NEW MODEL FOR VIBRATIONAL SPECTRA IN EVEN-EVEN NUCLEI*†

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The vibrating surface model,[1] with its prediction of equal level spacings and the spin sequence[2] 0, 2, and a triad 0, 2, and 4, is the simplest among those presented in explaining the vibrational spectra of even-even nuclei. Experimentally observed spectra, however, indicate deviations from it among which the more important ones are[3]: (*i*) The degeneracy of the triad is lifted and usually[4] $E_2(2)<E_1(4)$; (*ii*) the ratio $E_2(2)/E_1(2)$ is generally greater than two.

The first approach to explain these deviations was proposed by Scharff-Goldhaber and Weneser[3] who considered the coupling of single particles to the simple surface vibrational motions. This model gives rule (*ii*) above, but for (*i*) it predicts $E_2(2)>E_1(4)$. Recently Raz[5] found, by extending the GW calculation, that it was possible to satisfy also (*i*); however, as will be discussed below, this treatment appears to possess other difficulties.

Wilets and Jean[6] proposed a model very similar to the simple surface vibrational model, but which differs in that a finite, shape-independent equilibrium deformation is assumed; (the γ-unstable model). It explains (*ii*) but gives $E_2(2)=E_1(4)$.

Figure 1(a) shows the Cd^{114} spectrum obtained experimentally by Motz[7] in which all members of the triad as well as the 3_1 state were observed; (this latter may be considered as a member of the third excited surface vibrational state). This spectrum bears out (*i*) and (*ii*) and in addition indicates that (*iii*) $E_2(0)\approx E_1(4)\approx E_2(2)$, and (*iv*) the ratio of $E_1(3)/E_1(2)$ is close to, but somewhat larger than three. The calculation of Raz predicts too large a value for $E_2(0)$ and for $E_1(3)$ and thus fails to reproduce the new rules (*iii*) and (*iv*).

The WJ model is consistent with (*ii*) and (*iv*), but is inconsistent with (*i*) and (*iii*). The inconsistency, however, is not too drastic and this fact suggests that with a slight modification of the model these discrepancies might well be removed. By examining the explicit form of the wave functions of each state in the WJ model, it is not hard to see that the desired change in the spectrum will be obtained by introducing a potential energy term of the form[8]

$$\sum_n k_n(\beta-\beta_n)^n\cos^n 3\gamma, \qquad (1)$$

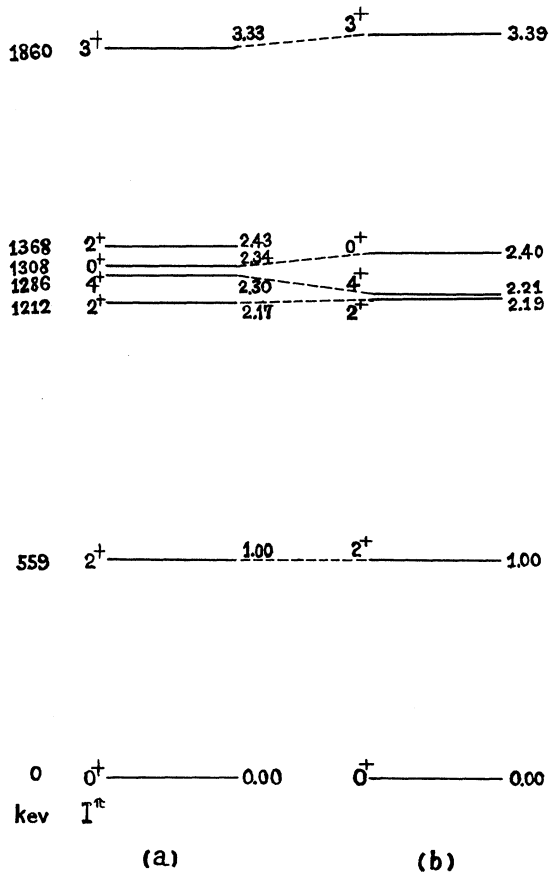

FIG. 1. Spectrum of the lower states of Cd^{114}. (a) Experimental; (b) theoretical. On the left-hand side of (a) are given the excitation energy in kev and the spin of each state, while on the right-hand side the ratio $E_n(I)/E_1(2)$ is given. The 2+ state at 1368 kev seems to be due to a single-particle excitation and the absence of the corresponding state in (b) is without significance since we here consider only collective states.

to the WJ potential

$$\tfrac{1}{2}C(\beta-\beta_0)^2. \qquad (2)$$

The spectrum obtained with the appropriate choice of the parameters in (1) and (2) is shown in Fig. 1(b) and it is seen that, in fact, good agreement with experiment is found.[9] The relative intensities of electromagnetic transitions between several states are also calculated and compared with experiment[10] in Table I, and again good agreement is obtained.

