dicted by Eq. (4) for a saturation parameter $s \simeq 0.3$, that is for the smallest grains.

The dependence of the line shape on the direction of the sweep is in qualitative agreement with the prediction of Kaplan[3] if the finite nuclear T_1 (10 sec at 4.2°K) and a spread in the size of the grains are taken into account.

The existence of a distribution of electron spin resonance shifts was also exhibited directly: after saturating the electron spin resonance line for several T_1, a large field modulation (5 gauss) was introduced suddenly. The electron line being then saturated during a small fraction of the modulation cycle only, the enhanced nuclear polarization could relax back to approximately its normal value. The electron line observed on the scope presented immediately after the introduction of the modulation, an asymmetrical broadening of the order of 0.4 gauss, which collapsed into a narrow line with a time constant of the order of $T_1 \simeq 10$ sec.

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ELECTRON COLLISION FREQUENCIES IN NITROGEN AND IN THE LOWER IONOSPHERE*

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Attempts[1,2] to correlate laboratory measurements of electron collision frequencies in nitrogen with the results of measurements using rockets passing through the D-layer of the ionosphere have not been very successful. Our measurements of the electron collision frequencies for thermal electrons in nitrogen, combined with an improved analysis of the data obtained with the rockets, appear to remove the discrepancies.

An improved version of the electron drift velocity tube used by Bradbury and Nielsen[3] has been used to measure the mobility of electrons in nitrogen at such low electric fields that the electrons are in thermal equilibrium with the gas.[4] The measured values of the product of electron mobility, μ, and gas density, N, are $\mu N = 1.10\times10^{24}$, 3.5×10^{23}, and 2.8×10^{23} cm^{-1} volt^{-1} second^{-1} at 77°K, 300°K, and 373°K, respectively. Following Phelps, Fundingsland, and Brown[5] (PFB) these results are analyzed by expressing the reciprocal of the momentum transfer collision frequency, $\nu_m(u)$, as a power series in the electron energy, u, and substituting into the standard expressions for electron mobility[6] to obtain a power series in the most probable electron energy, kT/e. The power series for the mobility is then fitted to the experimental data to obtain the coefficients of the series for $\nu_m(u)$. The resulting $\nu_m(u)$ is shown by the lower solid curve in Fig. 1. The dashed curves show

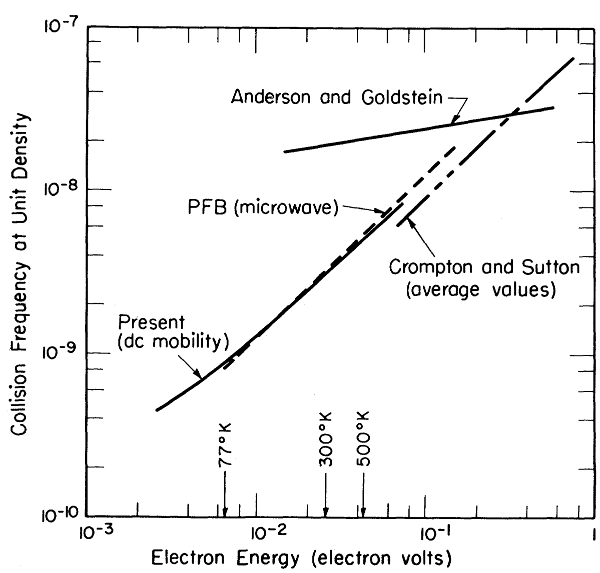

FIG. 1. Momentum transfer collision frequencies for monoenergetic electrons in nitrogen as a function of electron energy. The energy range of the thermal equilibrium experiments is shown by the values of kT/e at which the measurements were made and serves to indicate the range of validity of the curves of collision frequency as a function of energy. The mobility data are found to fit the relation $\mu N = 9.6\times10^{21}\,(kT/e)^{-1} - 1.39\times10^{19}\,(kT/e)^{-2}$ cm^{-1} volt^{-1} sec^{-1} within the experimental error so that $N/\nu_m(u) = 8.2\times10^{6}\,(u)^{-1} - 5.9\times10^{3}\,(u)^{-2}$ cm^3/sec.

the results of analysis of (a) thermal equilibrium measurements of microwave conductivity by PFB and (b) drift velocity and diffusion measurements at average electron energies above thermal by Crompton and Sutton.[7] The upper solid curve shows the results of Anderson and Goldstein.[8] Figure 1 shows very good agreement between the present results and those of PFB and satisfactory agreement with the data from Crompton and Sutton. These data show that the electron collision frequency in nitrogen is very nearly directly proportional to the electron energy. We shall use this approximation in the following calculations.

