where

$$\alpha = m^2 N_p / 2\pi n e^2 \hbar^2 K^3,$$

N_p being the number of pairs per cc, n the electron concentration, and K the Fermi wave number. The factors $B_\pm$ are the following integrals of the Fourier transform $J(q)$ of $J(\vec{r})$:

$$B_\pm = \int_0^{2K} |J(q)|^2 [1 \pm (\sin qR/qR)] q^3 dq. \tag{5}$$

The second term in the square brackets of (5) represents the interference of the scattered waves from the two atoms of a pair (averaged over random directions of $\vec{R}$). One should observe that the interference is constructive for elastic scattering, whereas it is destructive for inelastic scattering. It is precisely this feature which allows the elastic scattering to predominate over the inelastic. The remaining factors, $f_\pm$, of (3) and (4) contain the temperature dependence of the resistivities and are

$$f_+ = \sum_{I=0}^{2S} p_I (2I+1) I(I+1)/4, \tag{6}$$

$$f_- = \sum_{I=1}^{2S} p_I p_{I-1} I[(2S+1)^2 - I^2]/(p_I + p_{I-1}), \tag{7}$$

where p_I is the equilibrium probability that each state of energy E_I is occupied.

Consider now the temperature dependence of the resistivity contributions. For $W/kT < 1$, Eqs. (6) and (7) become

$$f_\pm \cong \tfrac{1}{2} S(S+1)[1 \pm S(S+1) W/3kT].$$

Therefore, the net temperature-dependent contribution to the resistivity is

$$\Delta\rho = \alpha S^2 (S+1)^2 (B_+ - B_-) W/6kT. \tag{8}$$

Since $B_+ > B_-$ [assuming only that $J(q)$ is well behaved], the resistivity will increase with decreasing temperature. One can show that this increase will be monotonic all the way to 0°K as long as

$$B_+/B_- > 2(4S+1)/(4S-1). \tag{9}$$

If the inequality (9) is not satisfied, the resistivity will go through a maximum before reaching its 0°K value. In either case the competition between (8) and the phonon resistivity should produce a minimum.

The magnitude of the anomalous resistivity increase also agrees with experiment for reasonable values of J and W. A detailed account of this work will be published elsewhere.

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ATTENUATION OF SOUND IN A GERMANIUM CRYSTAL AT ULTRA-HIGH FREQUENCIES AND LOW TEMPERATURES*

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We have measured the ultrasonic attenuation of compressional and shear waves in a high-purity crystal of germanium at frequencies up to 650 Mc/sec and at temperatures down to 1.5°K. At room temperature there is some evidence for the dislocation loss mechanism, but at low temperatures the attenuation is very small.

Previous measurements of ultrasonic attenuation at high frequencies in germanium have been at room temperature, and limited to 90 Mc/sec in the guided-wave method[1] and to 300 Mc/sec in the usual, unbounded medium, method.[2] We have added an ultra-high-frequency pulsed oscillator to our equipment to extend the frequency range to 700 Mc/sec. The low temperatures were achieved in a conventional liquid helium cryostat, the electrical energy passing down a bifilar line to the transducer, which was bonded to the parallel-sided specimen.

The specimen was of n-type germanium, with room temperature resistivity 45 ohm-cm and net donor concentration 1×10^{12}/cc. Its flat faces were oriented to be normal, within half a degree, to the [100] direction. They were both polished to a mirror finish and were parallel within 25×10^{-6} in./in.

The measured (or apparent) attenuation coefficient includes, in addition to the absorption in the specimen, a loss on reflection at the end faces[3,4] and a loss due to diffraction of the beam.[5] In the room temperature measurements we have applied a measured reflection correction[3] and a calculated diffraction correction[5] to both sets of results, giving the intrinsic attenuation curves of Fig. 1. For the compressional waves, the small thermoelastic loss[6] in the specimen has been subtracted, leaving only losses due to other intrinsic mechanisms.

The results for compressional waves are in agreement with those of Redwood at the lower frequencies and show an approximately square-law dependence on frequency—the measured slopes being 1.85 for the compressional waves and 1.90 for the shear waves. The dislocation-damping theory[7,2] predicts a square-law frequency dependence up to a frequency ω_m, above which the attenuation due to this mechanism should become nearly independent of frequency. The results at the highest frequencies seem to confirm the presence of a dislocation resonance, with ω_m about 500 Mc/sec, corresponding to a lower resonant frequency.