We now discuss the justification for introducing the term (1). For this purpose we first note that the WJ potential (2) can be described by curve (a) of Fig. 2, where the right and left halves, respectively, correspond to $\gamma=0°$ and $\gamma=60°$ (i.e., prolate and oblate deformations). Since the potential (2) is γ-independent, curve (a) has a right-left symmetry. A potential energy curve which is consistent with the parametric values (summarized in Table II) used in the present calculation is shown as curve (b) in Fig. 2. This curve has a right-left asymmetry which is clearly the result of the addition of the term (1) to the WJ potential. If we were to derive this type of potential from "first" principles, we should have taken, e.g., the Nilsson scheme[11] and added the single-particle energies for various values of the nuclear deformations. With such a procedure

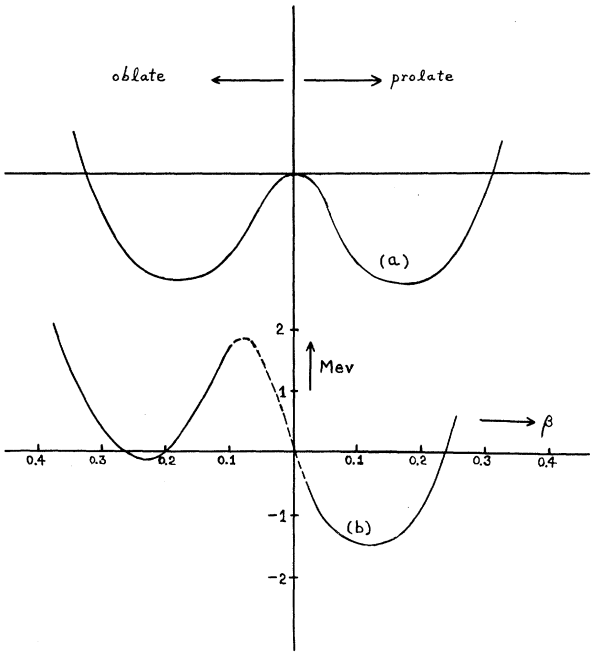

FIG. 2. Potential energy curve in various approximations. (a) Schematic representation of the WJ potential (2). (b) Potential energy curve which is consistent with the parametric values given in Table II, and which produces the theoretical spectrum (b) of Fig. 1.

an asymmetric energy curve is expected and thus would seem more natural than a symmetric one. To get a curve which can be compared directly with curve (b) of Fig. 2, however, it is necessary to take into account the residual interaction and such a calculation is now under way.

Regarding vibrational nuclei other than Cd^{114}, we first note that the ratio $E_2(2)/E_1(2)$ is less than two in several Hg and Pt isotopes. The recent theory of Davydov and Filippov[12] predicts values for this ratio that always exceed two, but in our model ratios less than two are easily obtained by allowing negative values for β_0 in (2). The DF theory further predicts that $E_1(2)+E_2(2)=E_1(3)$, while in our theory $E_1(2)+E_2(2)$ is greater than or less than $E_1(3)$ for $E_2(2)/E_1(2)$ greater than or less than 2, respectively. To the extent that only for Cd^{114} and Pt^{192} are data available [in the region where $E_2(2)/E_1(2)$ does not deviate very much from two], we find that this latter prediction agrees with experiment.

Finally, it should be mentioned that the DF theory can say nothing about the 0_2 state, and while $E_2(2)$ is correctly given, it predicts a value for $E_1(4)$ in Cd^{114} which is somewhat too

Table I. Comparison of the experimental and the theoretical values of the relative reduced transition probabilities of γ rays between several states. All the γ rays involved are $E2$ and therefore are not specified.

