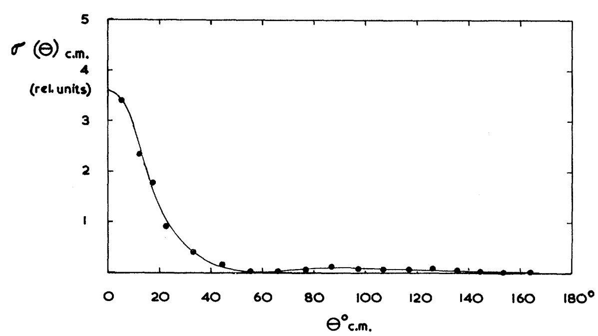

FIG. 2. Angular distribution of protons from the reaction $C^{12}(d,p)C^{13*}$ (3.09-Mev state) $E_d = 2.1$ Mev. The points represent experimental observations: the solid curve represents a calculated stripping distribution with $R = 7.4 \times 10^{-13}$ cm, and $l = 0$.

both reactions, $Li^7(d,p)Li^8$ (ground state) and $C^{12}(d,p)C^{13*}$ (3.09 Mev). In the lithium case one resonance was closely studied. Angular distributions were measured at the resonance energy, and at energies closely flanking the resonance.

No change—other than the increase in intensity associated with the resonance—in the marked degree of agreement between experimental and theoretical distributions was found.

In the carbon experiment a number of resonances were found and the effect of a particularly strong one in the angular distributions was studied in the same way as in the lithium case. Here too there was no significant change in the degree of goodness of fit with the simple stripping formalism used.

On the basis of these two experiments it would indeed appear that for small-Q reactions at low bombarding energies there is particularly little influence on the shape of the angular distributions from Coulomb and nuclear effects.

The physical picture underlying this proposal is as follows.[11] The momentum of the outgoing proton in a (d,p) reaction is made up of about half the momentum of the ingoing deuteron and of a part of the internal momentum of the deuteron at the instant at which stripping occurs. In the case of a large-Q reaction the total proton momentum must be high. If, however, the proton's share of the momentum of the ingoing deuteron is small—when the deuteron energy is small—the large deficit in momentum must be obtained from the ground-state wave function of the deuteron. In order to get this large latter contribution, the separation of proton and neutron in the deuteron must be small at the instant of stripping. For a small-Q reaction at low energy, on the other hand, the contribution to the outgoing proton momentum from the ground-state deuteron wave function need not be large, so that the separation of proton and neutron can be far greater than in the high-Q case.

The clear implication of this is that for small-Q stripping at low energies the proton can be well removed from the target nucleus when stripping occurs, reducing radically thereby the perturbing nuclear and Coulomb effects.

Wilkinson[11] has pointed out that in such small-Q, low-energy reactions, competition through the formation of a compound nucleus will in fact be discouraged, since the proton which—as has been indicated—can be well removed from the target nucleus for stripping, will have to penetrate the Coulomb barrier if a compound nucleus is to be formed. As in the high-Q case the proton must at least partially penetrate the Coulomb barrier in order to get sufficiently near to the neutron, this effect is much reduced.

The examples presented in this paper indicate an unusually high degree of agreement between experimental angular distributions and a simple stripping theory uncorrected for Coulomb and other effects. A physical description can give a clear connection between this agreement and the fact that such reactions have low Q values and are performed at low bombarding energies.

A full description and analysis of both experiments and their results will be published in the near future.

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