Most of this high-frequency attenuation disappears at liquid helium temperatures, however, as is shown in Fig. 2, for compressional waves at 340 and 500 Mc/sec and for shear waves at 333 Mc/sec. The apparent attenuation is independent of temperature below 15°K and in this region is so low that it may be largely due to the reflection losses, which we have not yet measured at low temperatures. The points shown above the dashed curve in Fig. 2 were observed on warming the sample, but not on cooling, and so may be due to changes in the bond. That a satisfactory high-frequency bond was achieved at helium temperatures is shown by the photograph (Fig. 3) of the compressional waves at 340 Mc/sec.

The rise in attenuation that begins at about 20°K is similar to that found in quartz by Bömmel and Dransfeld.[8] This rise suggests that the phonon-phonon scattering associated with Umklapp processes, first discovered in thermal conductivity measurements in dielectric crystals, may be present here. It is of interest to note that the phonon mean free path for Umklapp collisions in germanium, as estimated by the method of Berman _et al._,[9] is about 12 microns at 20°K, or

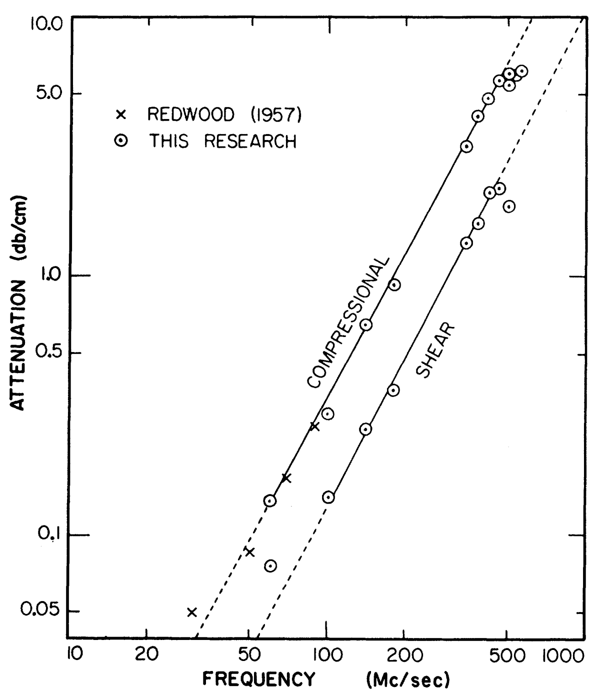

FIG. 1. Intrinsic attenuation of sound in high-purity germanium as a function of frequency at room temperature.

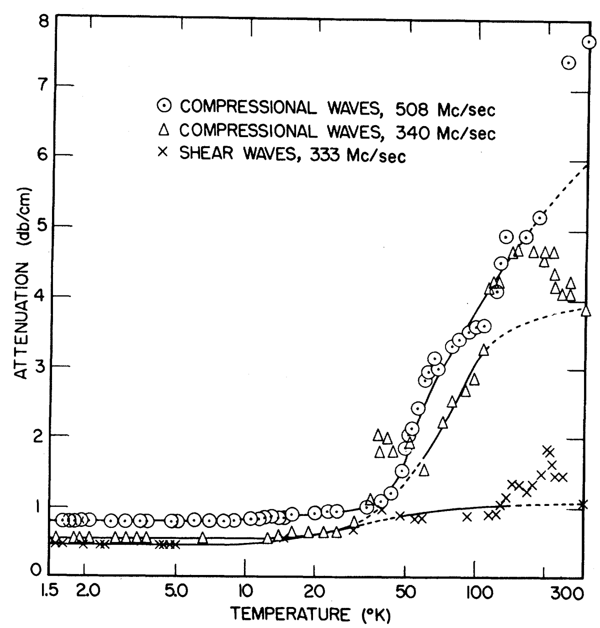

FIG. 2. Apparent attenuation of sound in high-purity germanium as a function of temperature. The dashed portions indicate regions where experimental effects, such as bond transformations, may be causing excess attenuation.