	Experimental	Theoretical
$T(2_2\rightarrow 2_1)/T(2_1\rightarrow 0_1)$	1.37 ± 0.31	1.70
$T(2_2\rightarrow 0_1)/T(2_1\rightarrow 0_1)$	0.020 ± 0.003	0.014

Table II. The parametric values used to obtain the theoretical spectrum of Fig. 1(b). In determining these values use has been made of the half-life of the 2_1 state (see reference 10) as well as the energy $E_1(2)$ shown in Fig. 1(a). For the $n=2$ term of (1), it is found that a very small value for k_2 yields good agreement with experiment; therefore, we show here the extreme case of $k_2=0$.

C	k_1	β_0	β_1
197 Mev	11.6 Mev	0.176	0.235

large. DF stress the agreement of their theory with experiment regarding the relative intensities of the $M1$ and $E2$ transitions between the 2_1 and 2_2 states, in addition to the relative intensities of other $E2$ transitions. However, it might be much more reasonable to consider the effect of single-particle excitations which begin to assume importance at this point.[13] This matter, however, will be discussed elsewhere together with a more detailed account of the context of the present note.

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[1]A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 26, No. 14 (1952); A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 27, No. 16 (1953).

[2]In this note we consider exclusively states which have even parity; therefore, we will not specify the parity of any state here and in the following.

[3]G. Scharff-Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955); referred to as GW; see also M. Nagasaki and T. Tamura, Progr. Theoret. Phys. (Kyoto) 12, 248 (1954).

[4]Throughout this note $E_n(I)$ is the energy, above the ground state, of a state I_n, n labelling the position, energy-wise, of the particular state in question among all the others with the spin I.

[5]B. J. Raz, Bull. Am. Phys. Soc. 3, 224 (1958), and Phys. Rev. 114, 1116 (1959).

[6]L. Wilets and M. Jean, Phys. Rev. 102, 788 (1956); referred to as WJ.

[7]H. T. Motz, Phys. Rev. 104, 1353 (1956).

[8]For the notation β and γ see, e.g., reference 6. k_n, β_n, and C are constant parameters whose magnitudes are adjusted to produce agreement of the spectrum with experiment.

[9]In this calculation we considered only the first two terms of (1); i.e., we set $k_n = 0$ for $n \geq 3$. This is not, however, too bad an approximation because as is easily seen all the n = odd (and/or even) terms act, in so far as the lower states are concerned, in a similar way to produce the desired modification of the WJ spectrum. The two terms in (1) remaining are treated as the perturbations to the WJ Hamiltonian with the states $N \leq 4$ treated explicitly, and those with $N \geq 5$ accounted for by a closure argument. (The definition of N is the same as in reference 6.)

[10]P. H. Stelson and T. K. McGowan, Phys. Rev. 110, 489 (1958); Bull. Am. Phys. Soc. 2, 267 (1957).

[11]S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, No. 16 (1955).

[12]A. S. Davydov and G. F. Filippov, Nuclear Phys. 8, 237 (1958); referred to as DF.

[13]In the derivation of Eq. (3.2) of DF, they expressed the nuclear shape by taking up to the quadratic term in α to obtain a nonvanishing $M1$ transition probability. This might not be consistent with the assumption that the deformation is described by an ellipsoid. If the latter had included also the 2^4-th pole deformation, DF would have obtained a vanishing $M1$ transition probability. We are indebted to Professor B. R. Mottelson for a discussion on this point.

ANGULAR DISTRIBUTIONS FROM STRIPPING REACTIONS OF LOW Q VALUES*

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Proton groups from the reactions $Li^7(d,p)Li^8$ and $C^{12}(d,p)C^{13*}$ (3.09-Mev state) for a range of deuteron energies between 0.5 and 2.5 Mev have been studied using high-resolution magnetic analysis. In fitting the measured angular distributions for the protons to a simple stripping theory, uncorrected for Coulomb and other perturbing effects, an unusually high degree of agreement was found.

Wilkinson[1] suggested that such agreement might be expected for reactions of fairly low Q values.

It is well known that the deviations of measured stripping (deuteron-induced) reactions from the simple Butler[2] and Born-approximation[3] theories for moderate energies (≥ 10 Mev, say) take a characteristic form. In general the first maximum can be unambiguously matched, but with increasing angle the agreement soon deteriorates. In particular the well-defined first minimum in the theoretical curves is filled in and subsequent maxima are poorly defined. In fact some 20 or so degrees after the first maximum the measured intensity is normally persistently higher than the theoretical prediction. This is attributed to competing compound-nucleus formation. The perturbing effects of nuclear and Coulomb inter-