quency directly proportional to energy and independent of energy. The values of $\nu_m(u = kT/e)$ inferred from a given observed value of $Q$ for the two different energy dependences of $\nu_m(u)$ are in the ratio of 2.5:1 at very small values of $\nu_m/\omega$. Note, however, that since this ratio changes with $\nu_m/\omega$ one cannot use the $Q$ curve for constant $\nu_m$ shifted to the left by a factor of 2.5.

The points of Fig. 3 show the collision frequencies for monoenergetic electrons in the $D$-layer computed from Fig. 2 and from Kane's values of $Q$. The curves give the collision frequencies for monoenergetic electrons in nitrogen at the gas densities and temperatures given by Nicolet[2] using our $\nu_m(u)$ (lower curve) and using Nicolet's extrapolation of results of Anderson and Goldstein[2] (upper curve). The agreement between the lower curve and the points is within the scatter of the rocket data. This agreement can be considered as evidence for the correctness of the gas density and temperature data obtained from other rocket studies.[2]

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## MOMENTS OF INERTIA OF EVEN-EVEN RARE EARTHS*

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Belyaev[1] has considered the effects of nuclear pairing interactions in analogy with the theory of superconductivity.[2-4] His treatment leads to the following formula[1] for the moment of inertia of an even-even nucleus:

$$\mathfrak{J} = \hbar^2 \sum_{kl} \frac{|\langle k|J_x|l\rangle|^2}{E_k + E_l}\left[1 - \frac{\Delta^2 + (\tilde{\epsilon}_k - \lambda)(\tilde{\epsilon}_l - \lambda)}{E_k E_l}\right], \quad (1)$$

where

$$E_k = [(\tilde{\epsilon}_k - \lambda)^2 + \Delta^2]^{1/2}, \quad (2)$$

and the sums, $l$ and $k$, run over all single particle states of a self-consistent spheroidal well. The "chemical potential" $\lambda$ is the solution of the equation

$$\sum_k \{1 - [(\tilde{\epsilon}_k - \lambda)^2 + \Delta^2]^{-1/2}[\tilde{\epsilon}_k - \lambda]\} = N. \quad (3)$$

$N$ here is the number of neutrons or protons outside of closed shells. The quantity $\Delta$ is equal to one-half the induced energy gap.

Numerical computations based on Eq. (1) have been made for twenty-six rare earth nuclei which

Table I. Rotational energies of even-even nuclei. Columns one to three list the nuclei considered, their deformations,[a] and their assigned energy gap parameters[b] which yield the calculated energy of the first rotational (2+) state in kev (Column 5). Columns 4 and 6 give the observed rotational energy[a, c] and the percent difference between experiment and theory. The energy gap is given in units of $\hbar\omega_0^0$, which is approximately $41A^{-1/3}$ Mev for a nucleus of mass number $A$.

| Nucleus | $\delta_{\text{Coulomb}}$ | $\Delta/\hbar\omega_0^0$ | $E_{\text{exp}}$ (kev) | $E_{\text{theor}}$ (kev) | % error |
|---|---|---|---|---|---|
| ${}_{60}Nd_{90}^{150}$ | 0.24 | 0.139 | 130 | 139.5 | + 6.8 |
| ${}_{62}Sm_{90}^{152}$ | 0.27 | 0.139 | 122 | 123.0 | + 0.8 |
| $Sm_{92}^{154}$ | 0.31 | 0.128 | 83 | 99.7 | +16.7 |
| ${}_{64}Gd_{90}^{154}$ | 0.28 | 0.139 | 123 | 120.0 | - 2.5 |
| $Gd_{92}^{156}$ | 0.39 | 0.128 | 89 | 85.6 | - 4.0 |
| $Gd_{94}^{158}$ | 0.44 | 0.117 | 79 | 68.3 | -15.7 |
| $Gd_{96}^{160}$ | 0.45 | 0.108 | 76 | 66.1 | -15.0 |
| ${}_{66}Dy_{94}^{160}$ | 0.33 | 0.117 | 86 | 92.1 | + 6.6 |
| $Dy_{96}^{162}$ | 0.34 | 0.108 | 82 | 84.4 | + 2.8 |
| $Dy_{98}^{164}$ | 0.39 | 0.100 | 73 | 70.0 | - 4.3 |
| ${}_{68}Er_{96}^{164}$ | 0.31 | 0.108 | 90 | 88.3 | - 1.9 |
| $Er_{98}^{166}$ | 0.31[d] | 0.100 | 80 | 85.9 | + 6.9 |
| $Er_{100}^{168}$ | | 0.093 | 80 | 83.7 | + 4.4 |
| $Er_{102}^{170}$ | | 0.088 | 79 | 77.7 | - 1.7 |
| ${}_{70}Yb_{100}^{170}$ | 0.28 | 0.093 | 84 | 86.1 | + 2.4 |
| $Yb_{102}^{172}$ | 0.29[d] | 0.088 | 78 | 80.1 | + 2.6 |
| $Yb_{104}^{174}$ | | 0.085 | 76 | 77.1 | + 1.4 |
| $Yb_{106}^{176}$ | | 0.085 | 82 | 74.7 | - 9.8 |
| ${}_{72}Hf_{104}^{176}$ | 0.28 | 0.085 | 89 | 83.8 | - 6.2 |
| $Hf_{106}^{178}$ | 0.29 | 0.085 | 91 | 78.5 | -15.9 |
| $Hf_{108}^{180}$ | 0.26 | 0.086 | 93 | 90.3 | + 3.0 |
| ${}_{74}W_{108}^{182}$ | 0.25 | 0.086 | 100 | 101.9 | + 1.9 |
| $W_{110}^{184}$ | 0.23 | 0.090 | 112 | 110.9 | - 1.0 |
| $W_{112}^{186}$ | 0.23 | 0.096 | 124 | 118.3 | - 4.8 |
| ${}_{76}Os_{110}^{186}$ | 0.19 | 0.090 | 137 | 130.9 | - 4.8 |
| ${}_{76}Os_{112}^{188}$ | 0.17 | 0.096 | 155 | 166.8 | + 7.1 |

[a]See reference 8.
[b]See reference 7.
[c]See reference 9.
[d]These deformations were obtained from measurements on unseparated isotopes.

exhibit rotational spectra. The eigenvalues, $\tilde{\epsilon}_\nu$, and wave functions of the self-consistent field were taken from the work of Nilsson,[5] except for certain modifications indicated by a recent study of the spectra of odd-mass nuclides.[6] The energy gaps were obtained from a smooth curve drawn through the experimental pairing energies of Johnson and Bhanot,[7] and the quadrupole deformations, from Coulomb excitation studies.[8]

The results are summarized in Table I. They indicate that Eq. (1), together with the Nilsson model, provides a remarkably good description of the moments of inertia in the rare earths, thereby lending support to the superconductor theory of nuclear pairing effects.

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