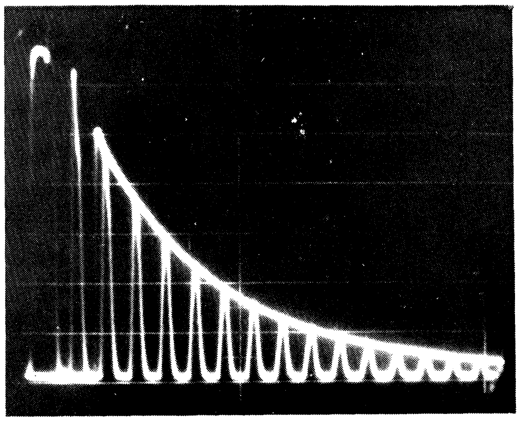

FIG. 3. Initial pulse and a series of echoes formed by 340-Mc/sec compressional waves in a germanium crystal at 2°K. The exponential curve is generated electronically and used in the measurement of the attenuation coefficient.

roughly the same as the acoustic wavelengths, which are 9.8, 14.5, and 10.4 microns for the waves shown in Fig. 2. Measurements at still higher frequencies are clearly possible at low temperatures and we are extending our results into the kilomegacycle region.

We are grateful to Dr. R. Logan of the Bell Telephone Laboratories for kindly supplying and analyzing the germanium crystal, and to N. Pitula and W. E. Oates for orientating and polishing it.

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HOT ELECTRONS AND CARRIER MULTIPLICATION IN SILICON AT LOW TEMPERATURE

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Under equilibrium conditions the mean energy of the electrons in a semiconductor is of the order of kT_0, where T_0 is the lattice temperature. In recent years considerable experimental work has shown that the electron energy can be increased substantially above kT_0 when an electric field is applied to the sample.[1] The electrons are heated while the temperature of the lattice remains constant. For very high electric fields the collision frequency between electrons and lattice increases and limits the electron drift velocity. These effects were first observed by Ryder and Shockley in n-type Ge at fields of 10^3 v/cm.[2] The increased carrier mobility at low temperatures reduces the electric field necessary for the observation of hot carriers. At temperatures where most of the carriers are frozen out on impurity levels, new effects are observed when the electrons become energetic enough to ionize donor impurities. Carrier multiplication by this process has been studied in Ge in the temperature range of liquid He.[3]

We have measured the electrical resistivity and the Hall coefficient as a function of the electric field in phosphorus-doped silicon ($E_i = 0.044$ ev) at 20°K. The samples (usual size $10 \times 2 \times 2$ mm^3) were provided with n^+ contacts. The carrier concentration, n, and the Hall mobility, μ, were calculated from the relations $n = 1/R_H e$ and $\mu = R_H/\rho$, where $\rho = E/j$. In Figs. 1 and 2 the current density j, n, and μ are plotted as a function of electric field for two samples containing less than 10^{16} oxygen atoms per cm^3. The room temperature resistivities and the donor and acceptor concentrations are listed on the figures.

To simplify the discussion we divide our field range into four parts:

(1) At low electric fields, $E \lesssim 5$ v/cm, the sam-

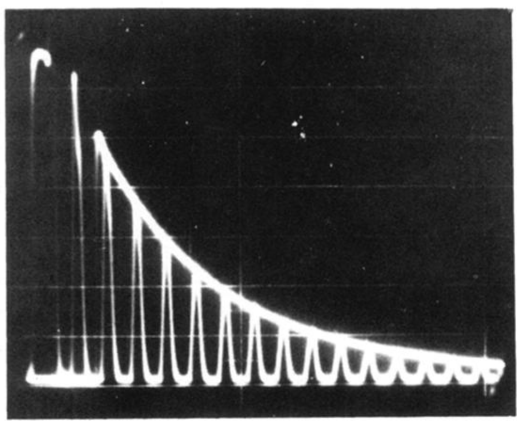

FIG. 3. Initial pulse and a series of echoes formed by 340-Mc/sec compressional waves in a germanium crystal at 2°K. The exponential curve is generated electronically and used in the measurement of the attenuation coefficient.