In order to evaluate the electron collision frequency in air, we must estimate the collision frequency for electrons in oxygen. The results of analysis of drift velocity and diffusion measurements in oxygen show that the collision frequency for electrons in oxygen is about two-thirds of that for nitrogen for energies down to 0.2 electron volt.[9] We shall assume that the ratio of collision frequencies for oxygen and nitrogen continues to be significantly less than unity down to thermal energies (0.026 ev) so that the error is less than 20% when $\nu_m(u)$ for air is taken equal to that for the nitrogen alone.

The application of the above results to the analysis of the properties of the ionosphere is illustrated by the following re-evaluation of recent data obtained from rockets fired during a polar blackout.[1] First, one notes that because of the energy dependence of the electron collision frequency in nitrogen, relations which assume that the collision frequency is independent of energy should not be used to analyze measurements made in air. In order to facilitate calculations, the function Q used by Kane[1] as a measure of the ratio of the differential absorption to the refractive index is rewritten in terms of functions which properly average $\nu_m(u)$ over the electron velocity distribution.[5,10] Thus,

$$Q(\gamma, Y) = [\sigma_r(\gamma_0) - \sigma_r(\gamma_x)][\sigma_i(\gamma_x) + 1.4\sigma_i(\gamma_0)]^{-1},$$

where $\gamma = \nu_m(u = kT/e)/\omega$, $\gamma_x = \gamma/(1-Y)$, $\gamma_0 = \gamma/(1+Y)$, σ_r and σ_i are the real and imaginary parts of the conductivity of the electrons in the gas, $\nu_m(u = kT/e)$ is the collision frequency for monoenergetic electrons at the energy $u = kT/e$, and Y is the ratio of the angular gyrofrequency due to the earth's magnetic field to the angular frequency of the radio wave. Plots of Q vs γ for the value of Y appropriate to Kane's data are shown in Fig. 2 for the cases of collision fre-

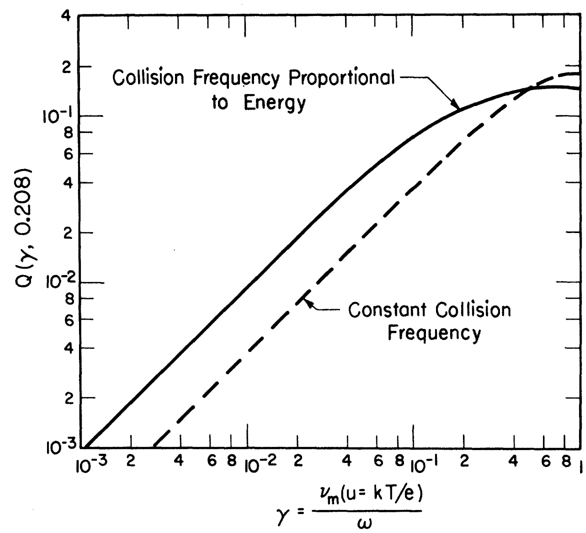

FIG. 2. The function $Q(\gamma, Y)$ vs γ for electron collision frequencies directly proportional to energy and independent of energy. The curves are calculated for $Y = 0.208$ which is the value appropriate to the earth's magnetic field at the site of the rocket measurements and to the frequency (7.75 megacycles/second) used. Values of Q for $\gamma > 1$ are not shown since, for $\gamma^2 \gg 1$, $Q(\gamma, Y)$ becomes extremely sensitive to the exact form of $\nu_m(u)$ at small u.

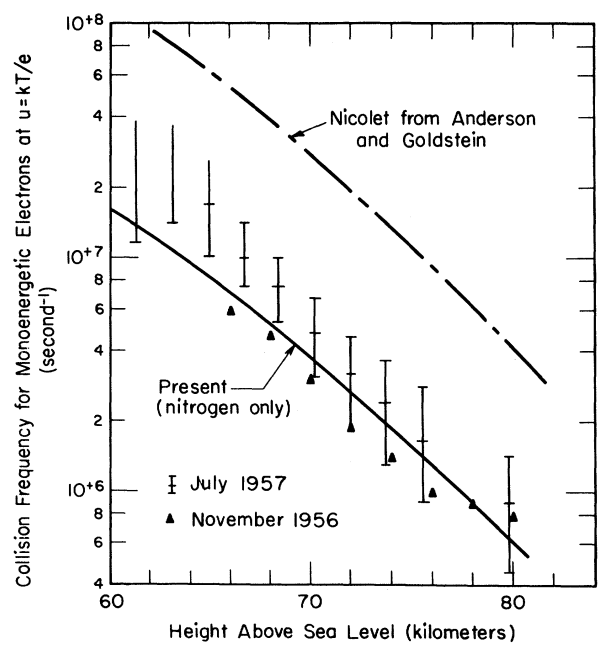

FIG. 3. Electron collision frequencies for monoenergetic electrons as a function of height above sea level. The error limits indicated are calculated from those given by Kane and would be approximately the same magnitude for the lower set of points. Note that since the mean values of Q computed from Kane's data for heights of 61.3 and 63.1 kilometers lie above the curve of $Q(\gamma)$ for $\nu_m(u) \approx u$, only the lower limits lead to real values of $\nu_m(u = kT/e)$.

quency directly proportional to energy and independent of energy. The values of $\nu_m(u=kT/e)$ inferred from a given observed value of Q for the two different energy dependences of $\nu_m(u)$ are in the ratio of 2.5:1 at very small values of ν_m/ω. Note, however, that since this ratio changes with ν_m/ω one cannot use the Q curve for constant ν_m shifted to the left by a factor of 2.5.

The points of Fig. 3 show the collision frequencies for monoenergetic electrons in the D-layer computed from Fig. 2 and from Kane's values of Q. The curves give the collision frequencies for monoenergetic electrons in nitrogen at the gas densities and temperatures given by Nicolet[2] using our $\nu_m(u)$ (lower curve) and using Nicolet's extrapolation of results of Anderson and Goldstein[2] (upper curve). The agreement between the lower curve and the points is within the scatter of the rocket data. This agreement can be considered as evidence for the correctness of the gas density and temperature data obtained from other rocket studies.[2]

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MOMENTS OF INERTIA OF EVEN-EVEN RARE EARTHS*

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Belyaev[1] has considered the effects of nuclear pairing interactions in analogy with the theory of superconductivity.[2-4] His treatment leads to the following formula[1] for the moment of inertia of an even-even nucleus:

$$\mathfrak{J} = \hbar^2 \sum_{kl} \frac{|\langle k|J_x|l\rangle|^2}{E_k + E_l}\left[1 - \frac{\Delta^2 + (\tilde{\epsilon}_k - \lambda)(\tilde{\epsilon}_l - \lambda)}{E_k E_l}\right], \quad (1)$$

where

$$E_k = [(\tilde{\epsilon}_k - \lambda)^2 + \Delta^2]^{1/2}, \quad (2)$$

and the sums, l and k, run over all single particle states of a self-consistent spheroidal well. The "chemical potential" λ is the solution of the equation

$$\sum_k \{1 - [(\tilde{\epsilon}_k - \lambda)^2 + \Delta^2]^{-1/2}[\tilde{\epsilon}_k - \lambda]\} = N. \quad (3)$$

N here is the number of neutrons or protons outside of closed shells. The quantity Δ is equal to one-half the induced energy gap.

Numerical computations based on Eq. (1) have been made for twenty-six rare earth nuclei which